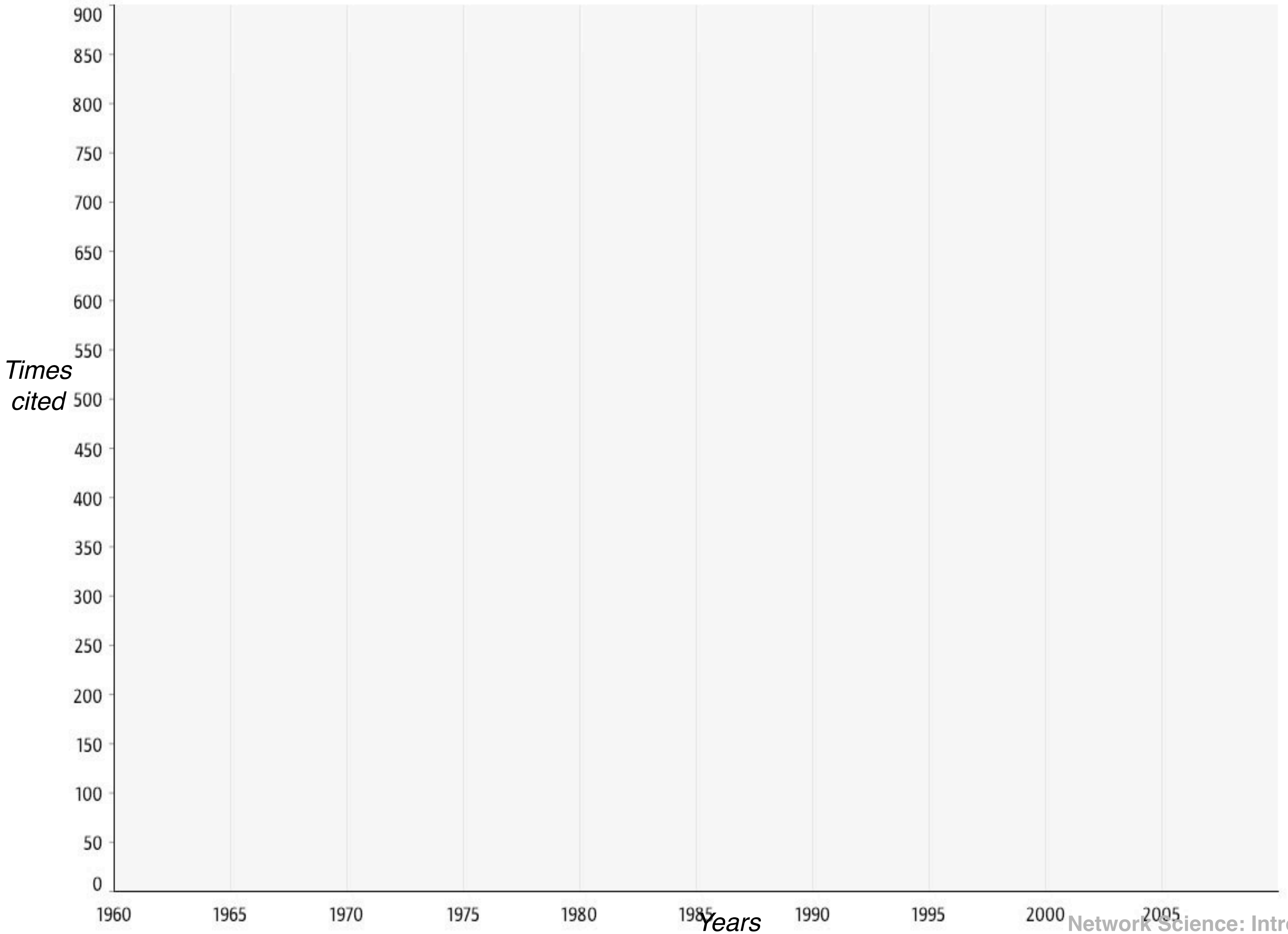
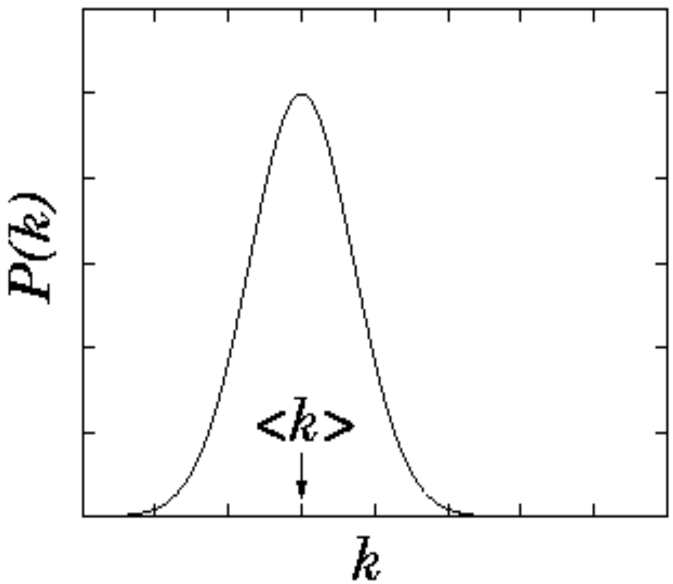
NETWORK SCIENCE | The science of the 21st century



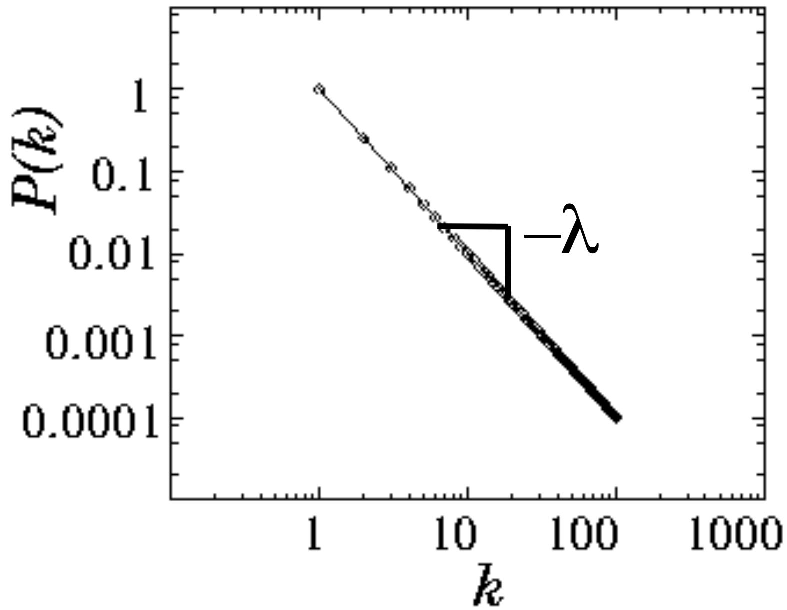
How to quantify? Number of nodes of degree *k [new]*

Courtesy: Barabasi 1999 plane

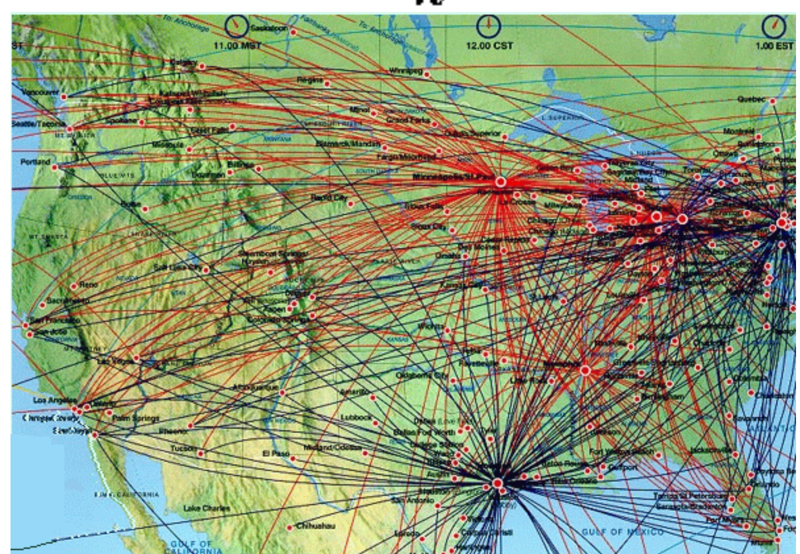
Erdos-Renyi distribution (exponential tail)



Scale-free distribution (power law tail)

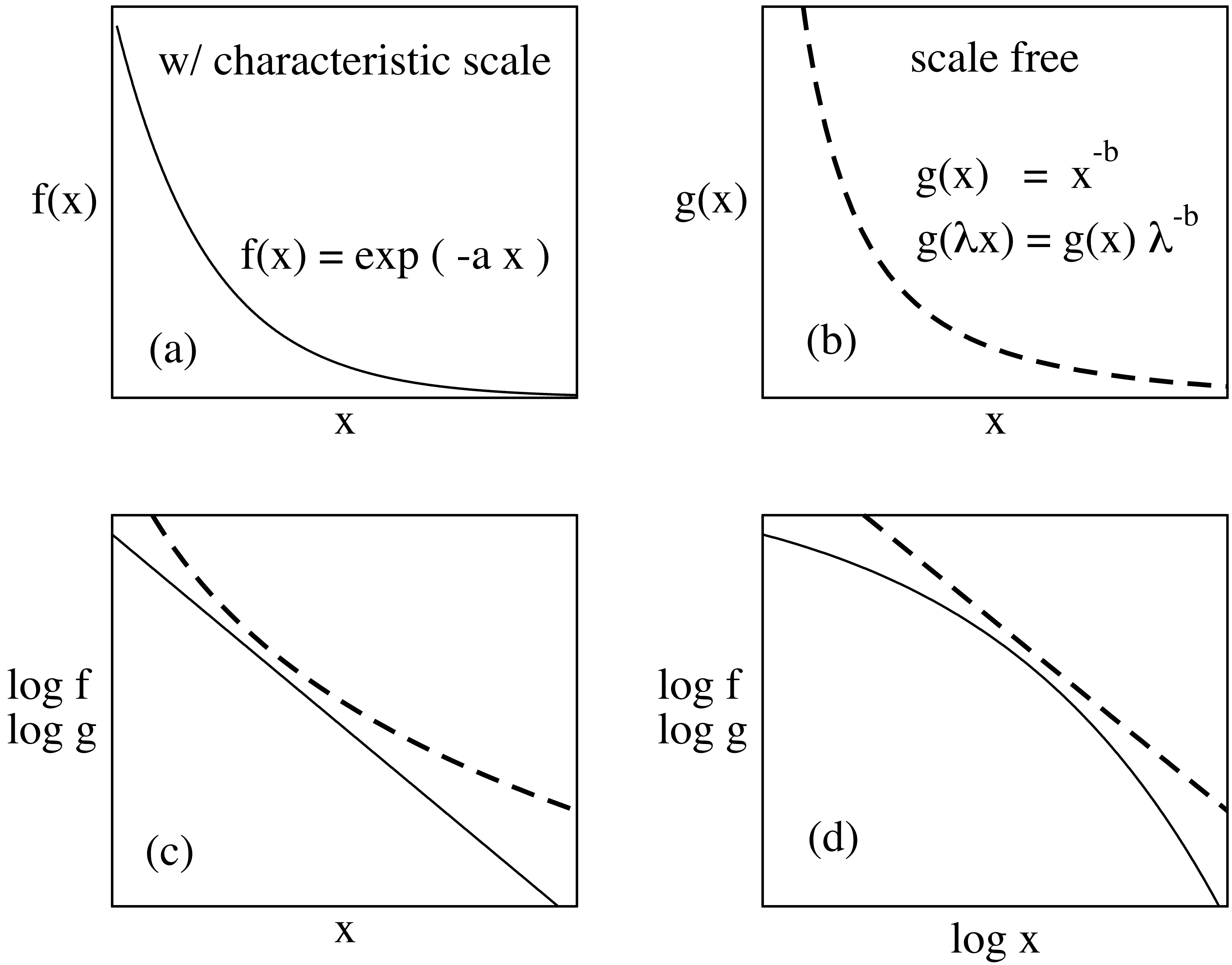


Exponential Tail

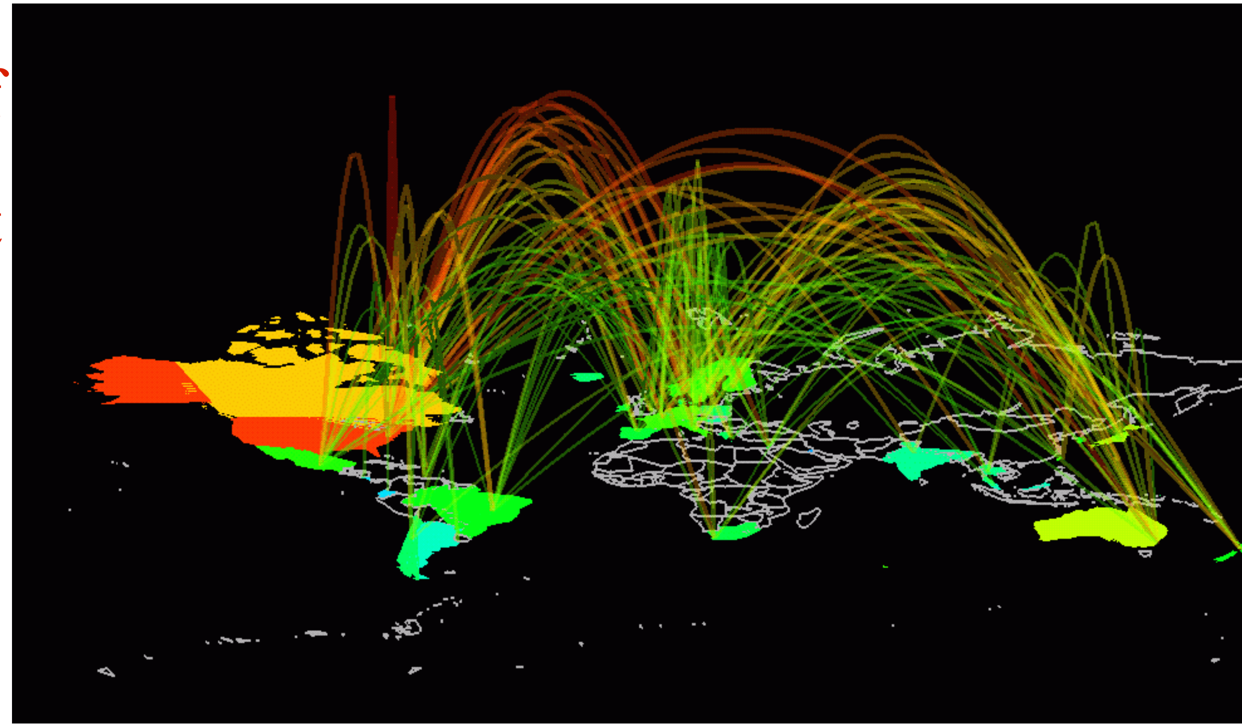


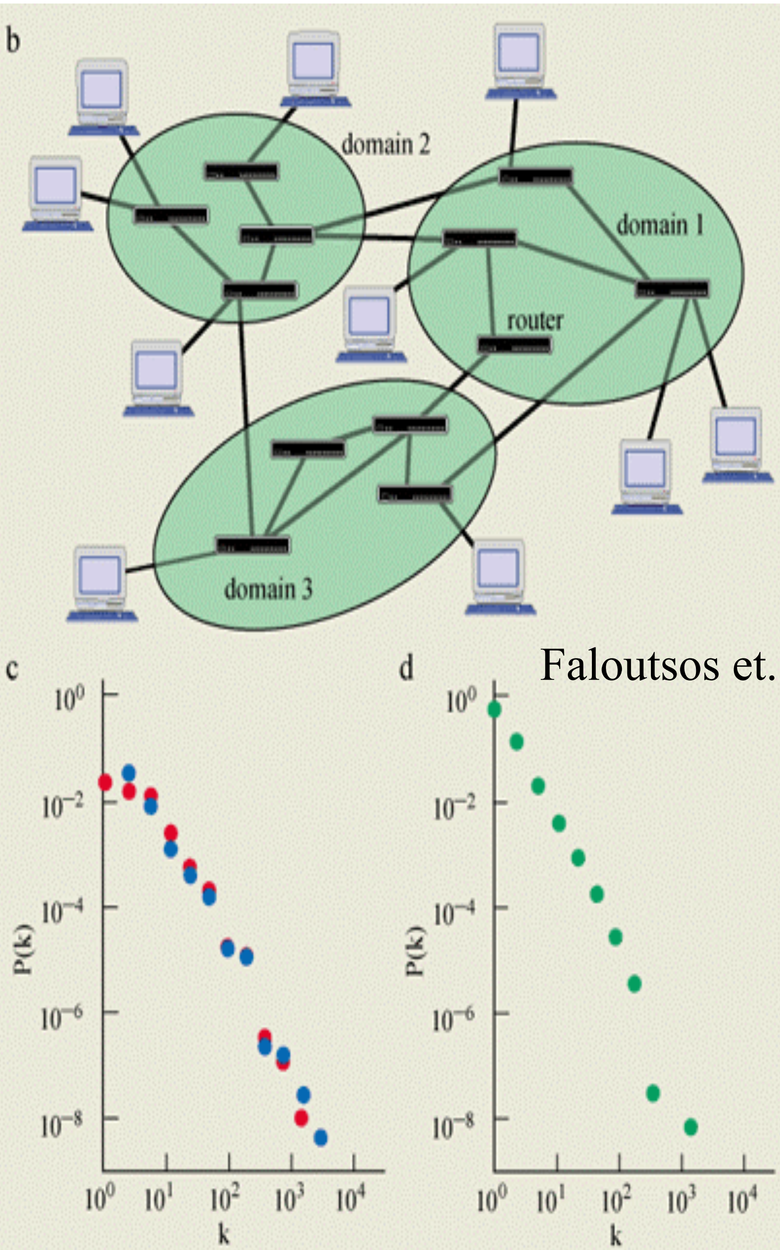
Power Law Tail

"Dummies Guide" to Discovering if a network is scale free:

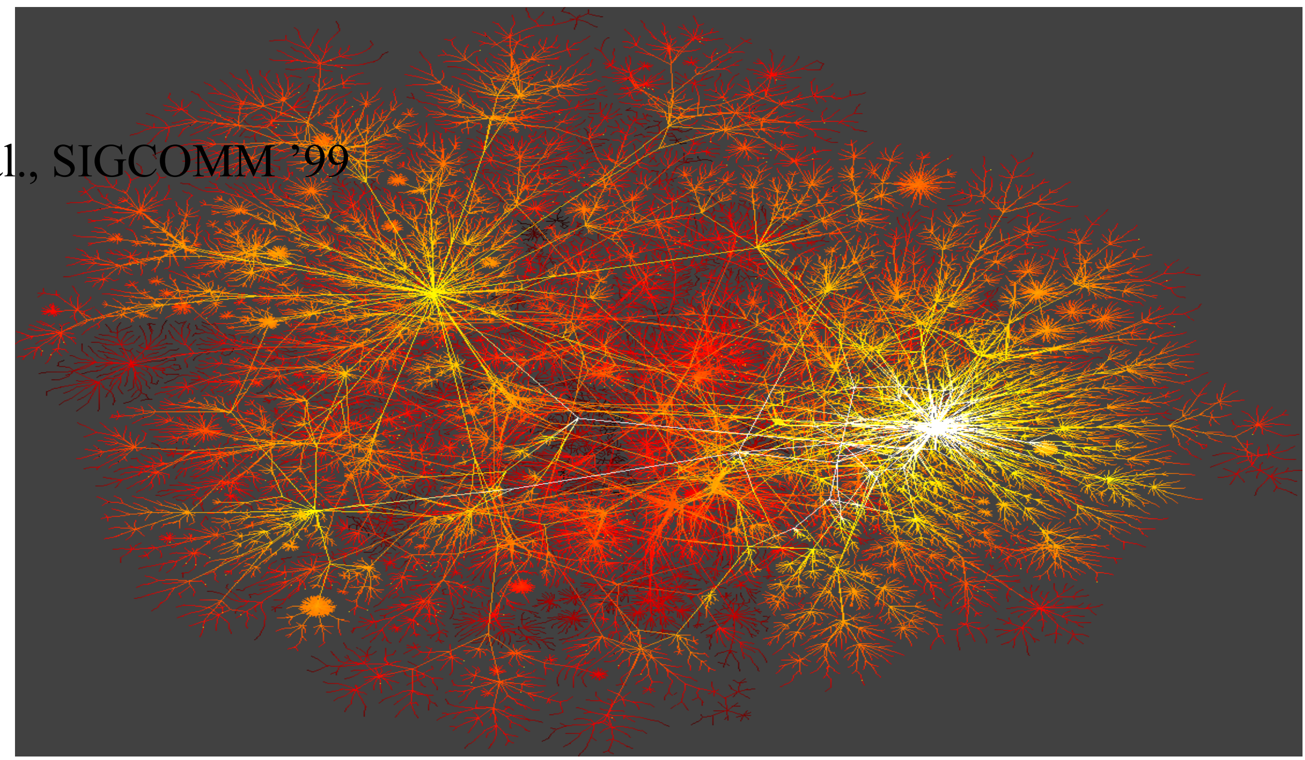


Real world example of scale-free networks: Internet

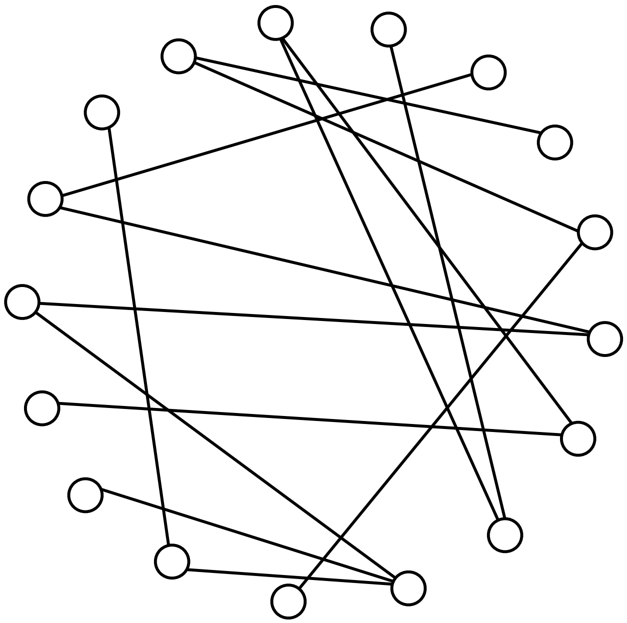




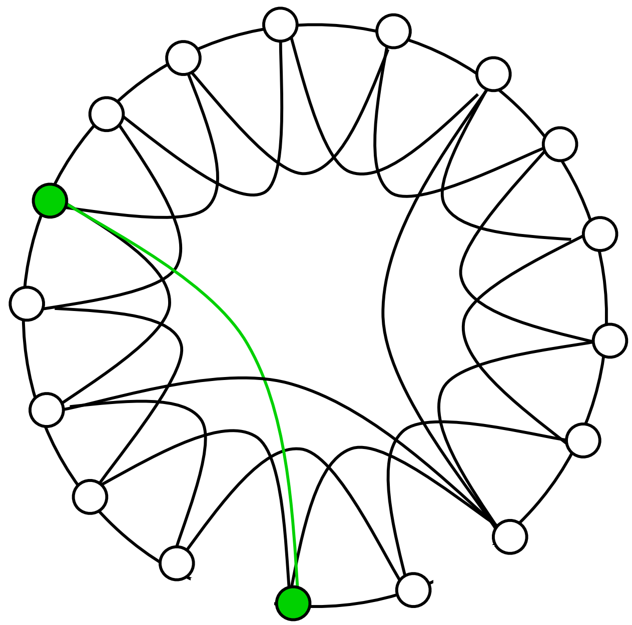
Faloutsos et. al., SIGCOMM ’99



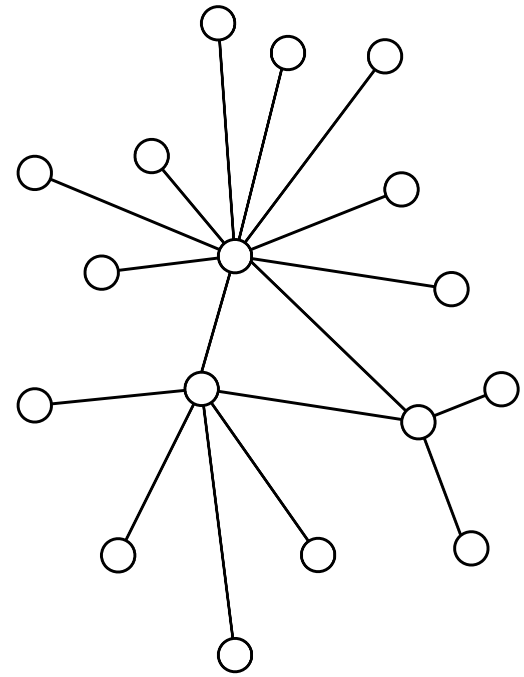
3 types of networks…



Erdös-Rényi
(Exponential tail)
Adv: solvable
Disadv: not realistic



Watts-Strogatz
(“re-wire”)
Adv: Small world
Disadv: not realistic



Scale-free
(Power law tail)
Adv: more realistic
Disadv: not solvable

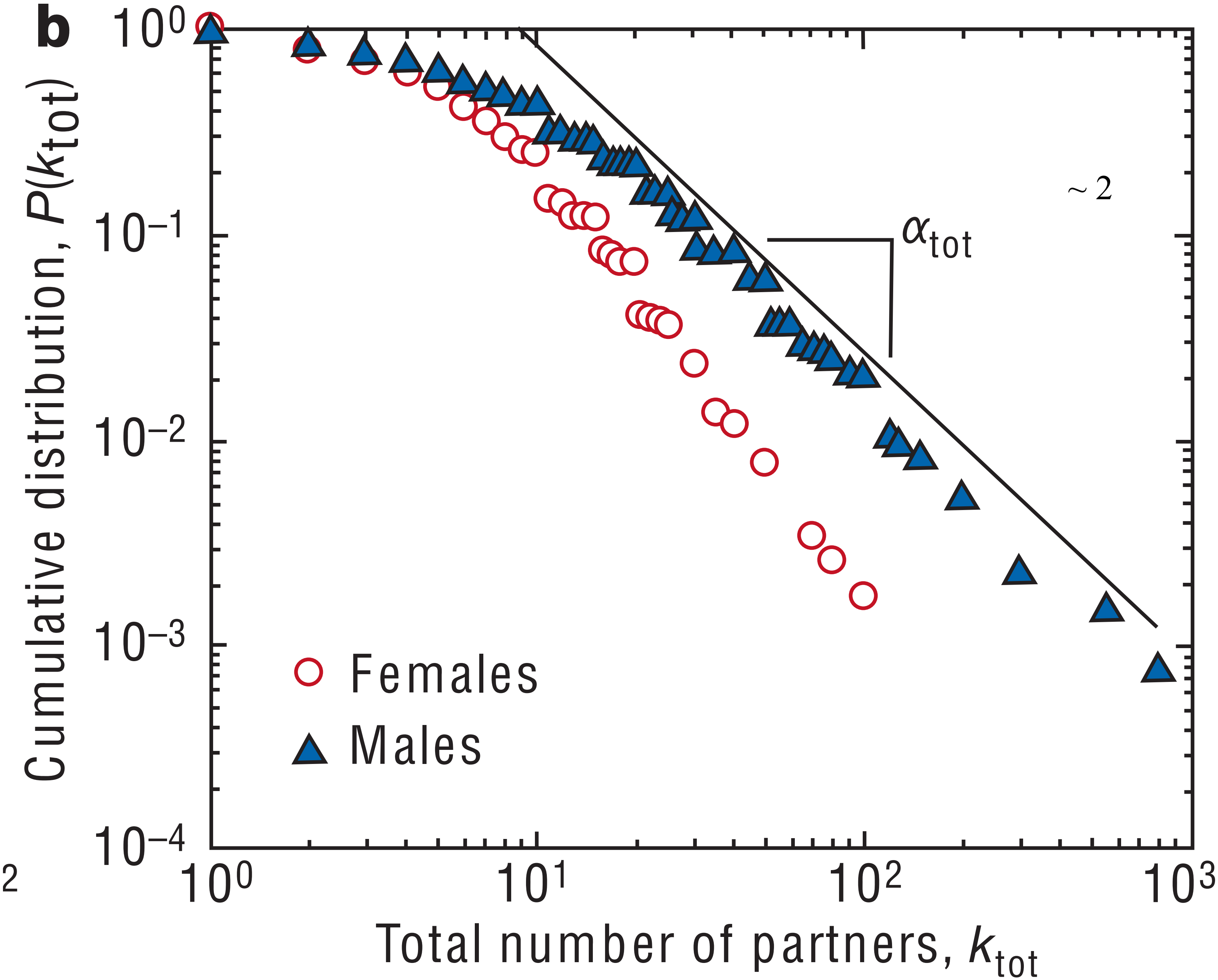
F. Liljeros, C. R. Edling, L. A. N. Amaral, H. E. Stanley, and Y. Aberg, "The Web of Human Sexual Contacts," Nature **411**, 907-908 (2001).

[Citations: 914 ISI, 1233 Google Scholar].

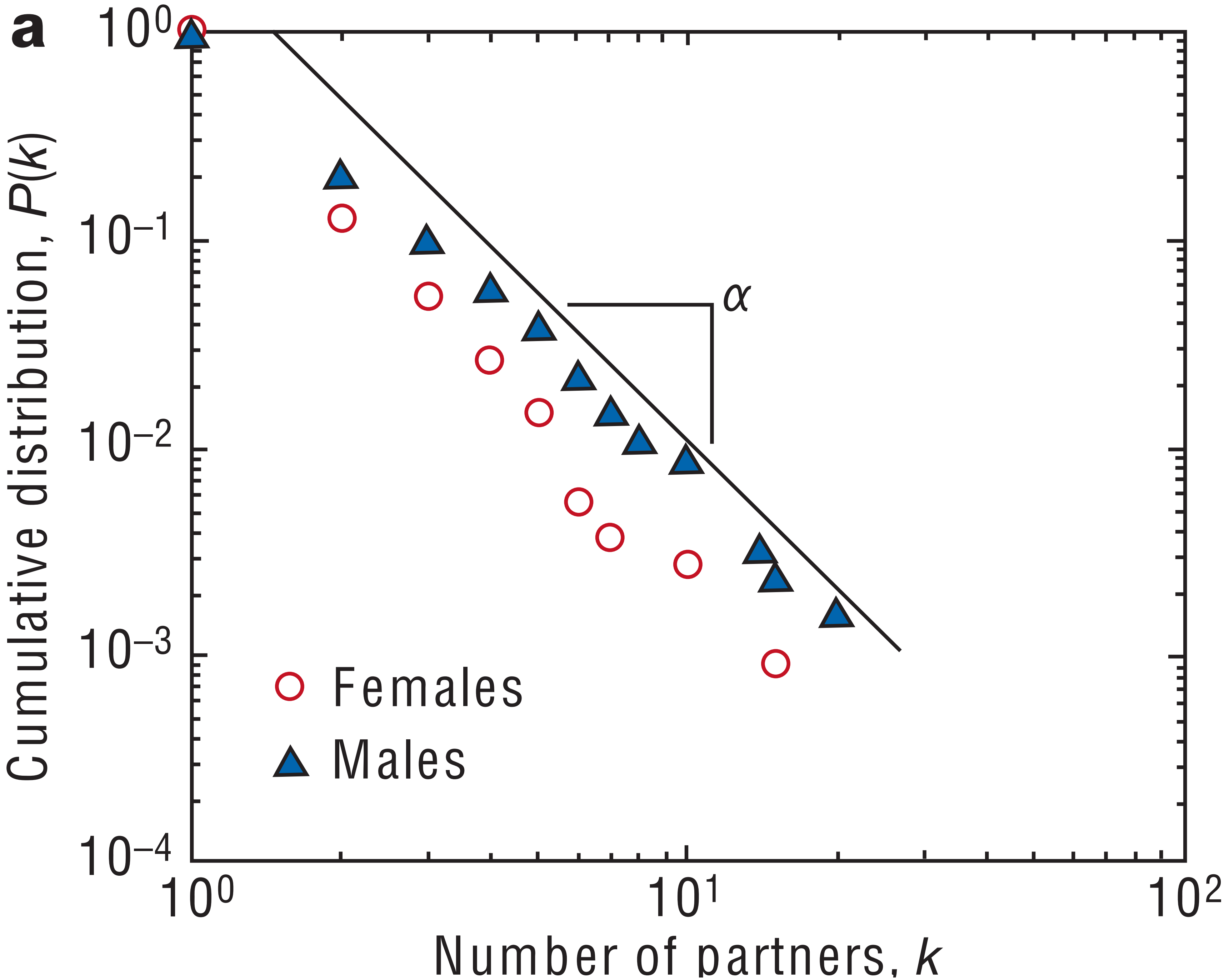
Unlike clearly defined 'real-world' networks[1], social networks tend to be subjective to some extent[2,3] because the perception of what constitutes a social link may differ between individuals. One unambiguous type of connection, however, is sexual contact, and here we analyse the sexual behaviour of a random sample of individuals[4] to reveal the mathematical features of a sexual-contact network. We find that the cumulative distribution of the number of different sexual partners in one year decays as a scale-free power law that has a similar exponent for males and females. The scale-free nature of the web of human sexual contacts indicates that strategic safe-sex campaigns are likely to be the most efficient way to prevent the spread of sexually transmitted diseases.

Q1: A “LAW” OF HUMAN BEHAVIOR? Q2: WHY CARE?

F. Liljeros, C. R. Edling, L. A. N. Amaral, H. E. Stanley, and Y. Aberg, Nature **411** (2001).



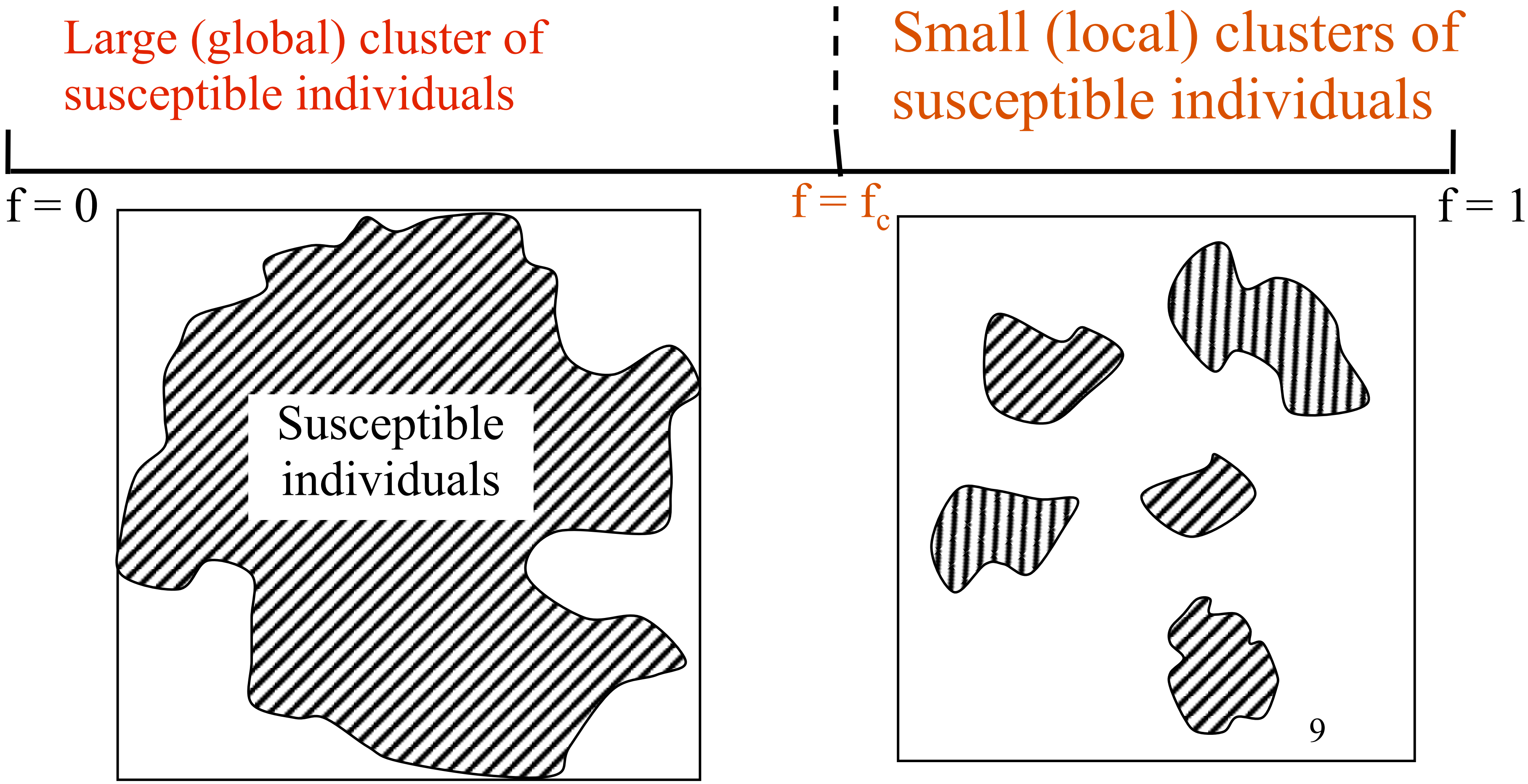
Worry: Artifact of "scale-free imagination"???



EXAMPLE: Network Immunization Strategies

REQUIREMENTS of an efficient immunization strategy:

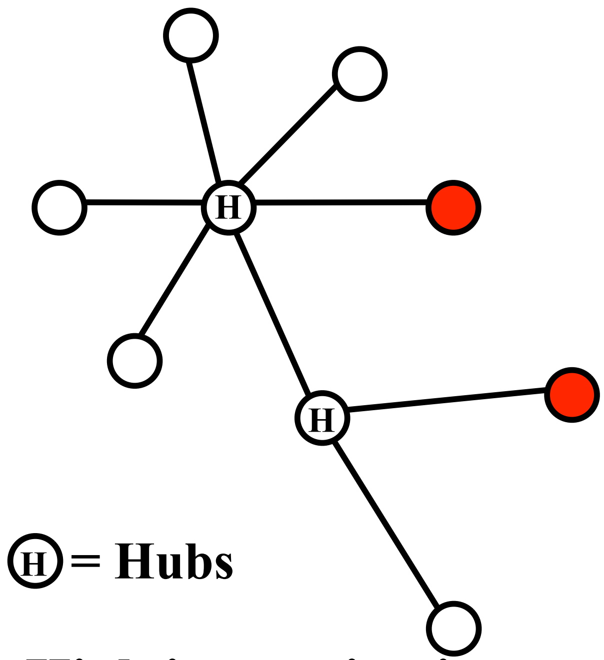
- Immunize at least a critical fraction f_c ("Immunization threshold") of the number of individuals so that only isolated clusters of susceptible individuals remain.
- Effective without detailed knowledge of the network.



Three immunization strategies

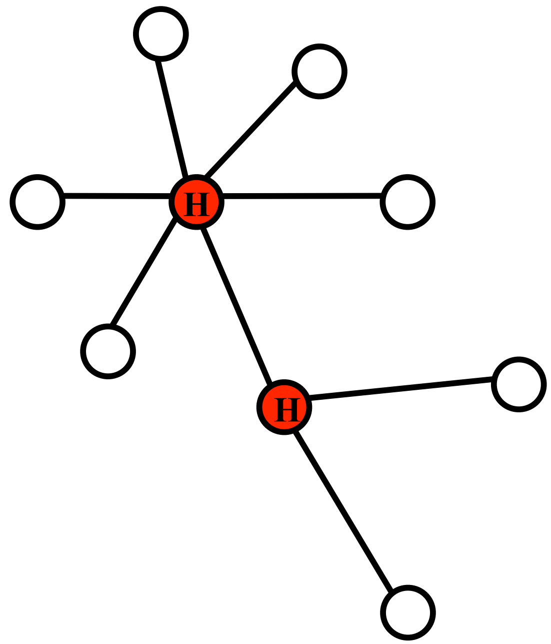
Example: Immunize 2 of the 9 nodes in a scale-free network
Question: What is chance to stop the spread?

Random:
2/9



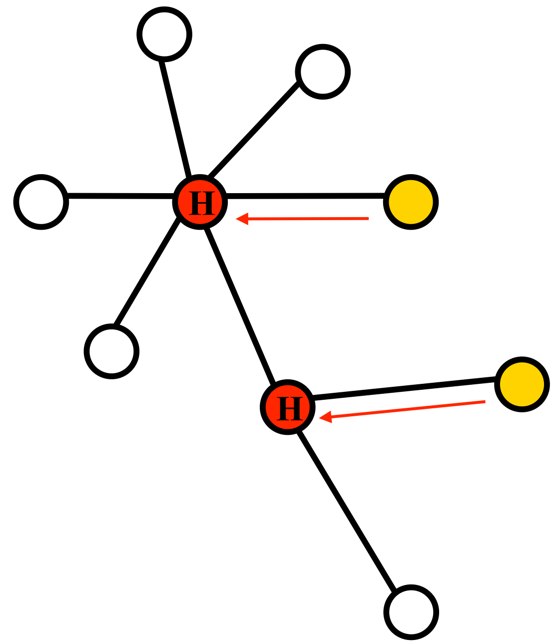
- **High immunization threshold**
- **No prior network information needed**

Targeted:
1



- **Low immunization threshold**
- **Need to know hubs (highly connected individuals)**

Acquaintance:
7/9



- **Low immunization threshold**
- **No prior network information needed**

Real world example: Stopping spread of heresy in Middle Ages
Ormerod/Roach -- The medieval inquisition: scale-free networks & the suppression of heresy

Real world example: Stopping spread of heresy in Middle Ages
Ormerod/Roach -- "The medieval inquisition: scale-free networks & the suppression of heresy"

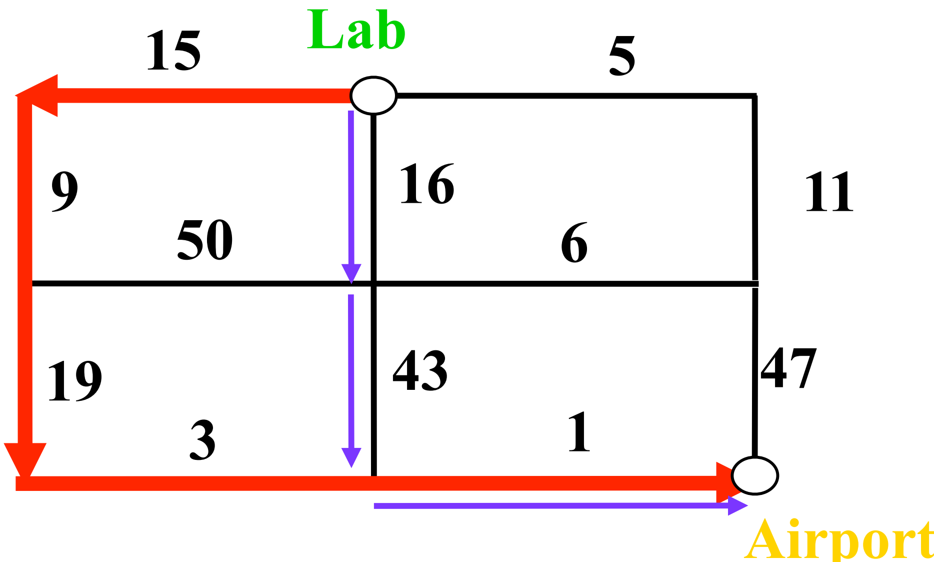
Knowing from the confessions of these Catholics that they were mixed up with heretics, [the crusaders] said to the abbot. 'What shall we do, lord? We cannot tell the good from the bad. The abbot, ……is said to have said: "Kill them. For God knows who are his." Thus innumerable persons were killed in that city.

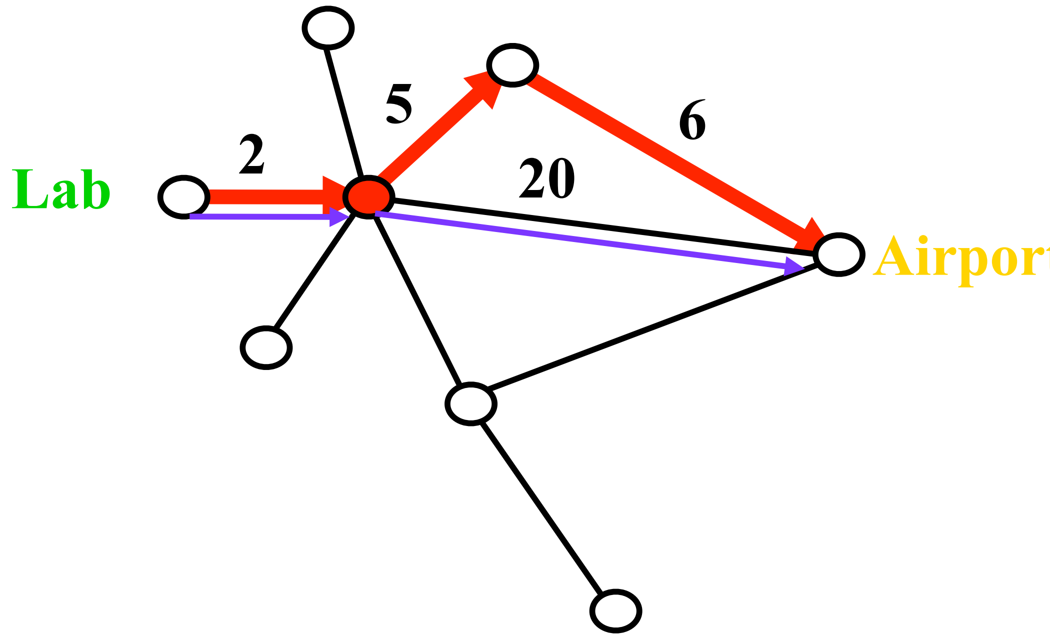
This singling out of guides and messengers, the contacts of the key heretics, rather than the heretics (*perfecti*) themselves is now known as 'acquaintance immunisation.' It is usually more efficient to inoculate one of the contacts of a node rather than the node itself [17].

R. Cohen et al. "Efficient Immunization Strategies for Computer Networks and Populations," Phys. Rev. Lett. **91**, 247901 (2003).

Optimal Path: Minimize total "cost"

L. A. Braunstein, S. V. Buldyrev, R. Cohen, S. Havlin, and H. E. Stanley,
``Optimal Paths in Disordered Complex Networks'' Phys. Rev. Lett. {\bf 91}, 168701.





For this example:

Shortest path: 3 (cost = 60)

Optimal path: 5 (cost = 47)

Shortest path: 2 (cost = 22)

Optimal path: 3 (cost = 13)

Generally:

Shortest path = $N^{0.50}$

Optimal path = $N^{0.61}$

$N^{0.50} < N^{0.61}$

ex: $(10^6)^{0.50} < (10^6)^{0.61}$

Shortest path = Log N

Optimal path = $N^{1/3}$

$\text{Log } N \ll N^{1/3}$

ex: $N=10^6$, $\log 10^6 \ll (10^6)^{1/3}$

For Want of a Nail

For want of a nail the shoe was lost.
For want of a shoe the horse was lost.
For want of a horse the rider was lost.
For want of a rider the battle was lost.
For want of a battle the kingdom was lost.
And all for the want of a horseshoe nail.

"For sparinge of a litel cost, Fulofte time a man hath lost, The large cote for the hod.";
For sparing a little cost often a man has lost the large shed for the head. (c 1390 Confessio Amantis)