Basics of the basics of options

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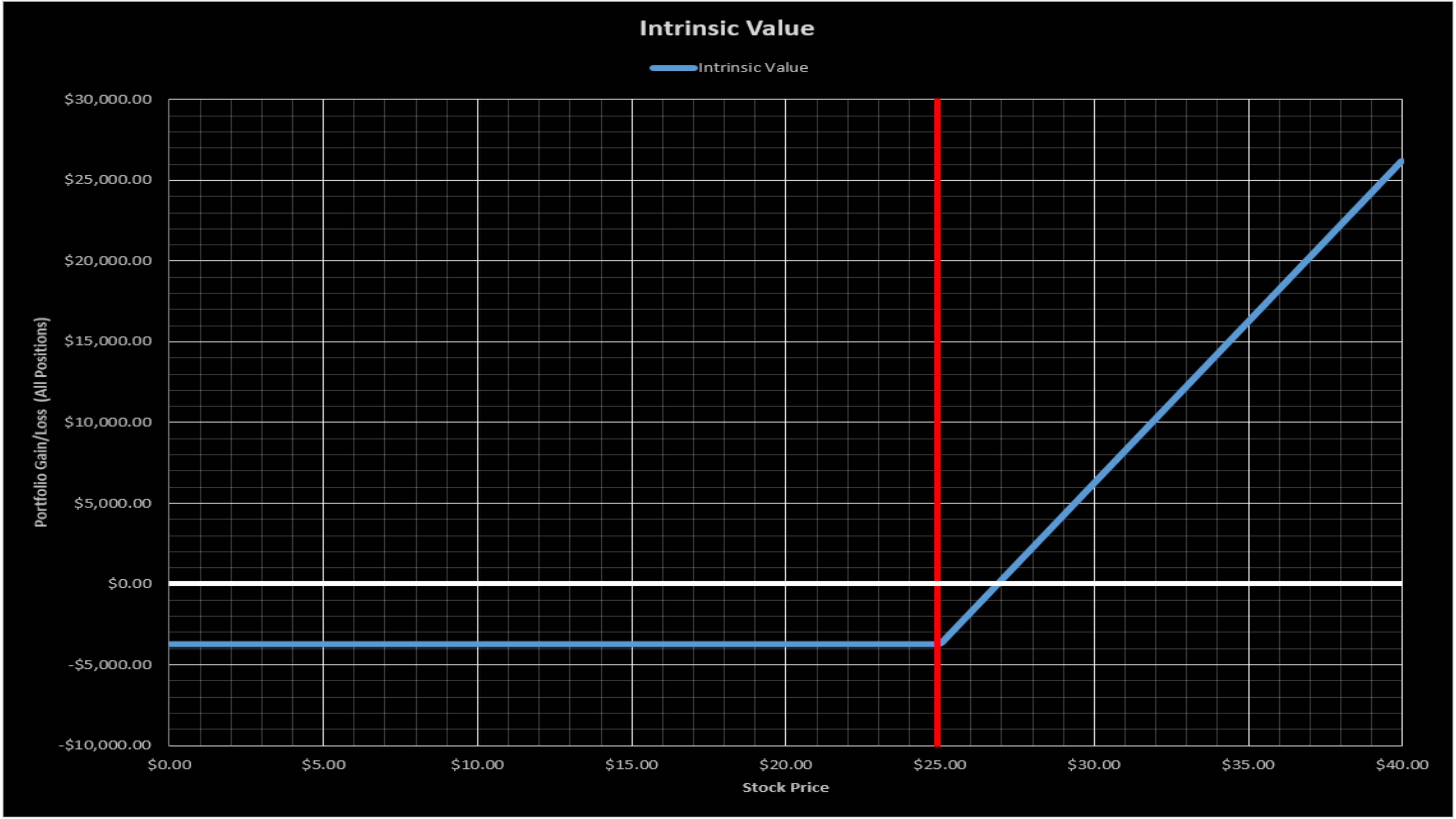
What is a call option

- The owner of a call option (said to be 'long') has the right, *but* **NOT** *the obligation,* to **buy** 100 shares of stock *at the specified strike price on or before the specified date.*
- We say the option is 'exercised' if the owner chooses to make the above transaction. Consider an example
- At t=0, the stock price is S=$30.00 per share. Let's say the strike price is K=$25.00 and you choose to exercise the option.
    1. Upon exercise you will buy 100 shares of the stock at $25 per share, so $2,500.00 is your total cost.
    2. The current stock price is $30, however, that means that you now have a stock position worth $3,000…..note that $3,000>$2,500
    3. In order to keep things simple, I'm going to only consider the case where you immediately sell the stock at the current market price[1]. This gives: $Gain = \$3{,}000 - \$2{,}500 = \$500$

1. Yes, you can choose not to sell the shares, but then we have to start talking about your portfolio and cash levels. There's enough complexity as it is....

Current value and intrinsic value

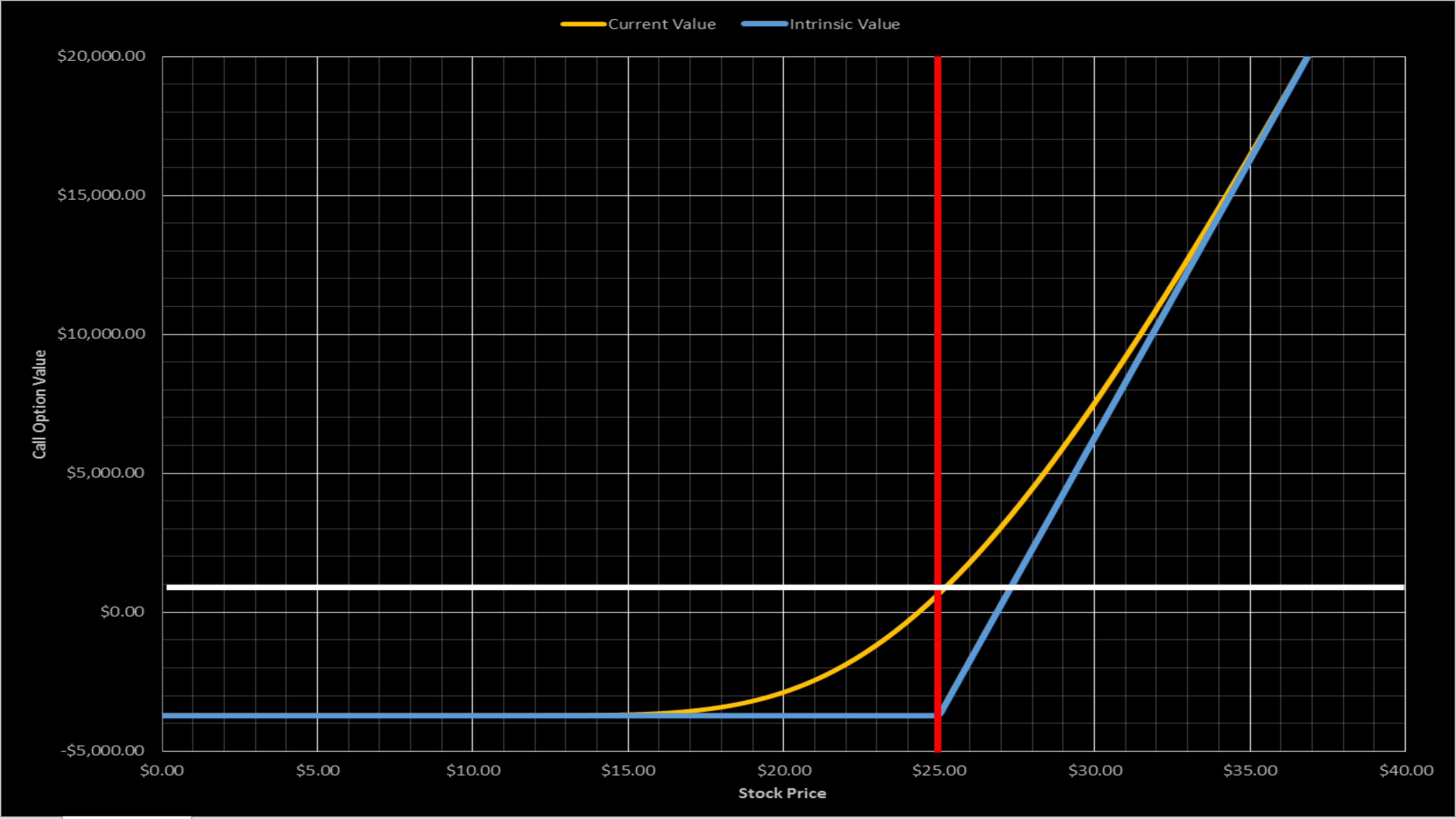
- When talking about options it is easy to lose track of the various 'prices' so let's take a minute to clarify exactly what the different terms are.
- Start with the **current market value** of the option. This is the price you would pay right now to buy the option.
- However, an option has two different 'types' of value. The current value is the sum of the two 'types'.
- The first 'type' of value is called the **intrinsic value.** The intrinsic value is how much the option would be worth if you exercise it right now.
- The formula for the intrinsic value of a call is simply $IV = Max\{0, S - K\}$
- S is the stock price
- K is the strike price



Intrinsic Value
Intrinsic Value
$30,000.00
$25,000.00
$20,000.00
$15,000.00
$10,000.00
$5,000.00
$0.00
-$5,000.00
-$10,000.00
Portfolio Gain/Loss (All Positions)
$0.00
$5.00
$10.00
$15.00
$20.00
$25.00
$30.00
$35.00
$40.00
Stock Price

Time Value

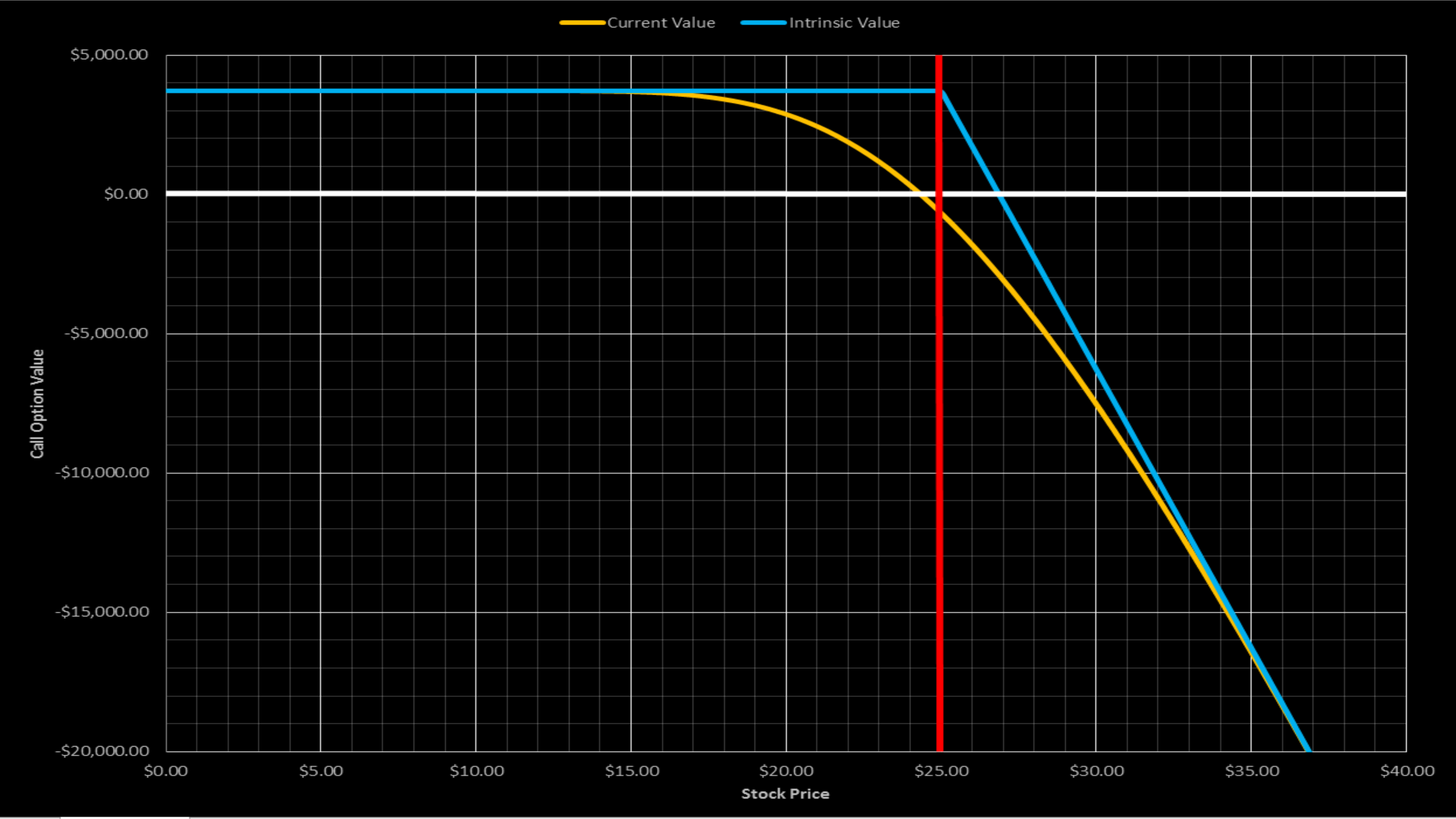
- The **time value** (TV) of the option is where things become terrifyingly complex.
- Keep in mind that: $Current\ Market\ Value = time\ value + intrinsic\ value$
- It is very easy to calculate the time value by simply subtracting from the current price of the option the intrinsic value of the option. But this uses market data and only changes the problem into “how do the market participants decide what the time value is/should be”?
- Because the intrinsic value is *entirely* determined by the stock price and the strike price, the time value is what option pricing models are really trying to determine.
- The key to understanding the time value of an option is to realize that the time value is a measurement of the ‘risk of the option’.



Current Value
Intrinsic Value
Call Option Value
Stock Price
$20,000.00
$15,000.00
$10,000.00
$5,000.00
$0.00
-$5,000.00
$0.00
$5.00
$10.00
$15.00
$20.00
$25.00
$30.00
$35.00
$40.00

Consider selling a call…

- When you sell a call, you are paid some amount of money (P). In return, you are offering to sell 100 shares of stock to the owner of the option at the strike price. The amount you are paid is called the **premium.**
- That means, if the option is exercised, they will pay you 100*K and you give them the 100 shares.
- But what if you don't own the shares? You would have to buy the shares at the current market price.
- $loss = 100 * Min\{0, K - S\}$
- To say it another way, you would lose $1.00 for every $0.01 increase in the stock price. To be more accurate, add the premium to the formula:
- $short\ call\ IV = 100 * P + 100 * Min\{0, K - S\}$



Current Value
Intrinsic Value
$5,000.00
$0.00
-$5,000.00
-$10,000.00
-$15,000.00
-$20,000.00
Call Option Value
$0.00
$5.00
$10.00
$15.00
$20.00
$25.00
$30.00
$35.00
$40.00
Stock Price

Risk…

- Obviously, the premium is simply the current value of the call **<u>at the time that you sell it.</u>**
- So you will only sell the option if the premium is high enough to account for (the risk of a loss) + (intrinsic value)
- The stock price is constantly changing, so you are always at risk of the price moving enough to cause a net loss….as long as there is still time before the option expires.
- On **the day the call expires** there is no longer any time for the stock price to move. Therefore, the *<u>time value is 0.</u>* That means the value of the call at expiration is simply its intrinsic value.
- On **any day before the call expires** the time value will **always be greater than zero.** The intrinsic value can be zero, so it is not uncommon for a call to get its entire value from its time value.

More about risk….

- If there is more time before expiration then there is more opportunity for the stock price to make large movements. It follows that the time value will decay as time passes.
- There is much less risk if $S<<<K$ because it will take a much larger increase in the stock price to cause a loss. Recall that the seller faces no loss if $S<K$.
- However, you will earn a much larger premium if you sell an option when $S>K$. Don't forget that the goal is to make money and not just to minimize risk. In this case, you would want the stock price to decrease, rather than simply remain static.
- If the stock has a very high volatility than it has a greater chance of making large movements in a shorter period of time. So higher volatility means higher risk.
- All of the risk factors are directly related to how the stock price changes with time and not on any outside factors.

Delta-hedged portfolio

- Consider a portfolio that is short one call option and long **δ** shares of the stock. Define Π to be the net value of the portfolio.
- $\Pi \equiv -C + \delta * S$
- Define δ by the following : $\delta \equiv \partial C / \partial S$ (where C is the value of the call)
- Why? Consider a small change in the stock price: ΔS
- Then $\partial C / \partial S \approx \Delta C / \Delta S$
- So: $\Delta \Pi \approx -\Delta C + \Delta S * \Delta C / \Delta S = -\Delta C + \Delta C = 0$
- That means that we have made a portfolio that can be adjusted at any time so that its value *<u>is not dependent on the price of the stock.</u>*
- **<u>Note:</u>** when I write $\Delta \Pi$ on this slide it refers to a change in the portfolio value due to a change in stock price. Later we consider changes in time (Δt) and the corresponding change in portfolio value…the convention is to also use the symbol $\Delta \Pi$ **so I want to make it clear that $\boldsymbol{\Delta \Pi}$ on this slide is NOT equal to $\boldsymbol{\Delta \Pi}$ on ANY other slide.**

Why delta-hedge?

- Well, because the stock price is the only source of risk being considered and because the delta-hedged portfolio is not dependent on the stock price, **therefore the delta-hedged portfolio is risk free.**
- Now I will state, without justification, the following assumption. If a portfolio is risk free, then it must gain value at the risk free rate.
- The risk free rate is assumed to be constant and positive (non-zero).
- The convention is to use the yield on the 10-year US Treasury bond as the risk free rate.
- So the risk free rate (symbol = r) is always a known value.
- On yahoo finance you can find the current value of (r) by entering the following ticker symbol “^TNX”.

Time evolution of the portfolio

- Consider the discrete time interval $[t, t+\Delta t]$
- The portfolio value changes as: $\boldsymbol{\Delta\Pi = r * \Pi * \Delta t}$
- The equation above is the central equation used to derive the Black-Scholes equation.
- If you expand the left hand side:
- $\Delta\Pi = -\Delta C + \Delta S * \partial C / \partial S$
- The details are involved, but in words….
- You need to specify how the stock price evolves over time. In the Black-Scholes model you assume that the stock price has a positive drift rate with a normally distributed random walk.
- C = C(t,S) is dependent on your choice for the stock price evolution.

Conclusions

- Using the black magic of stochastic calculus you can apply the chain rule to $C(t,S)$ and from there you move from continuous time to discrete time.
- Then you plug all of the results into the time evolution equation $\Delta\Pi = r\Pi\Delta t$
- The result is a friendly second order, non-linear, partial differential equation that determines the option price. You would apply the correct boundary conditions to derive the formula for the option value. Note that it gives the total current value including time and intrinsic value. The differential equation is shown below.
- $\partial C/\partial t + \sigma^2 S^2/2 \, \partial^2 C/\partial S^2 + rS\partial C/\partial S - rC = 0$