Methods of Pricing American Options

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Agenda

- Introduction/Recap of Options
- Binomial Options Pricing Model (BOPM) (in-depth)
- Quadratic Approximations
- Longstaff-Schwartz Method
- Future Work/Conclusion

Introduction/Recap of Options

- American v. European
  - Main difference is in the right to exercise
- Call
  - Right but not obligation to buy an asset
- Put
  - Right but not obligation to sell an asset
- Main uses of Options
  - To hedge risk
  - Speculation
- What is Arbitrage, why do we assume it doesn't exist?

Binomial Options Pricing Model (BOPM)

- Simple discrete-time model to value options.
- "the value of the call can be interpreted as the expectation of of its discounted future value in a risk-neutral world"
  (Cox, Ross, Rubinstein 1979)
- $u > r > d$

$$S \begin{cases} uS & \text{with probability } q, \\ dS & \text{with probability } 1-q. \end{cases}$$

$$C \begin{cases} C_u = \max[0, uS-K] & \text{with probability } q, \\ C_d = \max[0, dS-K] & \text{with probability } 1-q. \end{cases}$$

$$\Delta S + B \begin{cases} \Delta uS + rB & \text{with probability } q, \\ \Delta dS + rB & \text{with probability } 1-q. \end{cases}$$

Binomial Options Pricing Model (BOPM)

- We select Δ and B, so:
  - $\Delta uS + rB = Cu$
  - $\Delta dS + rB = Cd$
- Solving the above leads to:
  - $\Delta = C\downarrow u \ - \ C\downarrow d \ /(u-d)S$
  - $B = \ uC\downarrow d \ - \ dC\downarrow u \ /(u-d)r$
- This combination is known as the "hedging portfolio"
  - No riskless arbitrage

Binomial Options Pricing Model (BOPM)

- With the assumption of no riskless arbitrage:

  $C = \Delta S + B$

  $C = C\downarrow u \; - \; C\downarrow d \;/(u-d) \; + uC\downarrow d \; - \; d$
  $C\downarrow u \;/(u-d)r$

  $C = [(r-d/u-d\,)C\downarrow u + (u-r/u-d\,)$
  $C\downarrow d\,]1/r$

- To simplify, define:

  $p \equiv (r-d/u-d\,)$

  $(1-p) \equiv (u-r/u-d\,)$

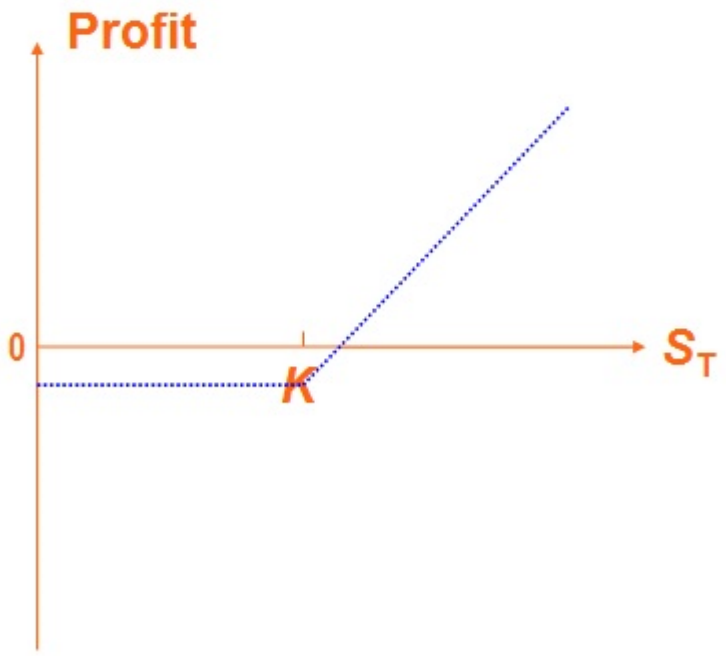
- Thus:

  $C = [pC\downarrow u + (1-p)C\downarrow d\,]1/r$

  if $C > (S-K)$

  otherwise; $C = S-K$

Call Option:



Binomial Options Pricing Model (BOPM)

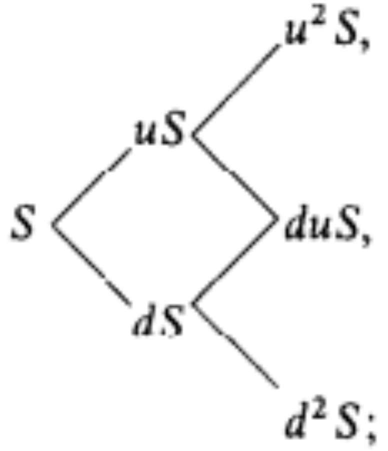
- The BOPM can be taken further to a more complicated scenario
- In this “2-phase” case:

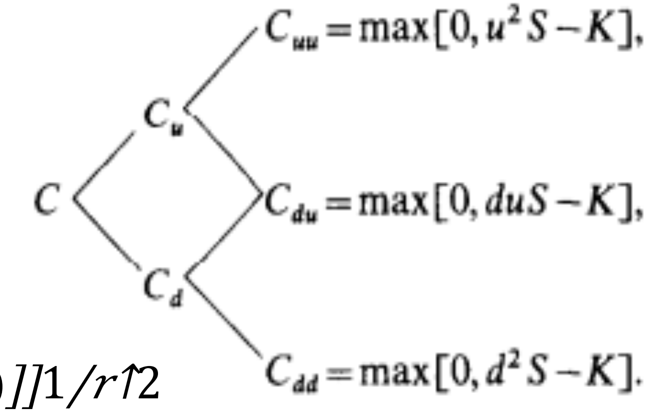
  $C_u = [pC_{uu} + (1-p)C_{ud}]1/r \quad C_d = [pC_{du}$
  $+ (1-p)C_{dd}]1/r$

- Δ,B chosen; and $C_{du} = C_{ud}$:

  $C = [p^2 C_{uu} + 2p(1-p)C_{ud} + (1-p)^2 C_{dd}]1/$
  r^2

- These get very complicated very quickly





$C = [p^2 max[0, u^2 (S-K)] + 2p(1-p) max[0, du(S-K)] + (1-p)^2 max[0, d^2 (S-K)]]1/r^2$

Quadratic Approximations

- First attempt to tackle options pricing from a heavily analytical sense
- Attempt to link American Options to Black-Scholes-Merton
- Directed economists to simulation methods (Longstaff-Schwartz)

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

Longstaff-Schwartz Method

- Use least-squares regression to estimate expected payoff of an option
- Regress discounted future option cash flows on the current price of the underlier associated with ***<u>in-the-money</u>*** sample paths
- The Risk-Neutral market model used in the simulation is the Stochastic Differential Equation:

  $dS = rSdt + \sigma SdZ$

  - r is the riskless rate; constant
  - σ is the exposure matrix; constant
  - Z follows a standard Brownian motion
- Simulation methods like the LSM are accurate, under assumptions known to be wrong (Risk-Neutral)

Future Work/Conclusion

- Multiple-Factor Models/Simulations
- Optimal Exercise
- Behavioral Economics
- Why bother??

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