Chapter 4

Physical and Chemical Branching Structures

In the first chapter we noted that "No two coastlines are ever exactly the same; neither are two snowflakes or two lightning bolts. Though two such patterns may have the same overall shape, if you look closely you will see that they differ in the details of their structure. The same is true for other natural fractals as well. Each example is unique..."

Now we make another observation. If two fractals are *of the same class* (e.g., two bolts of lightning), they will be somehow *similar* in appearance to each other.

What is the basis of that similarity? How can we describe, classify, and measure different random fractal patterns in nature? That is the subject of this chapter and the activities described on the following pages.

Q4.1: In the color insert there are several pictures of fractal structures. How are these objects similar to each other in appearance? In what ways do they look different?

Q4.2: There are many ways to rank objects. For example, you can line up students according to weight. Or you can line them up by height, shoe size, age, or hair color. Look at the photographs and figure out different ways to line up these images in order. Line them up according to as many different criteria as you can think of. How inventive can you be?

Q4.3: The pictures show very different objects, yet their images look rather similar in many ways. Why do you think these objects look so much alike? What are possible ways that their similarity might arise from a common cause?

The following set of activities will help you to: (i) describe aggregates and random fractals; (ii) model the aggregation process by which the structures grow; and (iii) recognize and measure the basic properties of random fractals.

4.1 Growing Rough Patterns: Electrodeposition

Is a coastline a line? Not really, because a line is one-dimensional, whereas a coastline is a *random fractal* that has a dimension whose value is between 1 and 2. Other random fractal patterns also have dimensions between 1 and 2: a snowflake, a nerve cell, a lightning stroke. The growth of these structures can be modeled by a process called *aggregation*, in which random walkers dance around the growing structure and stick to it when they touch it. The resulting jagged pattern is called an *aggregate*. Depending on the details of the growth process, some aggregates are natural fractals, others are not.

The shiny chromium surfaces on an expensive automobile are made by immersing the parts into a chemical bath that contains chromium

ions and running an electric current through the bath so that chromium metal attaches to the parts to be coated. This process is called *electroplating* or *electrodeposition*.

The goal in chromium electroplating is to produce a smooth surface so the car will look good and be protected against rust. In our pattern-forming experiment, the result will be quite different from this, as you will see.

To understand the experiment, you need to recall that:

(a) In the electrochemical deposition (ECD) experiment, a copper ion, for example, gains two electrons at the negative terminal and precipitates out as a copper atom, so that the negative terminal grows.

(b) Avogadro's number is 6.02×10^{23} molecules = 1 mole of molecules.

(c) The concentration of a solution is measured in molarity (symbol M). The molarity refers to how many moles of solute there are in a liter of solution. For the purposes of these experiments, we will not take into account volume expansion when the solute is added to water. So, we will interpret 0.1 M Cu_2SO_4 (aq) as being 0.1 mole of Cu_2SO_4 in 1 liter of water. (The abbreviation "aq" means "in aqueous solution." The Latin word "aqua" means "water" and is the root of the word "aquarium.")

(d) A coulomb is a measure of electric charge equal to 6.24×10^{18} elementary charges (the negative charge on the electron and the positive charge on the proton). Current is the flow of charge; an ampere is a measure of current: 1 ampere = 1 coulomb/second.

(e) It will help your reasoning during the experiment to remember the relationship between voltage V and electric field E. If a voltage V is applied between two electrodes, an electric field is set up that points away from the positive electrode, and towards the negative electrode. A positive charge moves in the direction of the electric field. For the geometry of the cell used in this experiment (a vertical-wire negative terminal circled by a positive terminal) the electric field is strongest near the central wire.

(f) When thinking about the flow of ions in the electrodeposition cell, take into account the resistance of the salt solution. Recall that the resistance R in ohms of an object of cross-section A, length L, and resistivity ρ is: $R = \rho L/A$.

(g) The applied voltage V, resistance of the circuit R, and the current that flows I, are related by the equation $V = IR$. For example, if the applied voltage across your cell (i.e., between the positive terminal and negative terminal) is 10 volts, and the resistance of the cell 100 ohms, then the current that flows through the cell is 0.1 ampere or 100 milliamperes.

HandsOn 13: Growing a Pattern in the Laboratory

In this experiment, supplied in the accompanying laboratory kit, you have the opportunity to grow a physical object and measure its fractal dimension by two independent methods. The electrochemical deposition experiment is abbreviated ECD. The ECD experiment is carried out in an ECD cell consisting of two parallel plastic plates (Figure 4.1). Between these plates a circular positive terminal surrounds a central negative terminal. The space between the plates is the thickness of the positive terminal wire, about 1/2 mm or 500 micrometers. Between the plates, and between the electrodes, is a salt solution. Most likely you will be using a solution of copper sulfate (Cu_2SO_4) or zinc sulfate (Zn_2SO_4). This apparatus is an *electrolytic cell*, one that requires a current input (as opposed to an *electrochemical* cell such as a battery—a galvanic cell—which spontaneously *produces* a current). The circuit is shown in Figure 4.2.

Q4.4: *Speculate:* What will happen at the negative terminal? If you expect something to grow, sketch what you expect it to look like as it grows. Draw three stages of its development in your laboratory notebook: early, half-way through, and completed. Explain why you expect it to appear the way you have drawn it.

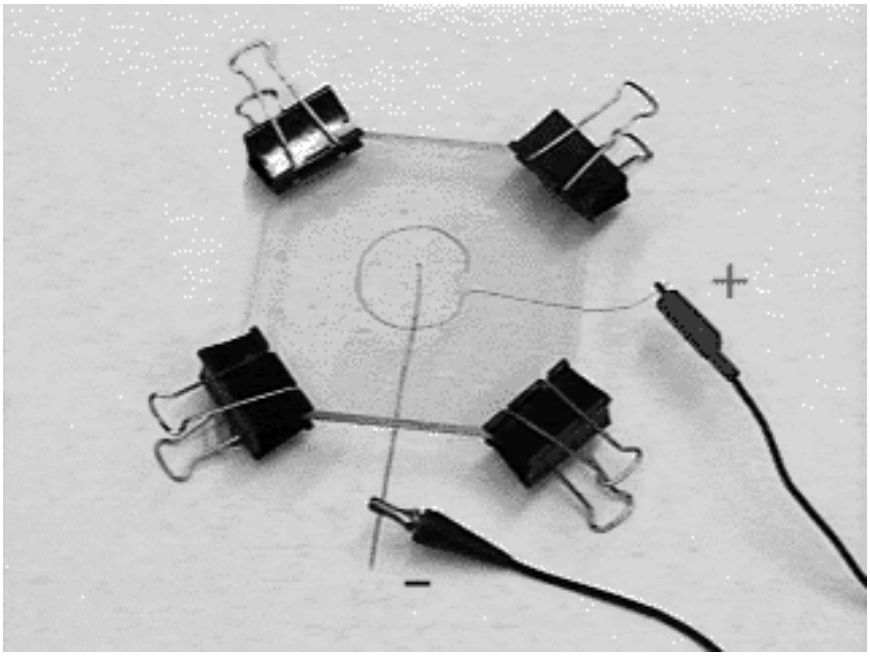

Figure 4.1: The electrodeposition cell. The circular loop of wire is the positive terminal, surrounding the central negative terminal. The space between the positive terminal and the negative terminal is filled with an electrolytic solution. The spacing between the plates is the same as the thickness of the positive terminal wire loop. The negative terminal wire extends vertically through both plates.

Q4.5: *Hypothesize:* What carries the current in the wires of the circuit? What carries the current through the cell? When the cell is hooked up, will current flow as long as the power supply stays turned on? Does the electrolytic solution develop a net charge?

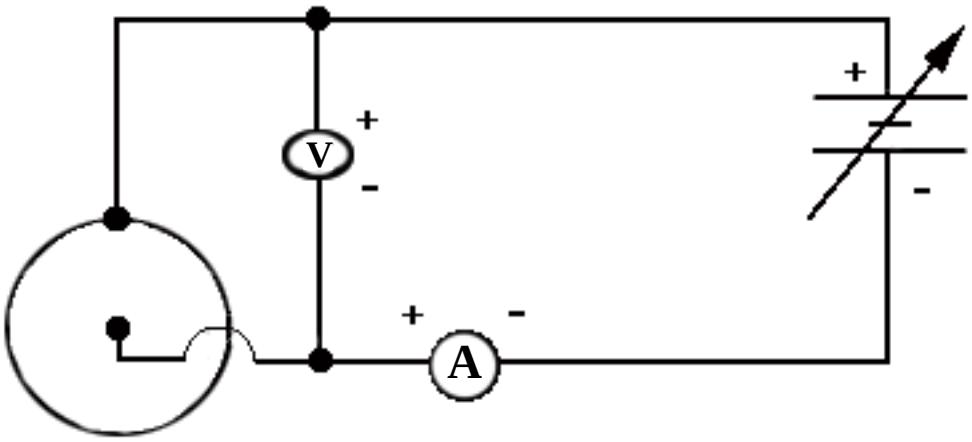

Figure 4.2: Circuit diagram for electrodeposition experiment. The electrodeposition cell is shown in the lower left-hand corner. The meters labeled A and V measure the current and voltage, respectively. An adjustable voltage power supply is represented by the battery symbol with an arrow across it.

Q4.6: How can you determine the *number of metal atoms* being deposited per second at the negative terminal? Data neeeded to answer this question can be obtained with the meters shown in the circuit of Figure 4.2.

Q4.7: Without using a balance, how can you determine experimentally the *mass* of the atoms deposited per second at the negative terminal? Is there a way to use the current meter to do this? Do you expect the mass deposited per second to vary with time?

If you are working in a classroom setting, you will need at least one partner, and preferably two partners, for this experiment. In addition

to the equipment provided in the kit, you will need a power source with variable voltage (e.g., between 1 and 20 Volts) and current (between 10 and 250 milliamps).

Setting Up the Experiment

1. Bend the the 1 mm wire into a right-angle about two centimeters from the end and insert it from below through the center hole in the bottom plate. From the center, this wire runs under the lower plate to the edge where it can be clipped to one lead from the power supply.

2. Place the lower plate on its rubber feet on a horizontal surface near the power supply, with the center wire sticking up.

3. Sandpaper the insulation from the thinner copper wire and bend it into a circle between 2 and 4 centimeters in diameter. Leave a small gap between the beginning and end of the circle, so the wire will not cross itself.

4. Lay the circular wire on the lower plate centered on the hole in the plate. Adjust the wire until it lies flat on the plate.

5. Pour the electrolytic liquid onto the center of the plate so that it spreads out to fill the wire circle. Be sure that there are no bubbles in the liquid.

6. Place the upper plate over the lower so that the vertical central wire passes through the holes in both plates. The upper plate should rest on the circular wire. There should be no bubbles in the space between the two wires (electrodes). Clamp each side with a 1/2 inch capacity binder clamp.

7. Now clip the power supply leads to the loose ends of the electrode wires. The negative terminal attaches to the center electrode.

8. Place a sheet of paper under the plates so you can see the pattern as it grows during the experiment.

Doing the Experiment

Divide the tasks among your partner(s). You will need to measure and record the following during the experiment on a copy of Table 4.1:

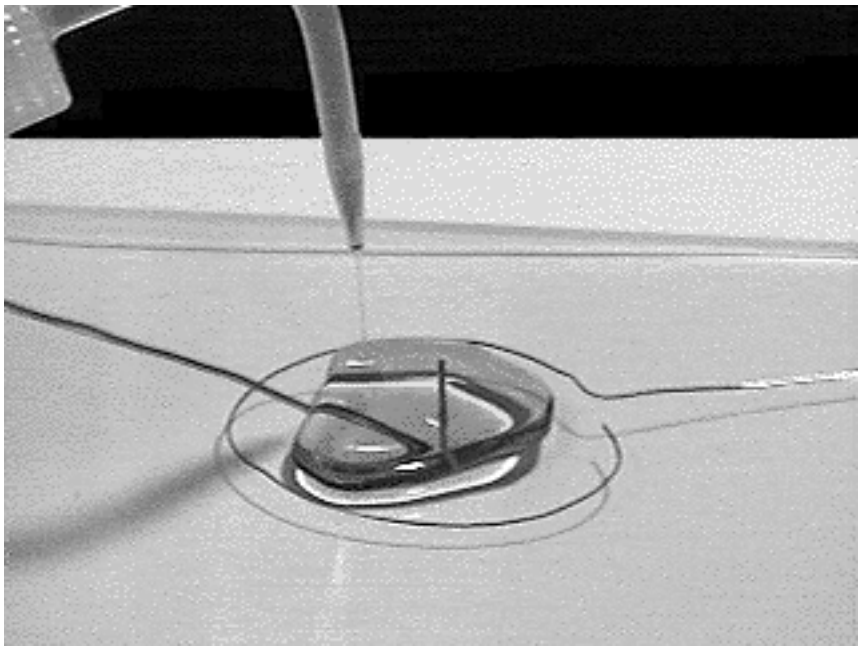

Figure 4.3: Filling the electrodeposition cell with electrolytic solution (as described in the section "Doing the Experiment."

1. The time at intervals of 20 seconds (longer or shorter depending on the way you assemble the cell; start with 20 seconds and after you have run the experiment once, reason whether you should use longer or shorter intervals).

2. The radius of the pattern at each recorded time.

3. The current at each recorded time.

With respect to the radius, the growth may not be symmetric, and some branches may stop growing during the experiment. The investigator recording the "radius" may choose to measure the overall radius of the entire pattern or the length of an "average" branch.

Table 4.1: Data Table for Electrodeposition experiment. Units are as follows: Δt and t in seconds; I in Amperes; ΔN and N_t in atoms. To calculate ΔN, use this algorithm: $\Delta N = I\Delta t/q$. For Cu^{2+} and ZN^{2+} ions, $q = 3.2 \times 10^{-19}$ Coulombs/ion.

Δt	t	I	r	ΔN	N_t
20	20				
20	40				
20	60				
20	80				
20	100				
20	120				
20	140				
20	160				
20	180				
20	200				
20	220				
20	240				
20	260				
20	280				
20	300				

A typical procedure might be that the timekeeper watches the clock and announces the time every 20 seconds. The second person reads the current on the ammeter, and the third measures the radius of the deposit. The timekeeper records these two numbers.

Hook up the circuit shown in Figure 4.2. Depending on the time available, you may want to try several experiments using different voltages. As a guide, 1 and 20 volts are probably the low and high values. Try a value between 10 and 15 volts to begin with.

Now it is time to grow your aggregate. The current will start at a low value, 10 milliamps or so, and rise to between 100 and 250 milliamps as the aggregate grows outward. Take care not to let the aggregate grow so large that it reaches the positive terminal. *What might happen if it does?*

Q4.8: Can you explain: asymmetries? branches, if your aggregate has them? size of the branches? color?

Digitize your electrodeposition aggregates by using a digital camera or a scanner. Since your aggregate is fragile, make sure you digitize the experiment immediately after you finish the experiment. You will need this digitized image for the section on measuring the fractal dimension.

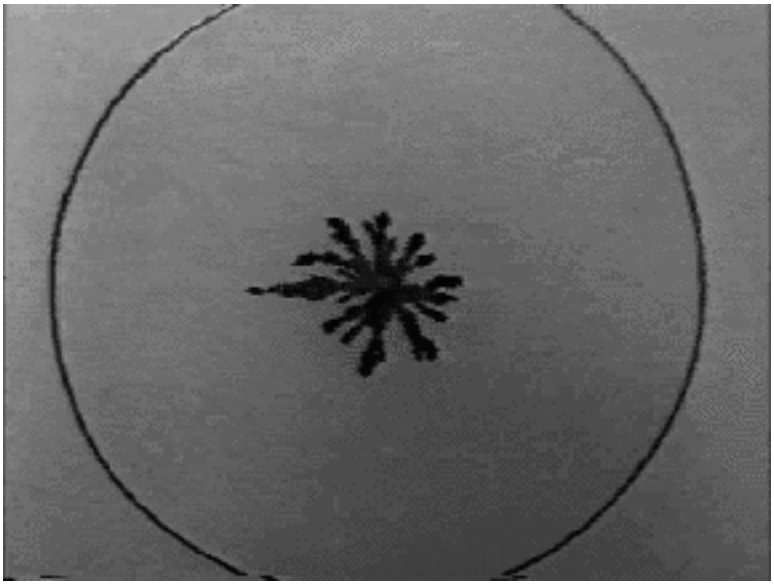

Figure 4.4: Sample copper electrodeposition aggregate in the early stages of growth.

Data Analysis

It is likely that the aggregate you grow does *not* appear to be a solid disk (most likely, they will look something like Figures 4.4 – 4.7). We want to measure the fractal dimension D of this aggregate. To estimate its dimension, we use the circle method, described in HandsOn 7. For the radius r of different circles, we substitute the approximate radius of the aggregate at different times during its growth. Instead of counting boxes inside a circle, we calculate the number N of copper atoms. If

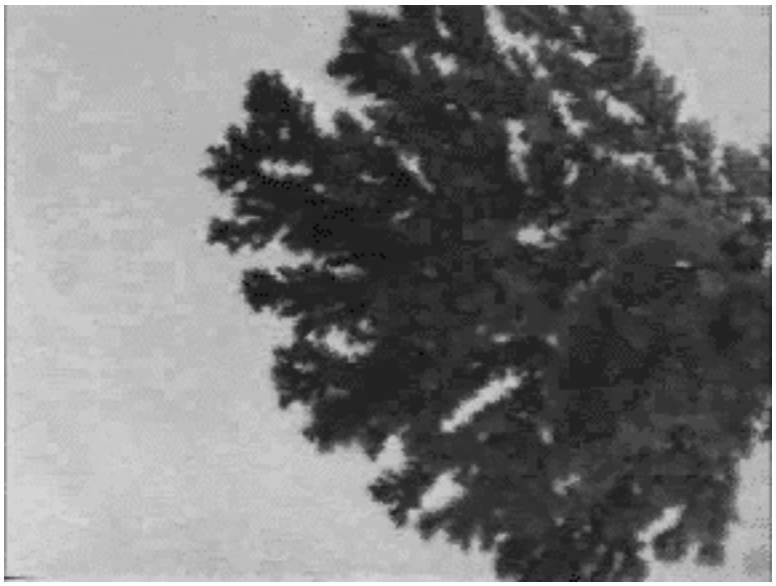

Figure 4.5: Zoom of the copper electrodepotion aggregate. Notice that the rough branches appear at both scales (e.g., at full scale and at the zoomed in scale).

the deposit is a fractal, we expect that the number of copper atoms N within a radius r to be

$$N = cr^D, \tag{4.1}$$

where c remains constant as the fractal grows and D is the dimension of the pattern. If we take the logarithm of both sides of this equation we get

$$\log N = \log(cr^D) = D \log r + \log c. \tag{4.2}$$

To make use of this equation to find the dimension D, you need to determine the radius r of the pattern at specific times during the growth and the total number N of copper atoms in the pattern at these times. You measured the radius directly several times during the experiment. But what about number N of copper atoms?

Before going further, discuss with your partners how you might measure the number of copper atoms that have been deposited at any time. Then read the following procedure:

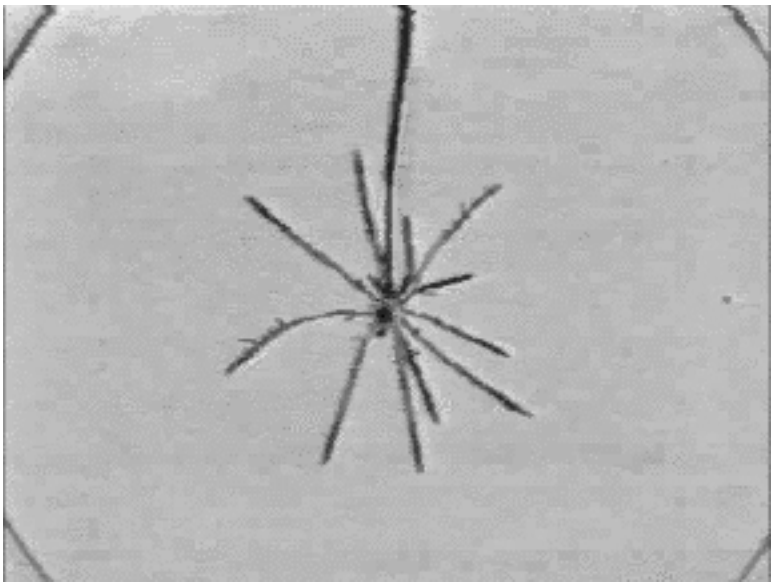

Figure 4.6: Sample zinc electrodeposition aggregate. Notice the differences in structure from the Copper experiment.

1. Use the following two steps to find the number ΔN of ions deposited in each time interval Δt (where Δ means "change"):

   (a) Find the charge deposited during that time interval. This is given by the expression $I\Delta t$, which has the units (coulomb/second) times second = coulomb.

   (b) Divide this charge deposited by the charge q on each copper atom (3.2×10^{-19} coulomb). The result is the number ΔN of ions deposited in the time interval Δt:

   $$\Delta N = \frac{I\Delta t}{q}.$$

   You probably noticed that the current varies over each time interval.

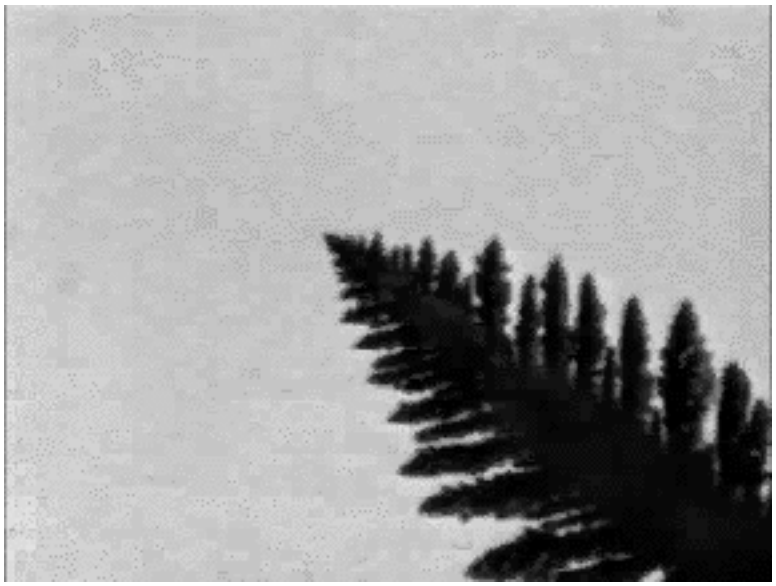

Figure 4.7: Zoom of the zinc electrodepositio aggregate. Do you notice any differences between the copper and zinc experiments?

Q4.9: Which is the "right" value of the current to use for a given interval? The value at the beginning? The value at the end? Would it make sense to use the average?

2. Since you can compute ΔN for each time interval, you can do a running sum, adding the increase Δt for the most recent time interval to the number summed from all previous time intervals. This gives the total number of ions deposited during any total time t. This is $N(t)$, the total number of ions deposited as a function of time. But you can also measure the radius r at time t. It follows that you know $N(r)$, the total number of ions deposited as a function of r.

   On log-log paper we plot the quantity N versus the quantity r. If you don't have log-log paper, you can use ordinary linear graph paper and press the logarithm button on your calculator to make

exactly this same plot. *If the resulting data appear to lie on a straight line*, the growth is *fractal* in nature, and the slope of the line D, is the fractal dimension, as shown in Eqn. 4.2.

3. Using log-log graph paper or a calculator and linear graph paper or a computer plotting program, plot $\log N$ versus $\log r$. If the data appear to lie on a straight line, measure the slope (the fractal dimension of the aggregate).

4. What is the total mass of the aggregate you grew?

5. In your experiment, the typical thickness of the aggregate is about 60 micrometers. The density of Cu metal is 8.92 gm/cm^3. What would be the radius of a solid copper disk with the same mass as your final deposit?

4.2 More on Fractal Dimension

How can one characterize these patterns we have created in the laboratory? Adjectives such as spiky, tenuous, ramified, or compact provide a qualitative nonscientific description of growth patterns. The observation that many of these growth patterns are *self-similar* (i.e., each piece of the pattern is geometrically similar to the whole) over many measurement scales leads to a clearer mathematical means of quantifying these patterns. Such self-similar objects are called *fractals*.

SimuLab 10: Measuring Fractal Dimension

We learned with the **Coastline** program (HandsOn 1 beginning on page 7) that a computer can quickly cover a pattern with boxes of various sizes, count the number of boxes, and plot the result in order to measure the dimension of that pattern. In this section we learn to use a new and even more powerful computer program called **Fractal Dimension** to analyze patterns grown in the laboratory, as well as patterns simulated by computer programs such as **Coastline**.

1. Open the **Fractal Dimension** program.

2. Select one of the images from the **Sample Images** menu.

3. Click on the red square at the left of the **Toolbar**. This covers the image with little boxes.

4. Click on the size control boxes at the top of the window to change the box size, then click again on the red square in the toolbar. The machine covers the coastline with boxes of the new size.

5. Change the box size and repeat the count with several box sizes.

6. Click on the graph icon, fifth from the left in the toolbar. This brings a log-log plot to the front. This plot gives the value of the slope, which is equal to the dimension D. If you wish to fill in the graph with more dots, go back and do additional counts with boxes of appropriate size.

7. Click on the table icon, sixth from the left in the toolbar. This opens a data table listing box lengths and counts. If you feel that one or more of the data points on the graph should be eliminated from the slope measurement, click on the number in the **# Boxes** column. This will "gray out" that row and the dot for that entry will disappear from the graph. (Restore the data point by clicking again on that item in the table.)

8. Analyze other images from the **Images** menu using the box method. In particular, you may want to check that a straight line (whether horizontal or diagonal) has a dimension of one. Is a solid square 2-dimensional, according to the **Fractal Dimension** program? You will also be interested in the Koch curve from the **Images** menu. Is the fractal dimension of the Koch curve determined by this program the same as your measurement in HandsOn 8 and the theoretical result of Eqn. 4.2?

   Remeasuring Coastline: Return to the **Images** menu and bring up the coastline image you saved earlier. Measure its dimension using the box method. Compare the result with the dimension measurement obtained using the **Coastline** program.

Measuring the Pattern from the Electro-Chemical Deposition Experiment: Now call up the image of the pattern you grew in the electrodeposition experiment and measure its dimension using the box method. How does this value compare with the value of the dimension you obtained using the current and radius values obtained during the experiment?

END ACTIVITY

4.3 Modeling Fractal Growth

How can we describe the process by which a pattern grows (aggregates)? Can we mimic the way a charged atom (ion) in a solution dances around, then deposits on the central electrode (becomes an uncharged metal atom)?

What does it mean for an ion "to dance?" Dancing means to stagger around randomly. A dancing ion is taking a random walk! We can use our understanding of random walks to mimic the process of electrochemical deposition.

Another word for *mimic* is *model* (see Section 3.8 on page 63 for a short discussion of models). We *model* the aggregation process using our knowledge of a random walker who staggers around and sticks to a growing structure. In the following activity you will use a 2-dimensional random walk to mimic (model) the aggregation process.

HandsOn 14: Building an Aggregate by Hand

Figure 4.8 shows a large random aggregation pattern. This pattern was grown using the random walk you are about to use. Your job is to start a random walker on the rim of the surrounding circle. Then let the walker stagger around the grid until it reaches the black pattern. When the walker reaches the black pattern, it sticks and becomes a black square. Then another walker starts from a *random* point on the rim of the surrounding circle and staggers around until it sticks to the pattern. And so forth. That's all there is to our model. This model is called *diffusion-limited aggregation*, or *DLA* for short and comes from the fact that the growth rate of the pattern depends on the rate at

which particles arrive at the surface by diffusion (the net motion due to their random walk).

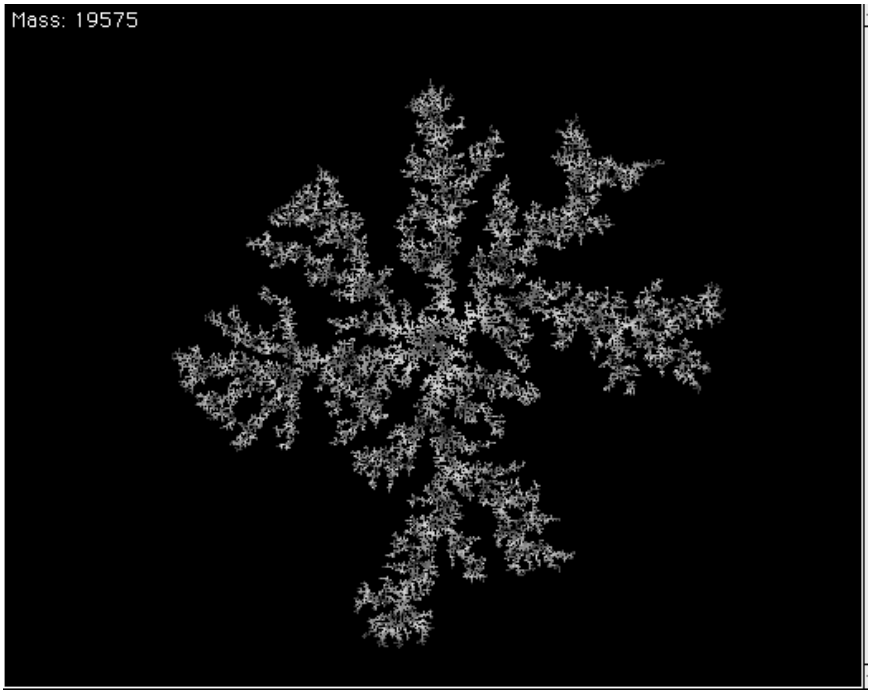

Figure 4.8: An aggregation resulting from a random-walk model. This pattern was built from a single "seed" (black square with white dot) by releasing 170 random walkers, one at a time, from the rim of the surrounding circle and allowing each to walk until it reaches and sticks to the growing pattern.

Q4.10: Do you think that the electrodeposition experiment can be described using this model? Discuss why or why not.

Carry out the following activity using simple linear graph paper.

1. Choose a small object to represent your walker: something small enough to fit within one square; something that will not roll.

2. Choose a starting place for the walker on the rim of the circle by spinning your pencil like a propeller as you drop it onto the figure. Place your walker wherever the tip of the pencil points on the rim of the circle.

3. Now roll a 4-sided die and move your walker one space right, left, up, or down. (You can also use a six-sided die; when you get a 5 or a 6, do not move the walker, just roll again. Some game stores sell tetrahedral, 4-sided, dice.)

   - If the die comes up with the number 1 $\rightarrow$ move one space right.
   - If the die comes up with the number 2 $\rightarrow$ move one space left.
   - If the die comes up with the number 3 $\rightarrow$ move one space up.
   - If the die comes up with the number 4 $\rightarrow$ move one space down.

   There is one exception to the rule above: If the next move takes the walker outside the circle, remove the walker and start a new one from another random location on the rim. This eliminates walkers that wander away from the pattern.

4. Repeat rolling the die and moving the walker until your walker lands next to a filled square of the pattern: right, left, above, or below the filled square (*not* a diagonal position). When this happens, fill in the square the walker is sitting on. That is the new addition to the aggregation.

   It may take quite a while to add one more square to the aggregation. You spend a lot of time rolling and reading the die. To shorten this time, we have programmed a computer to "roll the die" and print out a table, Figure 4.9. In this table L and R mean "left" and "right," respectively, while U and D mean "up" and "down." To read the table, start anywhere you want and read in a straight line horizontally or vertically, without repeating any line.

5. Spin the pencil again to point to a new starting point on the circle. This time one of you read out loud the letters from Figure 4.9 ("up," "right," "up," "down," etc.) while the other one moves the walker. How fast can you make your moves using the new method? Remember, whenever a walker moves outside the circle, remove it and start a new walker.

6. When the walker lands next to a black square of the aggregation (to the right, left, above, or below), fill in the square and start again.

7. Continue this process until you have added about 10 squares to the aggregation.

L U L U R U R R R U D D U D U U L R U L R R D L R
R D R R D L L D U D L U D U D U D R L L D D R U R
L R L D D R R D L R D D D D R U D U L R D R D U L
D U L L D R L R U U R R U D L U U D L L R R R D U
R L R D L R D U D D R U L U L D D R L R R U D D D
L R D L D R D U R L U D D D L L D R R L U U L U R U
U R U U R U U R L D R D U L U D U D U D D D U R D D
D R L L U U U L R L U U U L U R U R L L L U D L D
R L L R D R U R R U U U U U U D R L L U U U U U L D
R U D U L R R U U D L D D U L L L L R L R D L D L
R D L L D R U U L U U U L D L U D D L L D D R U L
D R R L L L D D R D R L D R R D R R R U D D D R L
U D L R U U L R D U D U R L D D D R D R U D U D R
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R D R R D U R D R U U R U L U U D D R L D D L U D
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L D D D U L D L D R U L U R D U L L D R D R L L U
R U D D D L L R R D L L L R U R L U L L U R U U L
U U U U D R U D L D D R D R R L U U L D R D L R L
D R L R U R R U L R L L L D U U L L R L U L R D R
U L U L L R U R L D L D U R U D R L D R D L L U R
R L D U L D L R D L U U D U U U U R D R L U L R D
L D D D D U R U L L L L D L L R R U L D R D L L L
R U U U U D R U D L U U D D D R R L D L U U L L L
U D D L L L R D R R U R R U D L U D D L U U D R U

Q4.11: Compare your results to those of others in the class. Are the new squares mostly added near the ends of the spidery legs or near the center of the existing pattern? Why should this be? Can you think of a simple explanation?

Q4.12: Compare this model to your picture of the motion of ions in the electrodeposition experiment. What component of the experiment does the walker represent? What component does the black pattern represent?

Q4.13: Who or what is doing the "die throwing" in the electrodeposition experiment? In what way do the ions in the electrodeposition experiment move differently than the walker in the aggregation model?

END ACTIVITY

SimuLab 11: Building an Aggregate by Computer

Aggregation Kit is available as a Java applet.

Can this process be made faster? Of course! Let the computer make random choices and move the walker. The computer moves the walker much faster than we can, but uses the same simple process.

Open the **Aggregation Kit** program. On the right is a black screen with a so-called "seed" molecule in the center.

1. The settings on the left regulate the process by which the seed will grow. For now, make these settings:
   - **Upper slider:** Speed. Set to a position near the left: 10 steps/sec.
   - **Cell size:** Set to a middle position.
   - **Particle movement buttons:** Select **Random**.

- **Fill gaps:** Slider to the left – Off

2. Press **Go** (upper left button).

   A random-walking "molecule" is released far away from the seed and staggers back and forth, up and down, until it finally touches and sticks to the seed. Immediately a second random walker is released, ultimately touching and sticking to the pair at the center. Then the third, fourth, fifth,..., hundredth walker is added to the growing pattern.[1]

3. Now try new settings: faster speed, smaller cell size.

4. Find settings for which the resulting pattern looks like the result of your electrodeposition experiment. When you grow such a pattern, choose **Save** under the **File** menu, using a name you can remember. You will need this saved pattern later.

Q4.14: Why is the DLA pattern so "leggy," so "spidery?" Why isn't it a compact blob? In your answer, think of the growth process in detail: the walker staggering back and forth as it passes inward between the ends of two legs.

END ACTIVITY

SimuLab 12: Changing Particle Movement

1. Start growing a new pattern by choosing **Clear** from the **Edit** menu, then choose **Split** from the same menu. This creates a new window showing a duplicate of the initial seed.

2. For this second pattern, click on the **Straight** button under **Particle Movement**. In the "straight" setting, each particle moves in a straight line but in a direction that is chosen randomly.

[1] In order to speed up the process, if the walker is too far from the pattern, the computer does not continue the random walk step by step. Instead, it places the particle at a point nearer the seed, as if it reached this nearer point by a random walk.

3. Click **Go** to start movement in this second window.

4. Now click on the first window and click **Go**. Click on one window then the other, changing the speed of each so that the two patterns grow at approximately the same rate. As each pattern grows, keep it all on screen by moving the **Cell Size** slider to the left.

5. Using the **Straight** setting, can you make a pattern that is similar to the result of your electrodeposition experiment? If so, save it as an image file for later analysis.

Q4.15: Compare the growths for the **Random** versus **Straight** settings: How is the movement of the particle different? Does the pattern grow in more steps or fewer steps? Is the pattern more leggy or more compact?

END ACTIVITY

SimuLab 13: Measuring the Computer-Generated Aggregation Pattern

Close the **Aggregation Kit** program and call up the **Fractal Dimension** program. Bring up the pattern you saved from the **Aggregation Kit** and measure its dimension using the box method. If you saved an image from the **Straight** setting of walker motion, call that up and measure its dimension also.

The **Fractal Dimension** program offers a second way to measure the dimension of an object, the so-called circle method of HandsOn 7 on page 22. Try out this second measurement method on your computer-generated pattern.

1. Click in the center of your pattern to set the center of the circles you are about to have the computer draw.

2. Click on the circle icon, second from the left in the toolbar. The computer counts the number of pixels inside the circle of that radius. (Pixels are the small dots that form the images on a computer screen.)

3. Now change the radius of the circle using the size tool at the top of the window. Click on the circle tool to count the pixels inside the new circle.

4. Select several more radii and have the computer count pixels for these circles also.

5. Click on the graph and data tools. You may want to choose additional values of the radius, so that points are more or less evenly spaced along the graph. Click on entries in the "# pixels" column of the table to discard points you do not want included in the graphical measurement of the slope.

6. You may want to use the circle method to re-measure the dimension of your saved coastline or the image saved from the electrodeposition experiment, comparing these measures of dimension to those obtained using the box method. To do this quickly, click on the fast circle icon, fourth from the left in the toolbar.

Q4.16: Do you get the same dimension using the circle method as using the box method?

Q4.17: Which settings of the **Aggregation Kit** program can produce a pattern with a fractal dimension near to that of the pattern from the electrodeposition experiment: the **Straight** setting or the **Random** setting for the way the walker moves?

Q4.18: There is always *some* difference between the dimension of the pattern grown in the electrodeposition experiment and the pattern grown with the **Aggregation Kit**. What does this difference mean? How large a difference is acceptable if the **Aggregation Kit** model of fractal growth is to be accepted?

END ACTIVITY

4.4 Viscous Fingering

How is pumping oil from the ground related to the development of ulcers? Is the path of a lightning bolt governed by the same laws of physics as the growth of a snowflake? And why can you understand how a lightning rod works by studying random walkers? or by studying the growth of a dust particle?

The experiment described in this section exhibits aspects of the fundamental physics of all the processes described above. This experiment, a study of "viscous fingering," was originally performed by H. S. Hele-Shaw, a naval architect, in 1898. The geometry of his apparatus was a little different from the one we use here. Nevertheless, we refer to the apparatus as the *Hele-Shaw cell* after his original design.

Consider this story. Let's say you have a large number of balloons to inflate. What kinds of balloons are hardest to inflate? Are long balloons harder or easier to inflate than round balloons? And when is any balloon hardest to blow up? Is it harder to blow air into a balloon when it is deflated? Or when it is close to full inflation?

What is the force that resists the air coming into the balloon as it is inflated? After the balloon is inflated, is the air under greater pressure inside the balloon or outside the balloon? If you think it is greater inside, what counter-force keeps the skin of the balloon from rupturing? And how is this problem related to stretching a rubber band?

Surface tension is a very common force which shapes much of nature around us. In effect, it is a force that resists the creation of surface area. Thus, when you fill a glass to its lip, and keep filling it some more, the curved surface (meniscus) that allows you to fill the glass beyond its top is maintained by surface tension. When it suddenly breaks, the flowing liquid has greater surface area. Surface tension keeps a balloon round instead of growing spikes.

Q4.19: Can you describe what might create surface tension at a molecular level? How do you envision the molecules at a liquid surface behaving? What about in the case of the balloon? Write a short paragraph comparing and contrasting a liquid surface with the surface of a balloon.

As another example, think of a straw with water flowing through it. Does the water flow more quickly near the walls of the straw? Or near the center? Or is the speed the same throughout?

It has been observed that when a fluid flows next to a surface, the speed of the fluid at the surface itself is zero. But at the center of the stream the fluid is moving! How can this be? What happens when you move one surface over another? Is it easy? What will stop a metal block from sliding over a metal surface? If you think it is friction, how do you imagine friction operating at a molecular level? Is heat generated? If so, where does that heat go? Can you apply your analysis to the fluid flowing in a straw, or over the surface of a plane's wing? What is happening at the molecular level that causes adjacent layers of fluid to move at different speeds?

The frictional property of fluids is called *viscosity*. The greater the viscosity of a fluid, the greater the force necessary to maintain fluid flow through a straw, or over a surface. Suppose two fluids have different viscosities; what differences in the molecules of the two fluids could give rise to the difference in viscosity?

Viscosity is a measure of the resistance of a liquid to flow. At room temperature, honey and molasses do not flow easily: they have high viscosity. Motor oil flows more easily than honey; it has a lower viscosity. Water has a still lower viscosity. Air flows so easily that one might be tempted to say that it has zero viscosity. But it does resist flow a little, as you can prove by trying to breathe through a drinking straw. The viscosity of air is approximately 50 times less than the viscosity of water.

Spread some wax paper or aluminum foil on the table. Pour a small puddle of honey in the middle. Place a plate with a flat bottom on the puddle of honey. Now drag the plate sideways along the surface

of the table. It takes a force to keep the dish moving. The viscosity of the honey is related to the force needed to drag the dish over the honey-puddle.

You will want to look up the viscosities of fluids you use in the Hele-Shaw experiments described in the following section. So you need to know about the unit in which viscosity is measured. Viscosity is measured in *poise*, whose plural is also poise. The viscosity of water is easy to remember; it is one centipoise, that is, one hundredth of a poise. Glycerol has a viscosity 1200 to 1400 times greater than water, or 12 to 14 poise. Air has a viscosity some 50 times smaller than that of water, or approximately 200 micropoise.

HandsOn 15: The Hele-Shaw Experiment with Glycerin

The viscosity of fluids plays an important part in the Hele-Shaw experiment, supplied in the accompanying laboratory kit. Figure 4.10 shows a Hele-Shaw cell with its two plates of plastic separated by cover slips (little squares of glass or plastic usually used to cover microscope slides). In each corner there are two cover slips, one on top of the other. Typically glass cover slips are between 130 and 160 micrometers in thickness. Plastic cover slips are roughly the same thickness. So the spacing between the plates is roughly 300 micrometers.

In the experiment you will first inject glycerol through the central hole to fill the cell. After the cell is filled with glycerol, you will inject colored water as shown in Figure 4.11.

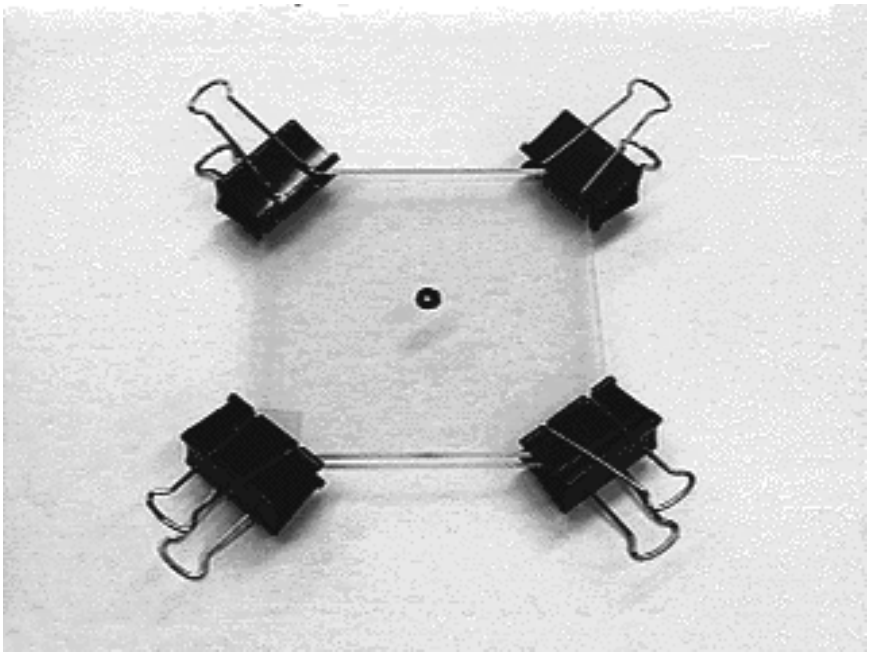

Figure 4.10: The Hele-Shaw cell. Fluid is injected through the center hole. The space between the plates is fixed by the thickness of two cover slips, one on top of the other placed in each corner of the cell.

Q4.20: *Speculate:* You will inject glycerol into the empty cell. As the glycerol spreads, what will be the *shape* of the boundary between glycerol and air? We call this boundary between two fluids the **interface**. Sketch the shape you predict for this interface, and provide a written argument to support your sketch. Was your prediction influenced by the viscosity difference between the glycerol and air? The symmetry of the cell? The narrowness of the cell spacing? The surface tension between the glycerol and air? Prior experience?

Figure 4.11: Hele-Shaw experiments showing the less viscous colored water injected into the more viscous glycerin.

Q4.21: The cell is now filled with glycerol. With another syringe, inject water through the central opening. The water should have food coloring in it so that you can distinguish it from the glycerol and trace the interface between the water and glycerol. As the water advances, what will be the shape of the interface between the water and the glycerol? Sketch your predicted shape and write a brief argument supporting your sketch. Does your argument depend on the difference in viscosity between the water and glycerol? The symmetry of the cell? The narrowness of the cell spacing? The surface tension between the water and glycerol? Prior experience?

Using the apparatus supplied in your laboratory kit, you can now perform the experiments suggested above. For data collection it will be easiest to work with one or two partners and a copy of Table 4.2. If possible, use a video camera (or snap digital pictures) to record the experiment as it progresses.

1. Assemble the cell as indicated in the instructions. Fill a syringe with approximately 20 cc of glycerol. Insert the syringe into the central opening, and slowly inject the glycerol. If you are patient,

Table 4.2: Data Table which should be used for both Hele-Shaw cell experiments.

ΔV	V_T	r
0.2ml	0.2ml	
0.2	0.4	
0.2	0.6	
0.2	0.8	
0.2	1.0	

you can let the glycerol flow by itself into the cell. Otherwise, you can press the plunger of the syringe slowly downward.

As the glycerol enters the cell, study its interface carefully. (The *interface* is the boundary between the two fluids.) If bumps develop on the interface, watch what happens to them. By a bump we mean a deviation from a symmetrical, smooth interface. Does a small bump grow? Or does it recede with time and the interface "heal" itself? What might give rise to the development of these bumps? What determines their lifetime?

2. The central opening on the plate should be filled with glycerol to the top in order to avoid introducing air bubbles. Fill the 1 cc syringe with water containing food color. Carefully attach the syringe to the nozzle. Inject roughly 0.2 cc of the water (at a rate of, say, 0.1 cc per second), watching the interface carefully. Measure and record the radius of the interface. If it is not symmetrical, approximate the average radius. Adding 0.2 cc at a time, repeat the measurement process until all the water has been injected.

3. Scan the interface with a flat bed scanner, or "grab" the image of the interface using a digital camera connected to your computer.

Q4.22: As the glycerol approaches the edge of the cell what is the shape of the interface?

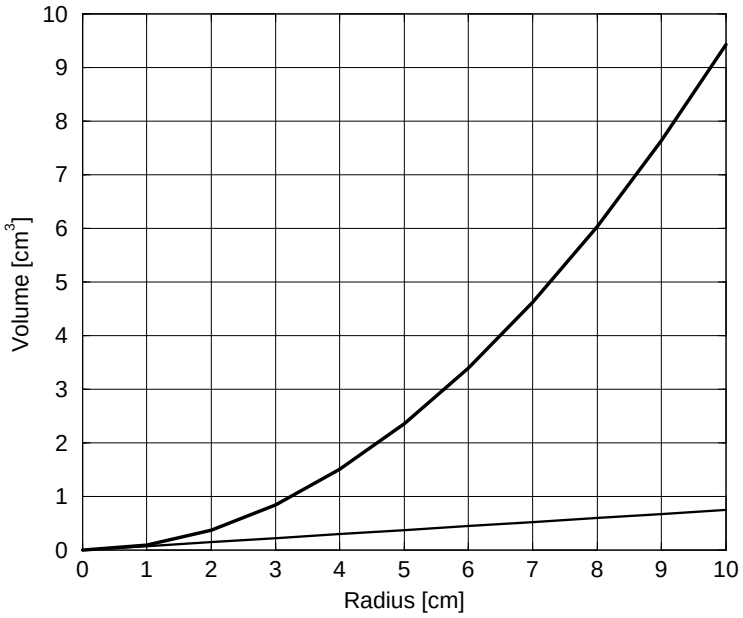

Figure 4.12: Plot your experimental measurements for the Hele-Shaw experiments on this graph. The two curves shown represent growing patterns of dimension 1 (the lower curve) and dimension 2 (the upper curve). Can you explain how these curves were drawn? Hint: The distance between plates is approximately 3mm.

Q4.23: Plot your radius versus volume measurements on the graph shown in Figure 4.12. What can you say about the "dimension" of the pattern you created?

END ACTIVITY

HandsOn 16: The Hele-Shaw Experiment with Carrageenan

The purpose of this experiment is to observe the viscous fingers that form when fluids of different viscosity interact with each other, *and*

when there is a chemical reaction due to differences in the pH of the two fluids. Specifically we will be working with carrageenan of varying concentrations and pH. Carrageenan is a polymer which in aqueous solution gels at high pH (basic conditions), but does not gel at low pH (acidic conditions). Carrageenan is added to ice cream, whipping cream, and some other foods because of its gelling properties.

The viscosity of aqueous carrageenan solutions is strongly dependent on both concentration and pH. In the previous experiments in the Hele-Shaw cell you worked with glycerol. The viscosity of glycerol is a simple function of its concentration. By contrast, the viscosity of carrageenan does not depend simply on its concentration, because it has a complicated inner structure.

pH is an important factor in carrageenan solutions. Long-chain molecules such as carrageenan can interact through electrostatic forces. The pH of the solution strongly influences this interaction. The pH reflects the ions present in the solution that mediate this interaction.

The experiments with carrageenan allow you to plan and carry out your own research project. You will repeat STEPS 1 AND 2 of the Hele-Shaw experiment above, using carageenan instead of glycerol and varying the pH of both the carageenan and the injected water. There are three different variables:

1. Concentrations of carageenan. We recommend starting with the values 22.5 mg/ml and 25 mg/ml.

2. pH of the carageenan. We recommend starting with values of 3 and 5.

3. pH of the injected water. We recommend starting with values of 3 and 5.

Don't forget to clean and dry the Hele-Shaw cell thoroughly between experiments with different solutions.

Preparing Carrageenan

Carrageenan is available from chemical supply companies (e.g., Sigma). Here, we describe how to prepare carrageenan solutions of various concentrations and pH. You will need:

- 2.5 g Carrageenan
- 100 ml Distilled Water
- 2 ml Hydrochloric Acid (HCl)
- Two 250 ml glass beakers
- pipette or dropper
- pH paper for the range pH 3 thru pH 5, or pH meter (see below)
- parafilm

1. Fixing the concentration of carrageenan solution. Carrageenan comes as a powder, and is slow to dissolve in water. Be prepared to let the solution stand for at least 24 hours to permit the powder to dissolve. The resulting solution is also an excellent medium for the growth of bacteria and mold, and will become contaminated. Hence, it should be used within a few days of preparation. To begin try a concentration of 25 mg/ml. Add 2.5 g of carrageenan to a quantity of water less than 100 ml of distilled water. Then add enough water to complete a volume of 100 ml. Cover the solution (e.g., with parafilm) and leave it overnight for the powder to dissolve. After the carrageenan has dissolved, mix the solution for a few seconds to make it homogeneous. Other concentrations may be made following these same guidelines but varying the amount of carrageenan per 100 ml distilled water.

2. Fixing the pH of the carrageenan solution. We recommend practicing the following procedure on a small quantity of your solution first. Initially the carrageenan solution will have a pH of between 8 and 9. Prepare a dilute solution of HCl acid (e.g., 2 ml of HCl (36.5–38%) in 100 ml of distilled water). Add the diluted HCl dropwise to your carrageenan solution, and mix the solution. Measure the pH of your solution using a pH meter (directions below) or with pH paper. By repeating this process, prepare carrageenan solutions of pH 3 and pH 5.

3. To adjust the pH of "pure" water, add 1 ml of HCl to 100 ml of distilled water then take its pH. Adjust the pH by adding distilled water.

Once you have the desired pH, add food coloring to the HCl solution. This will provide contrast between the HCl solution and the carrageenan when performing the Hele-Shaw experiment.

Observational Analysis

The reason that fluids flow is because of pressure differences across the fluid. For example, when you suck on a straw the fluid flows up the straw because atmospheric pressure at the bottom of the straw is greater than the pressure in your mouth. Similarly, when you apply pressure with the syringe at the central opening of the Hele-Shaw plates, fluid flows into the cell because the pressure at the center is greater than the atmospheric pressure surrounding the open edges of the cell.

To understand what follows it is useful to know that usually the speed of fluid flow in a pipe, or between two plates, is proportional to the pressure difference across the fluid, and inversely proportional to the distance over which that pressure difference is maintained. For example, if you suck harder on a straw, further reducing the pressure in your mouth, this increases the pressure difference across the straw (since atmospheric pressure remains constant), and the fluid flows faster. Similarly, if you use a shorter straw, the same pressure difference exists across a shorter length and the fluid flows faster. In the case of the Hele-Shaw cell, if your plates are smaller in diameter, then for the same applied pressure on the syringe, the fluid between the plates will flow faster.

Q4.24: Do you see an analogy between the applied *pressure* in the Hele-Shaw experiment and the applied *voltage* in the electrochemical deposition experiment?

For a given applied pressure difference, the speed of fluid flow between the plates of the Hele-Shaw cell is proportional to the square of

the spacing between the plates. Thus, if you double the spacing between the plates, you quadruple the flow rate if all other conditions are kept constant.

1. When you inject the glycerol or carrageenan into the air-filled cell, watch the interface for bumps. When a more viscous fluid invades a less viscous one (e.g., the glycerol or carrageenan invading air), the interface is "stable" in the sense that any bumps which form die away. The interface "heals" and returns to being smooth and symmetric. If you videotaped your experiment, or recorded it into a computer, play it back and watch this phenomenon again. Alternatively, repeat the experiment if you did not observe this phenomenon the first time.

2. By contrast, when you inject the water into the glycerol, or the acid into the carrageenan, the interface does not remain symmetric but breaks up into viscous *fingers*, the name given to the mitten-like protrusions that result when bumps on an interface grow. We say that the interface is "unstable" when a less viscous fluid is injected into a more viscous one.

Q4.25: Repeat the experiment, or study your video or pictures of it, and try to determine when the bumps start growing. Is there an initial symmetric interface at the outset? Or do viscous fingers originate directly beneath the nozzle?

Q4.26: Either by repeating the experiment or referring to your film, study the growth of individual fingers as they advance. Does a finger retain its shape and grow like an inflated long narrow balloon? Or do fingers break up and form multiple new fingers? Do all fingers grow, or do some stop advancing and become static? Where are the ones located that become static? Where do advancing fingers split?

The growth of the branching tree structure of viscous fingers is a primary example of how branching structures develop. A bump appears at the interface. The bump grows faster than adjacent areas of the interface and develops into a finger. The finger itself then splits and forms multiple branches growing from new bumps.

END ACTIVITY

SimuLab 14: The Fractal Dimension of Hele-Shaw Patterns

As already discussed in Section 2.2 beginning on page 22, a line has length $L = L^1$. A square of side L has an area equal to L^2. A cube of side L has volume L^3. In each case the exponent of L reveals the dimension D of the object. A fractal can have a fractional value of D.

1. During your experiment, for each volume V of fluid injected you obtained the radius R of the pattern and we plotted the results on the linear graph of Figure 4.12. If the fingering pattern is a fractal then we expect the relationship

$$V = cR^D, \tag{4.3}$$

where c is a constant of proportionality and D is the fractal dimension. If we take the logarithm of both sides of (4.3) we obtain

$$\log V = D \log R + \log c. \tag{4.4}$$

It follows that if we plot $\log V$ versus $\log R$ and obtain a straight line, then the slope of this line is D, the fractal dimension, and the intercept is $\log c$.

Using either a computer graphing program or log-log graph paper, plot $\log V$ versus $\log R$ and determine D from the slope.

2. You should also determine the fractal dimension of the final pattern by capturing the image using a digital camera, scanner, or a video camera if the computer has video capabilities. After the image has been converted into a digital image, use the **Fractal Dimension** program to determine the value of the dimension D.

END ACTIVITY

4.5 Why Do Viscous Fingers Branch? (Advanced)

Why do bumps grow faster than the rest of the interface? The answer to this question is fundamental to understanding why the Hele-Shaw experiment gives rise to a tree-like pattern when a less viscous fluid is injected into a more viscous one. Recall from our brief discussion above that the flow velocity is inversely proportional to the distance over which the pressure difference occurs. (You drink more water when you suck just as hard on a shorter straw.)

Consider a bump on a circular interface as in Figure 4.13. The pressure is the nearly the same at the bump interface as on the circular portion of the interface. On the other hand, the top of the bump is closer to the edge of the cell (R') than the rest of the interface (R). The same pressure difference over a shorter difference means greater flow velocity. As a result, the bump expands faster than the rest of the circular interface. *This is why the interface is unstable, bumps grow, and viscous fingers develop.* In a self-similar fashion, subsequent bumps grow on the fingers themselves causing the fingers to branch. And so on: a tree-like structure develops.

Q4.27: Can you make an analogy between this process of branching by viscous fingers and the branching process for electrochemical deposits? What corresponds to the pressure in the case of electrochemical deposition?

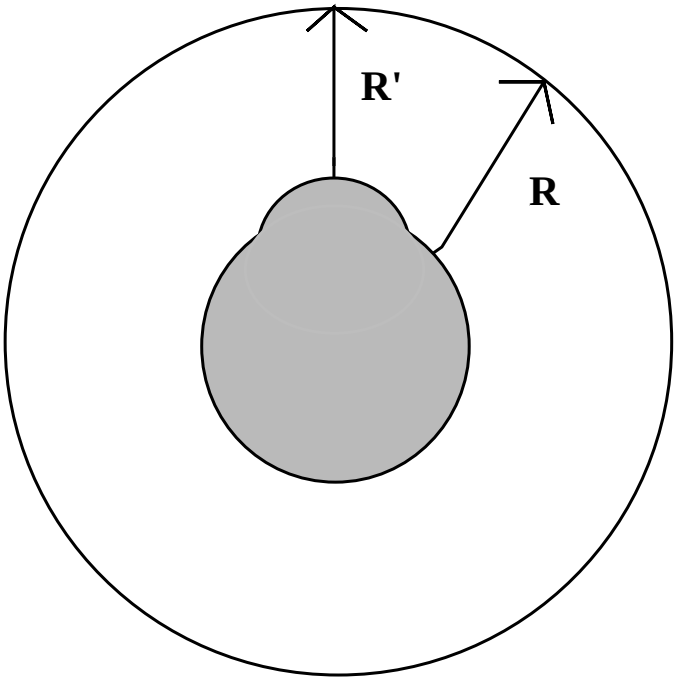

Figure 4.13: Bump on circular interface of injected fluid. R', the distance from the bump to the edge of the cell, is less than R, the distance from the circular interface to the edge of the cell. Because the velocity depends inversely on this distance, the bump grows faster.

Q4.28: What determines the thickness of the viscous fingers? Why are the viscous fingers in the carrageenan experiment narrower than those in the glycerol experiment? Predicting the minimum thickness of a viscous finger is still a matter for current research. A key factor is the difference in surface tension between the two fluids. Are your results consistent with your understanding of surface tension? Write a short argument saying why viscous fingers in glycerol should be thinner or thicker when you inject air rather than water, and explain your reasons. Try this experiment.

4.6 What Do You Think?

Q4.28: There are many naturally-occuring fractals. Read the following list of objects and give reasons for why each is (or is not) a fractal. SNOWFLAKE | PINE CONE | SOCCER BALL | LIGHTNING STROKE | LEAF | VEINS IN A LEAF | HUMAN LUNG SYSTEM | HUMAN SPINAL COLUMN | HUMAN CIRCULATORY SYSTEM | BLOCK OF WOOD

Q4.28: You have just discovered how a random walker can be used to grow spidery patterns similar to those found in nature. Does this mean that random processes are actually involved in forming these objects in nature? Or are lightning, nerve cells, termite tunnels, and electrodeposition patterns formed by totally different processes, processes that have nothing to do with random walkers?

Q4.28: The discovery that a random walker can be used to grow spidery patterns similar to structures found in nature is very important to scientists. Why do you think this discovery is important? In what ways could this discovery prove to be useful?

Q4.28: Write a brief statement predicting the value of the fractal dimension of the patterns that result under each of the following conditions and giving the reasons for your prediction:

(a) inject air into glycerol

(b) inject water at higher pressure into glycerol

(c) inject a 50-50 mix of water and glycerol into pure glycerol

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Research Projects

Try the suggestions below, design your own, or write an essay using any of the questions throughout this chapter as inspiration.

Pattern Formation

Now you have the tools needed to carry out your own investigations of electrodeposition and Hele-Shaw experiments and to investigate for yourself the computer programs. We suggest that you work with a small group and go through the following steps.

1 Think of a variation of the experiment or settings of the computer program (or, best of all, a comparison between the two).

2. Think about the results you expect to observe with your proposed experiment. Draw the expected pattern.

3. Submit a written proposal to your teacher, describing the experiment and the expected result. Your teacher will check if it is promising, the availability of needed components, and will evaluate its safety.

4. After approval, carry out the experiment and compare results with your predictions. Does your outcome warrant further investigations? Where will they lead?

Electrochemical Deposition

Here are some possible variations of the electrodeposition experiment:

(a) Variations of the solution concentration.

(b) Changing the salt in solution. Try using CuCl or ZnCl or $AgNO_3$ or tin salts.

(c) Varying the position of the negative terminal relative to the positive terminal.

(d) Using more than one negative terminal.

(e) Making uneven the spacing between the cell plates. This can be done by inserting a stack of cover slips, or equivalent, in one corner of the cell.

(f) Run at constant current rather than constant voltage. Can you estimate what range of currents will lead to a reasonable growth rate? See if your estimated current results in a change in aggregate shape or rate of growth.

Viscous Fingering

Here are some possible variations of the Hele-Shaw experiment that you can try:

(a) Inject air into as 50-50 mixture of air and glycerol.

(b) Inject oil into glycerol.

(c) Inject very concentrated sugar water into pure water, and vice versa. Do you observe anything in this case?

(d) Try placing a grooved plate between the plates of the Hele-Shaw cell and then performing the viscous fingering experiment. Can you predict what will happen?

(e) Try the Greased Lightning experiment. On a plate of plastic 3 or 4 cm on an edge, put a spot of lithium grease (a lubricant available at hardware and automobile supply stores). The spot should be about 1/2 cm in diameter. Place a second plastic plate over the first and squeeze the plates just above the spot as hard as you can. The grease will spread thinly between the plates. Can you predict the shape of the grease as it spreads? Will it finger? Now release the pressure on the grease slowly, and then gradually pull the plates apart. Can you explain the remaining pattern?

Repeat this experiment with a drop of model paint. Or with thick chocolate frosting. Or with toothpaste.

Other Research

Assemble a collection of images that may be fractals: river deltas, leaf veins, lightning strokes, nerve cells, root systems, and so forth. Estimate the fractal dimension of each one. If possible, scan the image into the computer and use the **Fractal Dimension** program to measure its dimension. Compare the result with your estimate.