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Critical market crashes

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Abstract

This review presents a general theory of financial crashes and of stock market instabilities that his co-workers and the author have developed over the past seven years. We start by discussing the limitation of standard analyses for characterizing how crashes are special. The study of the frequency distribution of drawdowns, or runs of successive losses shows that large financial crashes are "outliers": they form a class of their own as can be seen from their statistical signatures. If large financial crashes are "outliers", they are special and thus require a special explanation, a specific model, a theory of their own. In addition, their special properties may perhaps be used for their prediction. The main mechanisms leading to positive feedbacks, i.e., self-reinforcement, such as imitative behavior and herding between investors are reviewed with many references provided to the relevant literature outside the narrow confine of Physics. Positive feedbacks provide the fuel for the development of speculative bubbles, preparing the instability for a major crash. We demonstrate several detailed mathematical models of speculative bubbles and crashes. A first model posits that the crash hazard drives the market price. The crash hazard may sky-rocket at some times due to the collective behavior of "noise traders", those who act on little information, even if they think they "know". A second version inverses the logic and posits that prices drive the crash hazard. Prices may skyrocket at some times again due to the speculative or imitative behavior of investors. According the rational expectation model, this entails automatically a corresponding increase of the probability for a crash. We also review two other models including the competition between imitation and contrarian behavior and between value investors and technical analysts. The most important message is the discovery of robust and universal signatures of the approach to crashes. These precursory patterns have been documented for essentially all crashes on developed as well as emergent stock markets, on currency markets, on company stocks, and so on. We review this discovery at length and demonstrate how to use this insight and the detailed predictions obtained from these models to forecast crashes. For this, we review the major crashes of the past that occurred on the major stock markets of the planet and describe the empirical evidence of the universal nature of the critical log-periodic precursory signature of crashes. The concept of an "anti-bubble" is also summarized, with the Japanese collapse from the beginning of 1991 to present, taken as a prominent example. A prediction issued and advertised in January 1999 has been until

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recently born out with remarkable precision, predicting correctly several changes of trends, a feat notoriously difficult using standard techniques of economic forecasting. We also summarize a very recent analysis the behavior of the U.S. S&P500 index from 1996 to August 2002 and the forecast for the two following years. We conclude by presenting our view of the organization of financial markets.

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Contents

1. Introduction

The total world market capitalization rose from \$3.38 trillion (thousand billions) in 1983 to \$26.5 trillion in 1998 and to \$38.7 trillion in 1999. To put these numbers in perspective, the 1999 U.S. budget was \$1.7 trillion while its 1983 budget was \$800 billion. Market capitalization and trading volumes tripled during the 1990s. The volume of securities issuance was multiplied by six. Privatization has played a key role in the stock market growth (Megginson, 2000). Stock market investment is clearly the big game in town.

A market crash occurring simultaneously on most of the stock markets of the world as witnessed in October 1987 would amount to the quasi-instantaneous evaporation of trillions of dollars. In values of January 2001, a stock market crash of 30% indeed would correspond to an absolute loss of about 13 trillion dollars! Market crashes can thus swallow years of pension and savings in an instant. Could they make us suffer even more by being the precursors or triggering factors of major recessions as in 1929–1933 after the great crash of October 1929? Or could they lead to a general collapse of the financial and banking system as seems to have being barely avoided several times in the not-so-distant past?

Stock market crashes are also fascinating because they personify the class of phenomena known as "extreme events". Extreme events are characteristic of many natural and social systems, often referred to by scientists as "complex systems".

Here, we discuss how financial crashes can be understood by invoking the latest and most sophisticated concepts in modern science, i.e., the theory of complex systems and of critical phenomena. Our aim is to cover a territory bringing us all the way from the description of how the wonderful organization around us arises, to the holy grail of crash predictions.

This article is organized in eight parts. Section 2 introduces the fundamental questions: what are crashes? How do they happen? Why do they occur? When do they occur? Section 2 outlines the answers we propose, taking as examples some famous, or we should rather say, infamous historical crashes. Section 3 discusses first the limitation of standard analyses for characterizing how crashes are special. It presents then the study of the frequency distribution of drawdowns, or runs of successive losses, and shows that large financial crashes are "outliers": they form a class of their own as can be seen from their statistical signatures. If large financial crashes are "outliers", they are special and thus require a special explanation, a specific model, a theory of their own. In addition, their special properties may perhaps be used for their prediction. Section 4 reviews the main mechanisms leading to positive feedbacks, i.e., self-reinforcement, such as imitative behavior and herding between investors. Positive feedbacks provide the fuel for the development of speculative bubbles, preparing the instability for a major crash. Section 5 presents two versions of a rational model of speculative bubbles and crashes. The first version posits that the crash hazard drives the market price. The crash hazard may sky-rocket at some times due to the collective behavior of "noise traders", those who act on little information, even if they think they "know". The second version inverses the logic and posits that prices drive the crash hazard. Prices may skyrocket at some times again due to the speculative or imitative behavior of investors. According the rational expectation model, this entails automatically a corresponding increase of the probability for a crash. The most important message is the discovery of robust and universal signatures of the approach to crashes. These precursory patterns have been documented for essentially all crashes on developed as well as emergent stock markets, on currency markets, on company stocks, and so on. Section 5 also discusses two simple models of imitation and contrarian behavior of agents, leading to a chaotic dynamics of speculative bubbles and crashes and of the competition between value investors and technical analysts. Section 6 takes a step back and presents the general concept of self-similarity, with complex dimensions and their associated discrete self-similarity. Section 6 shows how these remarkable geometric and mathematical objects allow one to codify the information contained in the precursory patterns before large crashes. Section 7 analyzes the major crashes of the past that occurred on the major stock markets of the planet. It describes the empirical evidence of the universal nature of the critical log-periodic precursory signature of crashes. It also presents the concept of an "anti-bubble", with the Japanese collapse from the beginning of 1991 to present, taken as a prominent example. A prediction issued and advertised in January 1999 has been until now born out with remarkable precision, predicting correctly several changes of trends, a feat notoriously difficult using standard techniques of economic forecasting. We also summarize a very recent analysis the behavior of the U.S. S&P500 index from 1996 to August 2002 and the forecast for the two following years. Section 8 concludes.

2. Financial crashes: what, how, why and when?

2.1. What are crashes and why do we care?

Stock market crashes are momentous financial events that are fascinating to academics and practitioners alike. According to the academic world view that markets are efficient, only the revelation of a dramatic piece of information can cause a crash, yet in reality even the most thorough *post-mortem* analyses are typically inconclusive as to what this piece of information might have been. For traders and investors, the fear of a crash is a perpetual source of stress, and the onset of the event itself always ruins the lives of some of them.

Most approaches to explain crashes search for possible mechanisms or effects that operate at very short time scales (hours, days or weeks at most). We propose here a radically different view: the underlying cause of the crash must be searched months and years before it, in the progressive increasing build-up of market cooperativity or effective interactions between investors, often translated into accelerating ascent of the market price (the bubble). According to this "critical" point of view, the specific manner by which prices collapsed is not the most important problem: a crash occurs because the market has entered an unstable phase and any small disturbance or process may have triggered the instability. Think of a ruler held up vertically on your finger: this very unstable position will lead eventually to its collapse, as a result of a small (or absence of adequate) motion of your hand or due to any tiny whiff. The collapse is fundamentally due to the unstable position; the instantaneous cause of the collapse is secondary. In the same vein, the growth of the sensitivity and the growing instability of the market close to such a critical point might explain why attempts to unravel the local origin of the crash have been so diverse. Essentially, anything would work once the system is ripe. We explore here the concept that a crash has fundamentally an endogenous origin and that exogenous shocks only serve as triggering factors. As a consequence, the origin of crashes is much more subtle than often thought as it is constructed progressively by the market as a whole, as a self-organizing process. In this sense, this could be termed a systemic instability.

Systemic instabilities are of great concern to governments, central banks and regulatory agencies (De Bandt and Hartmann, 2000). The question that has often arisen in the 1990s is whether the new, globalized, information technology-driven economy has advanced to the point of outgrowing the set of rules dating from the 1950s, in effect creating the need for a new rule set for the New Economy. Those who make this call basically point to the systemic instabilities since 1997 (or even back to Mexico's peso crisis of 1994) as evidence that the old post-world war II rule set is now antiquated, thus endangering this second great period of globalization to the same fate as the first. With the global economy appearing so fragile sometimes, how big of a disruption would be needed to throw a wrench into the world's financial machinery? One of the leading moral authorities, the Basle Committee on Banking Supervision, advises (1997) that, "in handling systemic issues, it will be necessary to address, on the one hand, risks to confidence in the financial system and contagion to otherwise sound institutions, and, on the other hand, the need to minimize the distortion to market signals and discipline".

The dynamics of confidence and of contagion and decision making based on imperfect information are indeed at the core of the present work and will lead us to examine the following questions. What are the mechanisms underlying crashes? Can we forecast crashes? Could we control them? Or at least, could we have some influence on them? Do crashes point to the existence of a fundamental instability in the world financial structure? What could be changed to mollify or suppress these instabilities?

2.2. *The crash of October, 1987*

From the opening on October 14, 1987 through the market close on October 19, major indexes of market valuation in the United States declined by 30 percent or more. Furthermore, all major world markets declined substantially in the month, which is itself an exceptional fact that contrasts with the usual modest correlations of returns across countries and the fact that stock markets around the world are amazingly diverse in their organization (Barro et al., 1989).

In local currency units, the minimum decline was in Austria (−11.4%) and the maximum was in Hong Kong (−45.8%). Out of 23 major industrial countries (Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Ireland, Italy, Japan, Malaysia, Mexico, Netherland, New Zealand, Norway, Singapore, South Africa, Spain, Sweden, Switzerland, United Kingdom, United States), 19 had a decline greater than 20%. Contrary to a common belief, the U.S. was not the first to decline sharply. Non-Japanese Asian markets began a severe decline on October 19, 1987, their time, and this decline was echoed first on a number of European markets, then in North American, and finally in Japan. However, most of the same markets had experienced significant but less severe declines in the latter part of the previous week. With the exception of the U.S. and Canada, other markets continued downward through the end of October, and some of these declines were as large as the great crash on October 19.

A lot of work has been carried out to unravel the origin(s) of the crash, notably in the properties of trading and the structure of markets; however, no clear cause has been singled out. It is noteworthy that the strong market decline during October 1987 followed what for many countries had been an unprecedented market increase during the first nine months of the year and even before. In the U.S. market for instance, stock prices advanced 31.4% over those nine months. Some commentators have suggested that the real cause of October's decline was that over-inflated prices generated a speculative bubble during the earlier period.

The main explanations people have come up with are the following.

1. *Computer trading*. In computer trading, also known as program trading, computers were programmed to automatically order large stock trades when certain market trends prevailed, in particular sell orders after losses. However, during the 1987 U.S. Crash, other stock markets which did not use program trading also crashed, some with losses even more severe than the U.S. market.
2. *Derivative securities*. Index futures and derivative securities have been claimed to increase the variability, risk and uncertainty of the U.S. stock markets. Nevertheless, none of these techniques or practices existed in previous large and sudden market declines in 1914, 1929, and 1962.
3. *Illiquidity*. During the crash, the large flow of sell orders could not be digested by the trading mechanisms of existing financial markets. Many common stocks in the New York Stock Exchange were not traded until late in the morning of October 19 because the specialists could not find enough buyers to purchase the amount of stocks that sellers wanted to get rid of at certain prices. This insufficient liquidity may have had a significant effect on the size of the price drop, since investors had overestimated the amount of liquidity. However, negative news about the liquidity of stock markets cannot explain why so many people decided to sell stock at the same time.
4. *Trade and budget deficits*. The third quarter of 1987 had the largest U.S. trade deficit since 1960, which together with the budget deficit, led investors into thinking that these deficits would cause a fall of the U.S. stocks compared with foreign securities. However, if the large U.S. budget deficit was the cause, why did stock markets in other countries crash as well? Presumably, if unexpected changes in the trade deficit are bad news for one country, it should be good news for its trading partner.
5. *Overvaluation*. Many analysts agree that stock prices were overvalued in September, 1987. While Price/Earning ratio and Price/Dividend ratios were at historically high levels, similar Price/Earning

and Price/Dividends values had been seen for most of the 1960–1972 period over which no sudden crash occurred. Overvaluation does not seem to trigger crashes every time.

Other cited potential causes involve the auction system itself, the presence or absence of limits on price movements, regulated margin requirements, off-market and off-hours trading (continuous auction and automated quotations), the presence or absence of floor brokers who conduct trades but are not permitted to invest on their own account, the extent of trading in the cash market versus the forward market, the identity of traders (i.e., institutions such as banks or specialized trading firms), the significance of transaction taxes...

More rigorous and systematic analyses on univariate associations and multiple regressions of these various factors conclude that it is not clear at all what was the origin of the crash (Barro et al., 1989; Roll, 1988). The most precise statement, albeit somewhat self-referencing, is that the most statistically significant explanatory variable in the October crash can be ascribed to the normal response of each country's stock market to a worldwide market motion. A world market index was thus constructed (Barro et al., 1989; Roll, 1988) by equally weighting the local currency indexes of the 23 major industrial countries mentioned above and normalized to 100 on september 30. It fell to 73.6 by October 30. The important result is that it was found to be statistically related to monthly returns in every country during the period from the beginning of 1981 until the month before the crash, albeit with a wildly varying magnitude of the responses across countries (Barro et al., 1989; Roll, 1988). This correlation was found to swamp the influence of the institutional market characteristics. This signals the possible existence of a subtle but nonetheless present world-wide cooperativity at times preceding crashes.

2.3. How? Historical crashes

In the financial world, risk, reward and catastrophe come in irregular cycles witnessed by every generation. Greed, hubris and systemic fluctuations have given us the Tulip Mania, the South Sea bubble, the land booms in the 1920s and 1980s, the U.S. stock market and great crash in 1929, the October 1987 crash, to name just a few of the hundreds of ready examples (White, 1996).

2.3.1. The Tulip mania

The years of tulip speculation fell within a period of great prosperity in the republic of the Netherlands. Between 1585 and 1650, Amsterdam became the chief commercial emporium, the center of the trade of the northwestern part of Europe, owing to the growing commercial activity in newly discovered America. The tulip as a cultivated flower was imported into Western Europe from Turkey and it is first mentioned around 1554. The scarcity of tulips and their beautiful colors made them valuable and a must for members of the upper society.

During the build-up of the tulip market, the participants were not making money through the actual process of production. Tulips acted as the medium of speculation and its price determined the wealth of participants in the tulip business. It is not clear whether the build-up attracted new investment or new investment fueled the build-up, or both. What is known is that, as the build-up continued more and more, people were roped in to invest their hard won earnings. The price of the tulip lost all correlation to its comparative value with other goods or services.

What we now call the "tulip mania" of the seventeenth century was the "sure thing" investment during the period from mid-1500s to 1636. Before its devastating end in 1637, those who bought tulips rarely lost money. People became too confident that this "sure thing" would always make them money and, at its peak, the participants mortgaged their houses and businesses to trade tulips. The craze was so overwhelming that some tulip bulbs of a rare variety sold for the equivalent of a few tens of thousand dollars. Before the crash, any suggestion that the price of tulips was irrational was dismissed by all the participants.

The conditions now generally associated with the first period of a boom were all present: an increasing currency, a new economy with novel colonial possibilities, an increasingly prosperous country, all together had created the optimistic atmosphere in which booms are said to grow.

The crisis came unexpectedly. On february 4th, 1637, the possibility of the tulips becoming definitely unsalable was mentioned for the first time. From then to the end of May 1637, all attempts of coordination between florists, bulbgrowers as well as by the States of Holland were met with failure. Bulbs worth tens of thousand of U.S. dollars (in present value) in early 1637 became valueless a few months later. This remarkable event is often discussed in present days and parallels are drawn with modern speculation mania and the question is asked: does the tulip market's build-up and its subsequent crash has any relevance for today's times?

2.3.2. The South Sea bubble

The South Sea Bubble is the name given to the enthusiastic speculative fervor ending in the first great stock market crash in England in 1720 (White, 1996). The South Sea Bubble is a fascinating story of mass hysteria, political corruption, and public upheaval. It is really a collection of thousands of stories, tracing the personal fortunes of countless individuals who rode the wave of stock speculation for a furious six months in 1720. The "Bubble year" as it is referred to, actually involves several individual "bubbles" as all kinds of fraudulent joint-stock companies sought to take advantage of the mania for speculation. The following account borrows from (The) Bubble Project at *http://is.dal.ca/~dmcneil/bubble.html*.

In 1711, the South Sea Company was given a monopoly of all trade to the south seas. The real prize was the anticipated trade that would open up with the rich Spanish colonies in South America. In return for this monopoly, the South Sea Company would assume a portion of the national debt that England had incurred during the War of the Spanish Succession. When Britain and Spain officially went to war again in 1718, the immediate prospects for any benefits from trade to South America were nil. What mattered to speculators, however, were future prospects, and here it could always be argued that incredible prosperity lay ahead and would be realized when open hostilities came to an end.

The early 1700s was also a time of international finance. By 1719 the South Sea directors wished, in a sense, to imitate the manipulation of public credit that John Law had achieved in France with the Mississippi Company, which was given a monopoly of French trade to North America; Law had connived to drive the price of its stock up, and the South Sea directors hoped to do the same. In 1719 the South Sea directors made a proposal to assume the entire public debt of the British government. On April 12, 1720 this offer was accepted. The Company immediately started to drive the price of the stock up through artificial means; these largely took the form of new subscriptions combined with the circulation of pro-trade-with-Spain stories designed to give the impression that the stock could only go higher. Not only did capital stay in England, but many Dutch investors bought South Sea stock, thus increasing the inflationary pressure.

South Sea stock rose steadily from January through to the spring. And as every apparent success would soon attract its imitators, all kinds of joint-stock companies suddenly appeared, hoping to cash in on the speculation mania. Some of these companies were legitimate but the bulk were bogus schemes designed to take advantage of the credulity of the people. Several of the bubbles, both large and small, had some overseas trade or "New World" aspect. In addition to the South Sea and Mississippi ventures, there was a project for improving the Greenland fishery, another for importing walnut trees from Virginia. Raising capital sums by selling stock in these enterprises was apparently easy work. The projects mentioned so far all have a tangible specificity at least on paper if not in practice; others were rather vague on details but big on promise. The most remarkable was "A company for carrying on an undertaking of great advantage, but nobody to know what it is". The prospectus stated that "the required capital was half a million, in five thousand shares of 100 pounds each, deposit 2 pounds per share. Each subscriber, paying his [or her] desposit, was entitled to 100 pounds per annum per share. How this immense profit was to be obtained, [the proposer] did not condescend to inform [the buyers] at that time". As T.J. Dunning (1860) wrote: "Capital eschews no profit, or very small profit.... With adequate profit, capital is very bold. A certain 1% percent will ensure its employment anywhere; 20 percent certain will produce eagerness; 50 percent, positive audacity; 100 percent will make it ready to trample on all human laws; 300 percent and there is not a crime at which it will scruple, nor a risk it will not run, even to the chance of its owner being hanged". Next morning, at nine o'clock, this great man opened an office in Cornhill. Crowds of people beset his door, and when he shut up at three o'cock, he found that no less than one thousand shares had been subscribed for, and the deposits paid. He was thus, in five hours, the winner of 2000 pounds. He was philosophical enough to be contented with his venture, and set off the same evening for the Continent. He was never heard of again.

Such scams were bad for the speculation business and so largely through the pressure of the South Sea directors, the so-called "Bubble Act" was passed on June 11, 1720 requiring all joint-stock companies to have a royal charter. For a moment, the confidence of the people was given an extra boost, and they responded accordingly. South Sea stock had been at 175 pounds at the end of February, 380 at the end of March, and around 520 by May 29. It peaked at the end of June at over 1000 pounds (a psychological barrier in that four-digit number).

With credulity now stretched to the limit and rumors of more and more people (including the directors themselves) selling off, the bubble then burst according to a slow, very slow at first, but steady deflation (not unlike the 60% drop of the Japanese Nikkei index after its all time peak at the end of December 1990). By mid-August, the bankruptcy listings in the London Gazette reached an all-time high, an indication of how people bought on credit or margin. Thousands of fortunes were lost, both large and small. The directors attempted to pump-up more speculation. They failed. The full collapse came by the end of September when the stock stood at 135 pounds. The crash remained in the consciousness of the Western world for the rest of the eighteenth century, not unlike our cultural memory of the 1929 Wall Street Crash.

2.3.3. The Great crash of October 1929

The Roaring 1920s—a time of growth and prosperity on Wall Street and Main Street—ended with the Great Crash of October 1929 (for the most thorough and authoritative account and analysis, see (Galbraith, 1997)). Two thousand investment firms went under, and the American banking industry underwent the biggest structural changes of its history, as a new era of government regulation began. Roosevelt's New Deal politics would follow. The Great Depression that followed put 13 million Americans out of work (that the crash of October 1929 caused the Great Depression is a part of financial folklore, but nevertheless probably not fully accurate. For instance, using a regime switching framework, Coe (2002) finds that a prolonged period of crisis began not with the 1929 stock market crash but with the first banking panic of October 1930).

The October 1929 crash is a remarkable illustration of several remarkable features often associated with crashes. First, stock market crashes are often unforeseen for most people, especially economists. "In a few months, I expect to see the stock market much higher than today". Those words were pronounced by Irving Fisher, America's distinguished and famous economist, Professor of Economics at Yale University, 14 days before Wall Street crashed on Black Tuesday, October 29, 1929.

"A severe depression such as 1920–1921 is outside the range of probability. We are not facing a protracted liquidation". This was the analysis offered days after the crash by the Harvard Economic Society to its subscribers. After continuous and erroneous optimistic forecasts, the Society closed its doors in 1932. Thus, the two most renowned economic forecasting institutes in America at the time failed to predict that a crash and a depression were forthcoming, and continued with their optimistic views, even as the Great Depression took hold of America. The reason is simple: predictions of trend-reversals constitutes by far the most difficult challenge posed to forecasters and is very unreliable especially within the linear framework of standard (auto-regressive) economic models.

A second general feature exemplified by the October 1929 event is that a financial collapse has never happened when things look bad. On the contrary, macroeconomic flows look good before crashes. Before every collapse, economists say the economy is in the best of all worlds. Everything looks rosy, stock markets go up and up, and macroeconomic flows (output, employment, and so on) appear to be improving further and further. This explains why a crash catches most people, especially economists, totally by surprise. The good times are invariably extrapolated linearly into the future. Is it not perceived as senseless by most people in today's euphoria to talk about crash and depression?

During the build-up phase of a bubble such as the one preceding the October 1929 crash, there is a growing interest in the public for the commodity in question, whether it consists in stocks, diamonds or coins. That interest can be estimated through different indicators: increase in the number of books published on the topic (see Fig. 1), and increase in the subscriptions to specialized journals. Moreover, the well-known empirical rule according to which the volume of sales is growing during a bull market finds a natural interpretation: sales increases in fact reveal and pinpoint the progress of the bubble's diffusion throughout society. These features has been recently re-examined for evidence of a bubble, a 'fad' or 'herding' behavior, by studying individual stock returns (White and Rappoport, 1995). One story often advanced for the boom of 1928 and 1929 is that it was driven by the entry into the market of largely uninformed investors, who followed the fortunes of and invested in 'favorite' stocks. The result of this behavior would be a tendency for the favorite stocks' prices to move together more than would be predicted by their shared fundamental economic values. The comovement indeed increased significantly during the boom and was a signal characteristic of the tumultuous market of the early 1930s. These results are thus consistent with the possibility that a fad or crowd psychology played a role in the rise of the market, its crash and subsequent volatility (White and Rappoport, 1995).

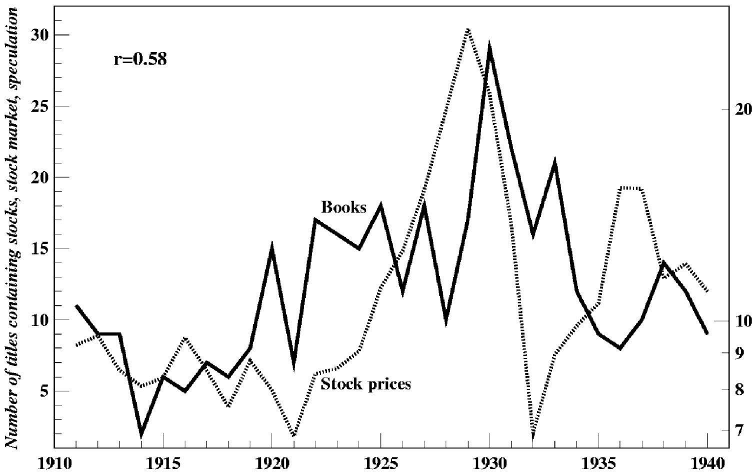

Fig. 1. Comparison between the number of yearly published books about stock market speculation and the level of stock prices (1911–1940). Black line: books at Harvard library whose titles contain one of the words "stocks", "stock market" or "speculation"; grey line: Standard and Poor index of common stocks. The curve of published books lags behind the price curve with a time-lag of about 1.5 years, which can be explained by the time needed for a book to get published. Source: The stock price index is taken from the Historical Abstract of the United States. Reproduced from (Roehner and Sornette, 2000).

The political mood before the October 1929 crash was also optimistic. In November 1928, Herbert Hoover was elected President of the United States in a landslide, and his election set off the greatest increase in stock buying to that date. Less than a year after the election, Wall Street crashed.

2.4. Why? Extreme events in complex systems

Financial markets are not the only systems with extreme events. Financial markets constitute one among many other systems exhibiting a complex organization and dynamics with similar behavior. Systems with a large number of mutually interacting parts, often open to their environment, self-organize their internal structure and their dynamics with novel and sometimes surprising macroscopic ("emergent") properties. The complex system approach, which involves "seeing" interconnections and relationships, i.e., the whole picture as well as the component parts, is nowadays pervasive in modern control of engineering devices and business management. It is also plays an increasing role in most of the scientific disciplines, including biology (biological networks, ecology, evolution, origin of life, immunology, neurobiology, molecular biology, and so on), geology (plate-tectonics, earthquakes and volcanoes, erosion and landscapes, climate and weather, environment, and so on), economy and social sciences (including cognition, distributed learning, interacting agents, and so on). There is a growing recognition that progress in most of these disciplines, in many of the pressing issues for our future welfare as well as for the management of our everyday life, will need such a systemic complex system and multidisciplinary approach. This view tends to replace the previous reductionist approach, consisting of decomposing a system in components, such that the detailed understand of each component was believed to bring understanding in the functioning of the whole.

A central property of a complex system is the possible occurrence of coherent large-scale collective behaviors with a very rich structure, resulting from the repeated nonlinear interactions among its constituents: the whole turns out to be much more than the sum of its parts. A part of the scientific community holds that most complex systems are not amenable to mathematical, analytic descriptions and can only be explored by means of "numerical experiments" (see for instance (Wolfram, 2002) from an extreme implementation of this view and (Kadanoff, 2002) for a enlightening criticism). In the context of the mathematics of algorithmic complexity (Chaitin, 1987), many complex systems are said to be computationally irreducible, i.e. the only way to decide about their evolution is to actually let them evolve in time. Accordingly, the "dynamical" future time evolution of complex systems would be inherently unpredictable. This unpredictability refers to the frustration to satisfy the quest for the knowledge of what tomorrow will be made of, often filled by the vision of "prophets" who have historically inspired or terrified the masses.

The view that complex systems are unpredictable has recently been defended persuasively in concrete prediction applications, such as the socially important issue of earthquake prediction (Geller et al., 1997a, b) (see the contributions in (Nature debates, 1999) for arguments put forward by leading seismologists and geophysicts either defending or fighting this view). In addition to the persistent failures at reaching a reliable earthquake predictive scheme, this view is rooted theoretically in the analogy between earthquakes and self-organized criticality (Bak, 1996). In this "fractal" framework, there is no characteristic scale and the power law distribution of earthquake sizes reflects the fact that the large earthquakes are nothing but small earthquakes that did not stop. They are thus unpredictable because their nucleation is not different from that of the multitude of small earthquakes which obviously cannot be all predicted.

Does this really hold for all features of complex systems? Take our personal life. We are not really interested in knowing in advance at what time we will go to a given store or drive to a highway. We are much more interested in forecasting the major bifurcations ahead of us, involving the few important things, like health, love and work that count for our happiness. Similarly, predicting the detailed evolution of complex systems has no real value and the fact that we are taught that it is out of reach from a fundamental point of view does not exclude the more interesting possibility of predicting phases of evolutions of complex systems that really count, like the extreme events.

It turns out that most complex systems in natural and social sciences do exhibit rare and sudden transitions, that occur over time intervals that are short compared to the characteristic time scales of their posterior evolution. Such extreme events express more than anything else the underlying "forces" usually hidden by almost perfect balance and thus provide the potential for a better scientific understanding of complex systems.

These crises have fundamental societal impacts and range from large natural catastrophes such as earthquakes, volcanic eruptions, hurricanes and tornadoes, landslides, avalanches, lightning strikes, meteorite/asteroid impacts, catastrophic events of environmental degradation, to the failure of engineering structures, crashes in the stock market, social unrest leading to large-scale strikes and upheaval, economic drawdowns on national and global scales, regional power blackouts, traffic gridlock, diseases and epidemics, and so on. It is essential to realize that the long-term behavior of these complex systems is often controlled in large part by these rare catastrophic events: the universe was probably born during an extreme explosion (the "big-bang"); the nucleosynthesis of all important heavy atomic elements constituting our matter results from the colossal explosion of supernovae (these stars more heavy than our sun whose internal nuclear combustion diverges at the end of their life); the largest earthquake in California repeating about once every two centuries accounts for a significant fraction of the total tectonic deformation; landscapes are more shaped by the "millennium" flood that moves large boulders rather than the action of all other eroding agents; the largest volcanic eruptions lead to major topographic changes as well as severe climatic disruptions; according to some contemporary views, evolution is probably characterized by phases of quasi-stasis interrupted by episodic bursts of activity and destruction (Gould and Eldredge, 1993); financial crashes, which can destroy in an instant trillions of dollars, loom over and shape the psychological state of investors; political crises and revolutions shape the long-term geopolitical landscape; even our personal life is shaped on the long run by a few key decisions or happenings.

The outstanding scientific question is thus how such large-scale patterns of catastrophic nature might evolve from a series of interactions on the smallest and increasingly larger scales. In complex systems, it has been found that the organization of spatial and temporal correlations do not stem, in general, from a nucleation phase diffusing across the system. It results rather from a progressive and more global cooperative process occurring over the whole system by repetitive interactions. For instance, scientific and technical discoveries are often quasi-simultaneous in several laboratories in different parts of the world, signaling the global nature of the maturing process.

Standard models and simulations of scenarios of extreme events are subject to numerous sources of error, each of which may have a negative impact on the validity of the predictions (Karplus, 1992). Some of the uncertainties are under control in the modeling process; they usually involve trade-offs between a more faithful description and manageable calculations. Other sources of errors are beyond control as they are inherent in the modeling methodology of the specific disciplines. The two known strategies for modeling are both limited in this respect: analytical theoretical predictions are still out of reach for many complex problems even if notable counter-examples exist (see for instance (Barra et al., 2002; Arad et al., 2001; Falkovich et al., 2001)). Brute force numerical resolution of the equations (when they are known) or of scenarios is reliable in the "center of the distribution", i.e., in the regime far from the extremes where good statistics can be accumulated. Crises are extreme events that occur rarely, albeit with extraordinary impact, and are thus completely under-sampled and thus poorly constrained. Even the introduction of teraflop (or even petaflops in the future) supercomputers does not change qualitatively this fundamental limitation.

Notwithstanding these limitations, we believe that the progress of science and of its multidisciplinary enterprises make the time ripe for a full-fledge effort towards the prediction of complex systems. In particular, novel approaches are possible for modeling and predicting certain catastrophic events, or "ruptures", that is, sudden transitions from a quiescent state to a crisis or catastrophic event (Sornette, 1999). Such ruptures involve interactions between structures at many different scales. In the present review, we apply these ideas to one of the most dramatic events in social sciences, financial crashes. The approach described here combines ideas and tools from mathematics, physics, engineering and social sciences to identify and classify possible universal structures that occur at different scales, and to develop application-specific methodologies to use these structures for prediction of the financial "crises". Of special interest will be the study of the premonitory processes before financial crashes or "bubble" corrections in the stock market.

For this, we will describe a new set of computational methods which are capable of searching and comparing patterns, simultaneously and iteratively, at multiple scales in hierarchical systems.

We will use these patterns to improve the understanding of the dynamical state before and after a financial crash and to enhance the statistical modeling of social hierarchical systems with the goal of developing reliable forecasting skills for these large-scale financial crashes.

2.5. When? Is prediction possible? A working hypothesis

Our hypothesis is that stock market crashes are caused by the slow buildup of long-range correlations leading to a global cooperative behavior of the market eventually ending into a collapse in a short critical time interval. The use of the word "critical" is not purely literary here: in mathematical terms, complex dynamical systems can go through "critical" points, defined as the explosion to infinity of a normally well-behaved quantity. As a matter of fact, as far as nonlinear dynamical systems go, the existence of critical points is more the rule than the exception. Given the puzzling and violent nature of stock market crashes, it is worth investigating whether there could possibly be a link between stock market crashes and critical points.

- Our key assumption is that a crash may be caused by *local* self-reinforcing imitation between traders. This self-reinforcing imitation process leads to the blossoming of a bubble. If the tendency for traders to "imitate" their "friends" increases up to a certain point called the "critical" point, many traders may place the same order (sell) at the same time, thus causing a crash. The interplay between the progressive strengthening of imitation and the ubiquity of noise requires a probabilistic description: a crash is *not* a certain outcome of the bubble but can be characterised by its hazard rate, i.e., the probability per unit time that the crash will happen in the next instant provided it has not happened yet.
- Since the crash is not a certain deterministic outcome of the bubble, it remains rational for investors to remain in the market provided they are compensated by a higher rate of growth of the bubble for taking the risk of a crash, because there is a finite probability of "landing smoothly", i.e., of attaining the end of the bubble without crash.

In a series of research articles, we have shown extensive evidence that the build-up of bubbles manifests itself as an over-all power law acceleration in the price decorated by "log-periodic" precursors, a concept related to fractals as will be become clear later. This article is to tell this story, to explain why and how these precursors occur, what do they mean? What do they imply with respect to prediction?

We claim that there is a degree of predictive skill associated with these patterns. This has already been used in practice and is investigated by our co-workers and us as well as several others, academics and most-of-all practitioners (see Sornette and Johansen, 2001, and Johansen and Sornette, 2002, for a recent review and assessment and Zhou and Sornette, 2002a, b, c for nonparametric tests using a generalization of the so-called q-derivative).

The evidence we shall discuss include:

- the Wall street October 1929, the World October 1987, the Hong-Kong October 1987, the World August 1998, the Nasdaq April 2000 crashes,
- the 1985 foreign exchange event on the U.S. dollar, the correction of the U.S. dollar against the Canadian dollar and the Japanese Yen starting in August 1998,

- the bubble on the Russian market and its ensuing collapse in 1997–1998,
- twenty-two significant bubbles followed by large crashes or by severe corrections in the Argentinian, Brazilian, Chilean, Mexican, Peruvian, Venezuelan, Hong-Kong, Indonesian, Korean, Malaysian, Philippine and Thai stock markets.

In all these cases, it has been found that log-periodic power laws adequately describe speculative bubbles on the western as well as on the emerging markets with very few exceptions.

Notwithstanding the drastic differences in epochs and contexts, we shall show that these financial crashes share a common underlying background as well as structure. The rationale for this rather surprising result is probably rooted in the fact that humans are endowed with basically the same emotional and rational qualities in the 21st century as they were in the 17th century (or at any other epoch). Humans are still essentially driven by at least a grain of greed and fear in their quest for a better well-being. The "universal" structures we are going to uncover may be understood as the robust emergent properties of the market resulting from some characteristic "rules" of interaction between investors. These interactions can change in details due, for instance, to computers and electronic communications. They have not changed at a qualitative level. As we shall see, complex system theory allows us to account for this robustness.

3. Financial crashes are "outliers"

In the spirit of Bacon in Novum Organum about 400 years ago, "Errors of Nature, Sports and Monsters correct the understanding in regard to ordinary things, and reveal general forms. For whoever knows the ways of Nature will more easily notice her deviations; and, on the other hand, whoever knows her deviations will more accurately describe her ways", we document in this section the evidences showing that large market drops are "outliers" and that they reveal fundamental properties of the stock market.

3.1. What are "abnormal" returns?

Stock markets can exhibit very large motions, such as rallies and crashes. Should we expect these extreme variations? Or should we consider them as anomalous?

Fig. 2 shows the distribution of daily returns of the DJIA and of the Nasdaq index for the period January 2nd, 1990 till September 29, 2000. For instance, we read in Fig. 2 that five negative and five positive daily DJIA market returns larger or equal to 4% have occurred. In comparison, 15 negative and 20 positive returns larger or equal to 4% have occurred for the Nasdaq index. The larger fluctuations of returns of the Nasdaq compared to the DJIA are also quantified by the so-called volatility (standard deviation of returns), equal to 1.6% (respectively, 1.4%) for positive (respectively, negative) returns of the DJIA, and equal to 2.5% (respectively, 2.0%) for positive (respectively, negative) returns of the Nasdaq index. The lines shown in Fig. 2 correspond to represent the data by an exponential function. The upward convexity of the trajectories defined by the symbols for the Nasdaq qualifies a stretched exponential model (Laherrère and Sornette, 1998) which embodies the fact that the tail of the distribution is "fatter", i.e., there are larger risks of large drops (as well as ups) in the Nasdaq compared to the DJIA.

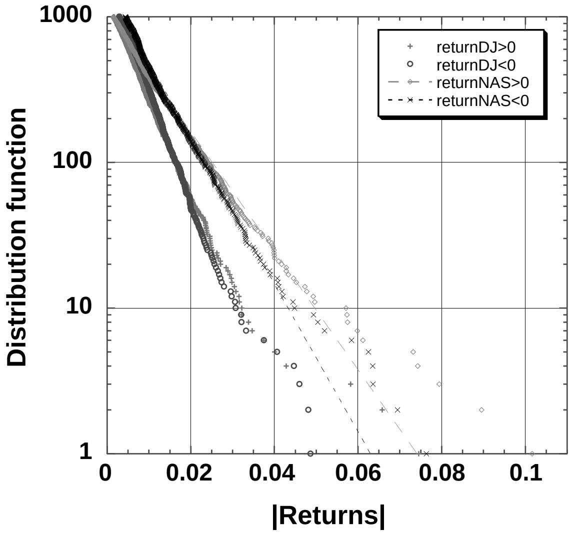

Fig. 2. Distribution of daily returns for the DJIA and the Nasdaq index for the period January 2, 1990 till September 29, 2000. The lines correspond to fits of the data by an exponential law. The branches of negative returns have been folded back onto the positive returns for comparison.

Let us use the exponential model and calculate the probability to observe a return amplitude larger than, say, 10 standard deviations (10% in our example). The result is 0.000045, which corresponds to 1 event in 22,026 days, or in 88 years. The drop of 22.6% of October 19, 1987 would correspond to one event in 520 million years, which qualifies it as an "outlier". Thus, according to the exponential model, a 10% return amplitude does not qualify as an "outlier", in a clear-cut and undisputable manner. In addition, the discrimination between normal and abnormal returns depends on our choice for the frequency distribution. Qualifying what is the correct description of the frequency distribution, especially for large positive and negative returns, is a delicate problem that is still a hot domain for research. Due to the lack of certainty on the best choice for the frequency distribution, this approach does not seem the most adequate for characterizing anomalous events. We now introduce another diagnostic that allows us to characterize abnormal market phases in a much more precise and nonparametric way, i.e., without referring to a specific mathematical representation of the frequency distribution.

3.2. Drawdowns (runs)

Extreme value theory (EVT) provides an alternative approach, still based on the distribution of returns estimated at a fixed time scale. Its most practical implementation is based on the so-called "peak-over-threshold" distributions (Embrechts et al., 1997; Bassi et al., 1998), which is founded on a limit theorem known as the Gnedenko–Pickands–Balkema–de Haan theorem which gives a natural limit law for peak-over-threshold values in the form of the Generalized Pareto Distribution (GPD), a family of distributions with two parameters based on the Gumbel, Weibull and Frechet extreme value distributions. The GPD is either an exponential or has a power law tail. Peak-over-threshold distributions put the emphasis on the characterization of the tails of distribution of returns and have thus been scrutinized for their potential for risk assessment and management of large and extreme events (see for instance, Phoa, 1999; McNeil, 1999). In particular, extreme value theory provides a general foundation for the estimation of the value-at-risk for very low-probability "extreme" events. There are however severe pitfalls (Diebold et al., 2001) in the use of extreme value distributions for risk management because of its reliance on the (unstable) estimation of tail probabilities. In addition, the EVT literature assumes independent returns, which implies that the degree of fatness in the tails decreases as the holding horizon lengthens (for the values of the exponents found empirically). Here, we show that this is not the case: returns exhibit strong correlations at special times precisely characterized by the occurrence of extreme events, the regime that EVT aims to describe. This suggests to re-examine EVT and extend it to variable time scales, for instance by analyzing the EVT of the distribution of drawdowns and drawups.

A drawdown is defined as a persistent decrease in the price over consecutive days. A drawdown is thus the cumulative loss from the last maximum to the next minimum of the price. Drawdowns embody a rather subtle dependence since they are constructed from runs of the same sign variations. Their distribution thus captures the way successive drops can influence each other and construct in this way a persistent process. This persistence is not measured by the distribution of returns because, by its very definition, it forgets about the relative positions of the returns as they unravel themselves as a function of time by only counting their frequency. This is not detected either by the two-point correlation function, which measures an *average* linear dependence over the whole time series, while the dependence may only appear at special times, for instance for very large runs, a feature that will be washed out by the global averaging procedure.

To demonstrate the information contained in drawdowns and contrast it with the fixed time-scale returns, let us consider the hypothetical situation of a crash of 30% occurring over three days with three successive losses of exactly 10%. The crash is thus defined as the total loss or drawdown of 30%. Rather than looking at drawdowns, let us now follow the common approach and examine the daily data, in particular the daily distribution of returns. The 30% drawdown is now seen as three daily losses of 10%. The essential point to realize is that the construction of the distribution of returns amounts to count the number of days over which a given return has been observed. The crash will thus contribute three days of 10% loss, *without* the information that the three losses occurred sequentially! To see what this loss of information entails, we consider a market in which a 10% daily loss occurs typically once every 4 years (this is not an unreasonable number for the Nasdaq composite index at present times of high volatility). Counting approximately 250 trading days per year, 4 years correspond to 1000 trading days and 1 event in 1000 days thus corresponds to a probability $1/1000 = 0.001$ for a daily loss of 10%. The crash of 30% has been dissected as three events which are not very remarkable (each with a relatively short average recurrence time of four years). The plot thickens when we ask what is, according to this description, the probability for three successive daily losses of 10%? Elementary probability tells us that it is the probability of one daily loss of 10% times the probability of one daily loss of 10% times the probability of one daily loss of 10%, giving 10^{-9}. This corresponds to a 1 event in 1 billion trading days! We should thus wait typically 4 millions years to witness such an event!

What has gone wrong? Simply, looking at daily returns and at their distributions has destroyed the information that the daily returns may be correlated, at special times! This crash is like a mammoth which has been dissected in pieces without memory of the connection between the parts and we are left with what look as mouses (bear with the slight exaggeration)! Our estimation that three successive losses of 10% are utterly impossible relied on the incorrect hypothesis that these three events are independent. Independence between successive returns is remarkably well-verified most of the time. However, it may be that large drops may not be independent. In other words, there may be "burst of dependence", i.e., "pockets of predictability".

It is clear that drawdowns will keep precisely the information relevant to identify the possible burst of local dependence leading to possibly extraordinary large cumulative losses. Our emphasis on drawdowns is thus motivated by two considerations: (1) drawdowns are important measures of risks used by practitioners because they represent their cumulative loss since the last estimation of their wealth. It is indeed a common psychological trait of people to estimate a loss by comparison with the latest maximum wealth; (2) drawdowns automatically capture an important part of the time dependence of price returns, similarly to the run-statistics often used in statistical testing (Knuth, 1969) and econometrics (Campbell et al., 1997; Barber and Lyon, 1997). As previously showed (Johansen and Sornette, 1998, 2002), the distribution of drawdowns contains an information which is quite different from the distribution of returns over a fixed time scale. In particular, a drawndown embodies the interplay between a series of losses and hence measures a "memory" of the market. Drawdowns exemplify the effect of correlations in price variations when they appear, which must be taken into account for a correct characterisation of market price variations. They are direct measures of a possible amplification or "flight of fear" where previous losses lead to further selling, strengthening the downward trend, occasionally ending in a crash. We stress that drawdowns, by the "elastic" time-scale used to define them, are effectively function of several higher order correlations at the same time.

Johansen and Sornette (2002) have shown that the distribution of drawdowns for independent price increments x is asymptotically an exponential (while the body of the distribution is Gaussian (Mood, 1940)), when the distribution of x does not decay more slowly than an exponential, i.e., belong to the class of exponential or super-exponential distributions. In contrast, for sub-exponentials (such as stable Lévy laws, power laws and stretched exponentials), the tail of the distribution of drawdowns is asymptotically the same as the distribution of the individual price variations. Since stretched exponentials have been found to offer an accurate quantification of price variations (Laherrère and Sornette, 1998; Sornette et al., 2000a, b; Andersen and Sornette, 2001) thus capturing a possible sub-exponential behavior and since they contain the exponential law as a special case, the stretched exponential law is a good null hypothesis.

The cumulative stretched distribution is defined by

$$N_c(x) = A \exp(-(|x|/\chi)^z) , \quad (1)$$

where x is either a drawdown or a drawup. When $z < 1$ (resp. $z > 1$), $N_c(x)$ is a stretched exponential or sub-exponential (resp. super-exponential). The special case $z = 1$ corresponds to a pure exponential. In this case, χ is nothing but the standard deviation of $|x|$.

Johansen and Sornette (2002) have analyzed the major financial indices, the major currencies, gold, the twenty largest U.S. companies in terms of capitalisation as well as nine others chosen randomly. They find that approximately 98% of the distributions of drawdowns is well-represented by an exponential or a stretched exponential, while the largest to the few ten largest drawdowns are occurring with a significantly larger rate than predicted by the exponential. This is confirmed by extensive testing on surrogate data. Very large drawdowns thus belong to a different class of their own and call for a specific amplification mechanism. Drawups (gain from the last local minimum to the next local maximum) exhibit a similar behavior in only about half the markets examined. We now present some of the most significant results.

3.3. Testing outliers

Testing for "outliers" or more generally for a change of population in a distribution is a subtle problem: the evidence for outliers and extreme events does not require and is not even synonymous in general with the existence of a break in the distribution of the drawdowns. Let us illustrate this pictorially by borrowing from another domain of active scientific investigation, namely the search for the understanding of the complexity of eddies and vortices in turbulent hydrodynamic flows, such as in mountain rivers or in the weather. Since solving the exact equations of these flows does not provide much insight as the results are forbidding, a useful line of attack has been to simplify the problem by studying simple toy models, such as "shell" models of turbulence, that are believed to capture the essential ingredient of these flows, while being amenable to analysis. Such "shell" models replace the three-dimensional spatial domain by a series of uniform onion-like spherical layers with radii increasing as a geometrical series $1, 2, 4, 8, \ldots, 2^n$ and communicating with each other typically with nearest and next-nearest neighbors.

As for financial returns, a quantity of great interest is the distribution of velocity variations between two instants at the same position or between two points simultaneously. Such a distribution for the square of the velocity variations has been calculated (L'vov et al., 2001) and exhibits an approximate exponential drop-off as well as a co-existence with larger fluctuations, quite reminiscent of our findings in finance (Johansen and Sornette, 1998, 2002). Usually, such large fluctuations are not considered to be statistically significant and do not provide any specific insight. Here, it turns out that it can be shown that these large fluctuations of the fluid velocity correspond to intensive peaks propagating coherently over several shell layers with a characteristic bell-like shape, approximately independent of their amplitude and duration (up to a re-scaling of their size and duration). When extending these observations to very long times so that the anomalous fluctuations can be sampled much better, one gets a continuous distribution (L'vov et al., 2001). Naively, one would expect that the same physics apply in each shell layer (each scale) and, as a consequence, the distributions in each shell should be the same, up to a change of unit reflecting the different scale embodied by each layer. It turns out that the three curves for three different shells can indeed by nicely collapsed, but only for the small velocity fluctuations, while the large fluctuations are described by very different heavy tails. Alternatively, when one tries to collapse the curves in the region of the large velocity fluctuations, then the portions of the curves close to the origin are not collapsed at all and are very different. The remarkable conclusion is that the distributions of velocity increment seem to be composed of two regions, a region of "normal scaling" and a domain of extreme events. The theoretical analysis of L'vov et al. (2001) further substantiate the fact that the largest fluctuations result from a different mechanism.

Here is the message that comes out of this discussion: the concept of outliers and of extreme events does not rest on the requirement that the distribution should not be smooth. Noise and the very process of constructing the distribution will almost always smooth out the curves. What is found by L'vov et al. (2001) is that the distribution is made of two different populations, the body and the tail, which have different physics, different scaling and different properties. This is a clear demonstration that this model of turbulence exhibits outliers in the sense that there is a well-defined population of very large and quite rare events that punctuate the dynamics and which cannot be seen as scale-up versions of the small fluctuations.

As a consequence, the fact that the distribution of small events might show up some curvature or continuous behavior does not tell anything against the outlier hypothesis. It is essential to keep this point in mind when looking at the evidence presented below for the drawdowns.

Other groups have recently presented supporting evidence that crash and rally days significantly differ in their statistical properties from the typical market days. For instance, Lillo and Mantegna investigated the return distributions of an ensemble of stocks simultaneously traded in the New York Stock Exchange (NYSE) during market days of extreme crash or rally in the period from January 1987 to December 1998. Out of two hundred distributions of returns, one for each of two hundred trading days where the ensemble of returns is constructed over the whole set of stocks traded on the NYSE, anomalous large widths and fat tails are observed specifically on the day of the crash of October 19, 1987, as well as during a few other turbulent days. Lillo and Mantegna document another remarkable behavior associated with crashes and rallies, namely that the distortion of the distributions of returns are not only strong in the tails describing large moves but also in their center. Specifically, they show that the overall shape of the distributions is modified in crash and rally days. Closer to our claim that markets develop precursory signatures of bubbles of long time scales, Mansilla has also shown, using a measure of relative complexity, that time sequences corresponding to "critical" periods before large market corrections or crashes have more novel informations with respect to the whole price time series than those sequences corresponding to periods where nothing happened. The conclusion is that, in the intervals where no financial turbulence is observed, that is, where the markets works fine, the informational contents of the (binary-coded) price time series is small. In contrast, there seems to be significant information in the price time series associated with bubbles. This finding is consistent with the appearance of a collective herding behavior modifying the texture of the price time series compared to normal times.

3.4. The Dow Jones industrial average

Fig. 3 shows the distribution of drawdowns and of drawups for the returns of the DJIA over this century.

The (stretched) exponential distribution has been derived on the assumption that successive price variations are independent. There is a large body of evidence for the correctness of this assumption for most trading days (Campbell et al., 1997). However, consider, for instance, the 14 largest drawdowns that have occurred in the Dow Jones Industrial Average in this century. Their characteristics are presented in Table 1. Only 3 lasted one or two days, whereas 9 lasted four days or more. Let us examine in particular the largest drawdown. It started on October 14, 1987 (1987.786 in decimal years), lasted four days and led to a total loss of −30.7%. This crash is thus a run of four consecutive losses: first day the index is down by 3.8%, second day by 6.1%, third day by 10.4% and fourth by 30.7%. In terms of consecutive losses this correspond to 3.8%, 2.4%, 4.6% and with 22.6% on what is known as the Black Monday of October 1987.

The observation of large successive drops is suggestive of the existence of a transient correlation as we already pointed out. For the Dow Jones, this reasoning can be adapted as follows. We use a simple functional form for the distribution of daily losses, namely an exponential distribution with

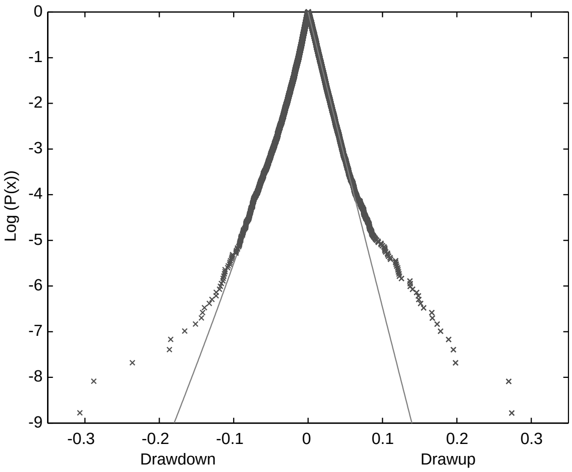

Fig. 3. Normalized natural logarithm of the cumulative distribution of drawdowns and of the complementary cumulative distribution of drawups for the Dow Jones Industrial Average index (U.S. stock market). The two continuous lines show the fits of these two distributions with the stretched exponential distribution. Negative values such as −0.20 and −0.10 correspond to drawdowns of amplitude respectively equal to 20% and 10%. Similarly, positive values corresponds to drawups with, for instance, a number 0.2 meaning a drawup of +20%. Reproduced from Johansen and Sornette (2001c).

Table 1
Characteristics of the 14 largest drawdowns of the Dow Jones Industrial Average in this century

Rank	Starting time	Index value	Duration (days)	Loss (%)
1	1987.786	2508.16	4	−30.7
2	1914.579	76.7	2	−28.8
3	1929.818	301.22	3	−23.6
4	1933.549	108.67	4	−18.6
5	1932.249	77.15	8	−18.5
6	1929.852	238.19	4	−16.6
7	1929.835	273.51	2	−16.6
8	1932.630	67.5	1	−14.8
9	1931.93	90.14	7	−14.3
10	1932.694	76.54	3	−13.9
11	1974.719	674.05	11	−13.3
12	1930.444	239.69	4	−12.4
13	1931.735	109.86	5	−12.9
14	1998.649	8602.65	4	−12.4

The starting dates are given in decimal years. Reproduced from (Johansen and Sornette, 2001c).

decay rate 1/0.63% obtained by a fit to the distribution of drawdowns shown in Fig. 3. The quality of the exponential model is confirmed by the direct calculation of the average loss amplitude equal to 0.67% and of its standard deviation equal to 0.61% (recall that an exact exponential would give the three values exactly equal: 1/decay = average = standard deviation). Using these numerical values, the probability for a drop equal to or larger than 3.8% is $\exp(-3.8/0.63) = 2.4 \times 10^{-3}$ (an event

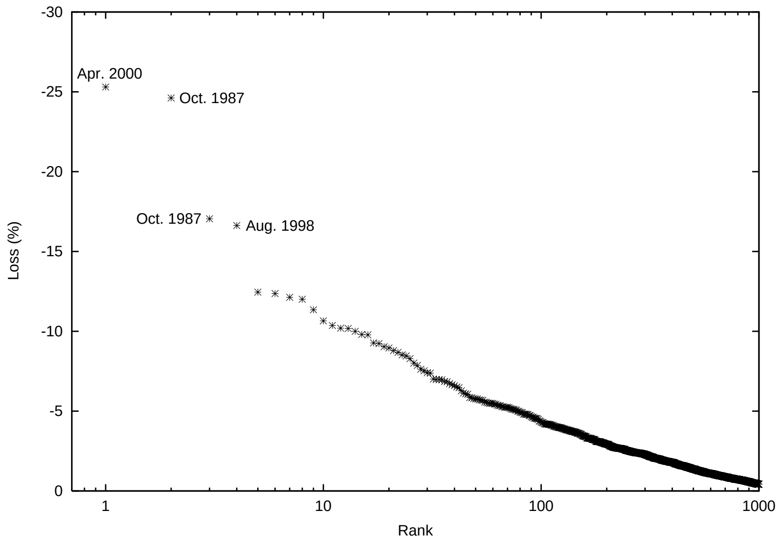

Fig. 4. Rank ordering of drawdowns in the Nasdaq Composite since its establishment in 1971 until 18 April 2000. Rank 1 is the largest drawdown. Rank 2 is the second largest, and so on. Reproduced from Johansen and Sornette (2000a).

occurring about once every two years); the probability for a drop equal to or larger than 2.4% is $\exp(-2.4/0.63) = 2.2 \times 10^{-2}$ (an event occurring about once every two months); the probability for a drop equal to or larger than 4.6% is $\exp(-4.6/0.63) = 6.7 \times 10^{-4}$ (an event occurring about once every six years); the probability for a drop equal to or larger than 22.6% is $\exp(-22.6/0.63) = 2.6 \times 10^{-16}$ (an event occurring about once every 10^{14} years). All together, under the hypothesis that daily losses are uncorrelated from one day to the next, the sequence of four drops making the largest drawdown occurs with a probability 10^{-23}, i.e., once in about 4 thousands of billions of billions years. This exceedingly negligible value 10^{-23} suggests that the hypothesis of uncorrelated daily returns is to be rejected: drawdowns and especially the large ones may exhibit intermittent correlations in the asset price time series.

3.5. The Nasdaq composite index

In Fig. 4, we see the rank ordering plot of drawdowns for the Nasdaq composite index, since its establishment in 1971 until 18 April 2000. The rank ordering plot, which is the same as the (complementary) cumulative distribution with axis interchanged, puts emphasis on the largest events. The four largest events are not situated on a continuation of the distribution of smaller events: the jump between ranks 4 and 5 in relative value is larger than 33% whereas the corresponding jump between ranks 5 and 6 is less than 1% and this remains true for higher ranks. This means that, for drawdowns less than 12.5%, we have a more or less "smooth" curve and then a larger than 33% gap to ranks 3 and 4. The four events are according to rank the crash of April 2000, the crash of October 1987, a larger than 17% "after-shock" related to the crash of October 1987 and a larger than 16% drop related to the "slow crash" of August 1998, that we shall discuss later on.

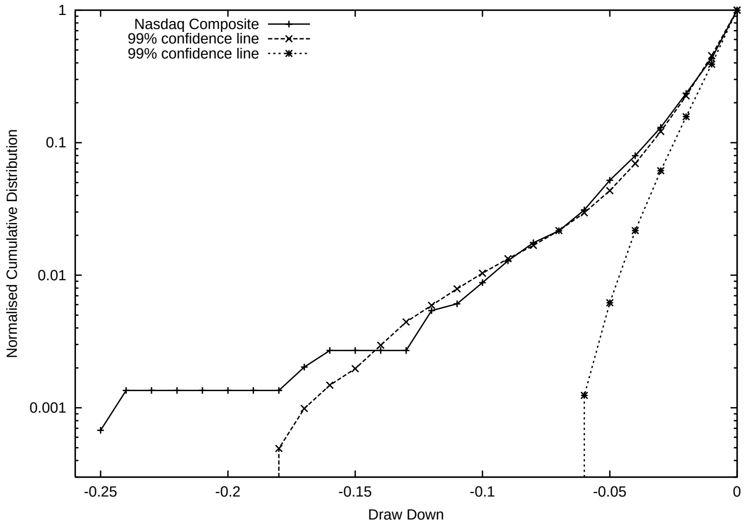

Fig. 5. Normalized cumulative distribution of drawdowns in the Nasdaq Composite since its establishment in 1971 until 18 April 2000. The 99% confidence lines are estimated from synthetic tests obtained by generating surrogate financial time series constructed by reshuffling the daily returns at random. Reproduced from Johansen and Sornette (2000a).

To further establish the statistical confidence with which we can conclude that the four largest events are outliers, the daily returns have been reshuffled 1000 times generating 1000 synthetic data sets. This procedure means that the synthetic data sets will have exactly the same distribution of daily returns. However, higher order correlations and dependence that may be present in the largest drawdowns are destroyed by the reshuffling. This "surrogate" data analysis of the distribution of drawdowns has the advantage of being *nonparametric*, i.e., independent of the quality of fits with a model such as the exponential or any other model. We will now compare the distribution of drawdowns both for the real data and the synthetic data. With respect to the synthetic data, this can be done in two complementary ways.

In Fig. 5, we see the distribution of drawdowns in the Nasdaq Composite compared with the two lines constructed at the 99% confidence level for the entire *ensemble* of synthetic drawdowns, i.e. by considering the individual drawdowns as independent: for any given drawdown, the upper (resp. lower) confidence line is such that 5 of the synthetic distributions are above (below) it; as a consequence, 990 synthetic times series out of the 1000 are within the two confidence lines for any drawdown value which define the typical interval within which we expect to find the empirical distribution.

The most striking feature apparent in Fig. 5 is that the distribution of the true data breaks away from the 99% confidence intervals at approximately 15%, showing that the four largest events are indeed "outliers". In other words, chance alone cannot reproduce these largest drawdowns. We are thus forced to explore the possibility that an amplification mechanism and dependence across daily returns might appear at special and rare times to create these outliers.

A more sophisticated analysis is to consider each synthetic data set *separately* and calculate the *conditional probability* of observing a given drawdown given some prior observation of drawdowns.

This gives a more precise estimation of the statistical significance of the outliers, because the previously defined confidence lines neglect the correlations created by the ordering process which is explicit in the construction of a cumulative distribution.

Out of 10,000 synthetic data sets that were generated, we find that 776 had a single drawdown larger than 16.5%, 13 had two drawdowns larger than 16.5%, 1 had three drawdowns larger than 16.5% and none had 4 (or more) drawdowns larger than 16.5% as in the real data. This means that, given the distribution of returns, by chance we have a 8% probability of observing a drawdowns larger than 16.5%, a 0.1% probability of observing two drawdowns larger than 16.5% and for all practical purposes zero probability of observing three or more drawdowns larger than 16.5%. Hence, we can reject the hypothesis that the four largest drawdowns observed on the Nasdaq composite index could result from chance alone with a probability or confidence better than 99.99%, i.e., essentially with certainty. As a consequence, we are lead again to conclude that the largest market events are characterised by a stronger dependence than is observed during "normal" times.

This analysis confirms the conclusion from the analysis of the DJIA shown in Fig. 3, that drawdowns larger than about 15% are to be considered as outliers with high probability. It is interesting that the same amplitude of approximately 15% is found for both markets considering the much larger daily volatility of the Nasdaq Composite. This may result from the fact that, as we have shown, very large drawdowns are more controlled by transient correlations leading to runs of losses lasting a few days than by the amplitude of a single daily return.

The statistical analysis of the Dow Jones average and the Nasdaq composite suggests that large crashes *are* special. In following sections, we shall show that there are other specific indications associated with these "outliers", such as precursory patterns decorating the speculative bubbles ending in crashes.

3.6. The presence of "Outliers" is a general phenomenon

To avoid a tedious repetition of many figures, we group the cumulative distributions of drawdowns and complementary cumulative distributions of several stocks in the same Fig. 6. In order to construct this figure, we have fitted the stretched exponential model (1) to each distribution and obtained the corresponding parameters A, χ and z given in Johansen and Sornette (2001c). We then construct the normalized distributions

$$N_C^{(n)}(x) = N_c((|x|/\chi)^z)/A \ , \tag{2}$$

using the triplet A, χ and z which is specific to each distribution. Fig. 6 plots the expression (2) for each distribution, i.e., N_c/A as a function of $y \equiv \text{sign}(x)(x/\chi)^z$. If the stretched exponential model (1) held true for all the drawdowns and all the drawups, all the normalized distributions should collapse exactly onto the "universal" functions e^y for the drawdowns and e^{-y} for the drawups. We observe that this is the case for values of $|y|$ up to about 5, i.e., up to typically 5 standard deviations (since most exponents z are close to 1), beyond which there is a clear upward departure observed both for drawdowns and for drawups. Comparing with the extrapolation of the normalized stretched exponential model $e^{-|y|}$, the empirical normalized distributions give about 10 times too many drawdowns and drawups larger than $|y| = 10$ standard deviations and more the 10^4 too many drawdowns and drawups larger than $|y| = 20$ standard deviations. Note that for AT& T, a crash of $\approx 73\%$ occurred which lies beyond the range shown in Fig. 6.

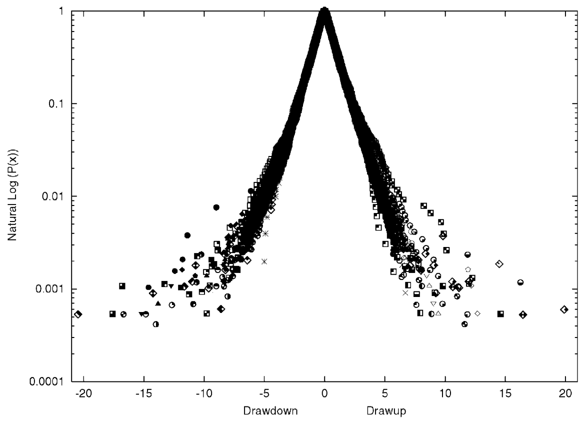

Fig. 6. Cumulative distribution of drawdowns and complementary cumulative distribution of drawups for 29 companies, which include the 20 largest USA companies in terms of capitalisation according to Forbes at the beginning of the year 2000, and in addition Coca Cola (Forbes number 25), Qualcomm (number 30), Appl. Materials (number 35), Procter & Gamble (number 38), JDS Uniphase (number 39), General Motors (number 43), Am. Home Prod. (number 46), Medtronic (number 50) and Ford (number 64). This figure plots each distribution N_c normalized by its corresponding factor A as a function of the variable $y \equiv \text{sign}(x)(|x|/\chi)^z$, where χ and z are specific to each distribution and obtained from the fit to the stretched exponential model. Reproduced from Johansen and Sornette (2001c).

The results obtained in Johansen and Sornette (1998, 2000a, 2001c, 2002) can be summarized as follows:

1. Approximately 1–2% of the largest drawdowns are not at all explained by the exponential null-hypothesis or its extension in terms of the stretched exponential (1). Large drawdowns up to three times larger than expected from the null-hypothesis are found to be ubiquitous occurrences of essentially all the times series that we have investigated, the only noticeable exception being the French index CAC40. We term "outliers" these anomalous drawdowns.
2. About half of the time series show outliers for the drawups. The drawups are thus different statistically from the drawdowns and constitute a less conspicuous structure of financial markets.
3. For companies, large drawups of more than 15% occur approximately twice as often as large drawdowns of similar amplitudes.
4. The bulk (98%) of the drawdowns and drawups are very well-fitted by the exponential null-hypothesis (based on the assumption of independent price variations) or by the stretched exponential model.

The most important result is the demonstration that the very largest drawdowns are outliers. This is true notwithstanding the fact that the very largest daily drops are *not* outliers, except for the exceptional and unique daily drop on October 29, 1987. Therefore, the anomalously large amplitude of the drawdowns can only be explained by invoking the emergence of rare but sudden persistences of successive daily drops, with in addition correlated amplification of the drops. Why such successions of correlated daily moves occur is a very important question with consequences for portfolio management and systemic risk, to cite only two applications, that we are going to investigate in the following sections.

3.7. Implications for safety regulations of stock markets

The realization that large drawdowns and crashes in particular may result from a run of losses over several successive days is not without consequences for the regulation of stock markets. Following the market crash of October 1987, in an attempt to head off future one-day stock market tumbles of historic proportions, the Securities and Exchange Commission and the three major U.S. stock exchanges agreed to install the so-called circuit breakers. Circuit breakers are designed to gradually inhibit trading during market declines, first curbing New York Stock Exchange program trades and eventually halting all U.S. equity, options and futures activity. Similar circuit breakers are operating in the other world stock markets with different specific definitions.

The argument is that the halt triggered by a circuit breaker will provide time for brokers and dealers to contact their clients when there are large price movements and to get new instructions or additional margin. They also limit credit risk and loss of financial confidence by providing a "time-out" to settle up and to ensure that everyone is solvent. This inactive period is of further use for investors to pause, evaluate and inhibit panic. Finally, circuit breakers clarify the illusion of market liquidity by spelling out the economic fact of life that markets have limited capacity to absorb massive unbalanced volumes. They thus force large investors, such as pension portfolio managers and mutual funds, to take even more account of the impact of their "size order", thus possibly cushioning large market movements. Others argue that a trading halt can increase risk by inducing trading in anticipation of a trading halt. Another disadvantage is that they prevent some traders from liquidating their positions, thus creating market distorsion by preventing price discovery (Harris, 1997).

For the October 1987 crash, countries that had stringent circuit breakers, such as France, Switzerland and Israel, had also some of the largest cumulative losses. According to the evidence presented here that large drops are created by transient and rare dependent losses occurring over several days, we should be cautious in considering circuit breakers as reliable crash killers.

4. Positive feedbacks

Since it is the actions of investors whose buy and sell decisions move prices up and down, any deviation from a random walk in the stock market price trajectory has ultimately to be traced back to the behavior of investors. We are in particular interested in mechanisms that may lead to positive feedbacks on prices, i.e., to the fact that, conditioned on the observation that the market has recently moved up (respectively down), this makes it more probable to keep it moving up (respectively down), so that a large cumulative move ensues. The concept of "positive feedbacks" has a long history in economics and is related to the idea of "increasing returns"—which says that goods become cheaper the more of them are produced (and the closely related idea that some products, like fax machines, become more useful the more people use them). "Positive feedback" is the opposite of "negative feedback", a concept well-known for instance in population dynamics: the larger the population of rabbits in a valley, the less they have grass per rabbit. If the population grows too much, they will eventually starve, slowing down their reproduction rate which thus reduces their population at a later time. Thus negative feedback means that the higher the population, the slower the growth rate, leading to a spontaneous regulation of the population size; negative feedbacks thus tend to regulate growth towards an equilibrium. In contrast, positive feedback asserts that the higher the price or the price return in the recent past, the higher will be the price growth in the future. Positive feedbacks, when unchecked, can produce runaways until the deviation from equilibrium is so large that other effects can be abruptly triggered and lead to rupture or crashes. Youssefmir et al. (1998) have stressed the importance of positive feedback in a dynamical theory of asset price bubbles that exhibits the appearance of bubbles and their subsequent crashes. The positive feedback leads to speculative trends which may dominate over fundamental beliefs and which make the system increasingly susceptible to any exogenous shock, thus eventually precipitating a crash.

There are many mechanisms in the stock market and in the behavior of investors which may lead to positive feedbacks. We describe a general mechanism for positive feedback, which is now known as the “herd” or “crowd” effect, based on imitation processes. We present a simple model of the best investment strategy that an investor can develop based on interactions with and information taken from other investors. We show how the repetition of these interactions may lead to a remarkable cooperative phenomenon in which the market can suddenly “solidify” a global opinion, leading to large price variations.

4.1. Herding

There are growing empirical evidences of the existence of herd or “crowd” behavior in speculative markets (see Shiller, 2000 and references therein). Herd behavior is often said to occur when many people take the same action, because some mimic the actions of others. The term “herd” obviously refers to similar behavior observed in animal groups. Other terms such as “flocks” or “schools” describe the collective coherent motion of large numbers of self-propelled organisms, such as migrating birds and gnus, lemmings and ants. In recent years, much of the observed herd behavior in animals has been shown to result from the action of simple laws of interactions between animals. With respect to humans, there is a long history of analogies between human groups and organized matter (Callen and Shapero, 1974; Montroll and Badger, 1974). More recently, extreme crowd motions such as under panic have been remarkably well quantified by models that treat the crowd as a collection of individuals interacting as a granular medium with friction such as the familiar sand of beaches (Helbing et al., 2000).

Herding has been linked to many economic activities, such as investment recommendations (Scharfstein and Stein, 1990; Graham, 1999; Welch, 2000s), price behavior of IPO’s (Initial Public Offering) (Welch, 1992), fads and customs (Bikhchandani et al., 1992), earnings forecasts (Trueman, 1994), corporate conservatism (Zwiebel, 1995) and delegated portfolio management (Maug and Naik, 1995). Researchers are investigating the incentives investment advisors face when deciding whether to herd and, in particular, whether economic conditions and agents’ individual characteristics affect their likelihood of herding. Although herding behavior appears inefficient from a social standpoint, it can be rational from the perspective of managers who are concerned about their reputations in the labor market, Such behavior can be rational and may occur as an information cascade (Welch, 1992; Bikhchandani et al., 1992; Devenow and Welch, 1996), a situation in which every subsequent actor, based on the observations of others, makes the same choice independent of his/her private signal. Herding among investment newsletters, for instance, is found to decrease with the precision of private information (Graham, 1999): the less information you have, the more important is your incentive to follow the consensus.

Research on herding in finance can be subdivided in the following nonmutually exclusive manner (Devenow and Welch, 1996; Graham, 1999).

1. *Informational cascades* occur when individuals choose to ignore or downplay their private information and instead jump on the bandwagon by mimicking the actions of individuals who acted previously. Informational cascades occur when the existing aggregate information becomes so overwhelming that an individual's single piece of private information is not strong enough to reverse the decision of the crowd. Therefore, the individual chooses to mimic the action of the crowd, rather than act on his private information. If this scenario holds for one individual, then it likely also holds for anyone acting after this person. This domino-like effect is often referred to as a cascade. The two crucial ingredients for an informational cascade to develop are: [1] sequential decisions with subsequent actors observing decisions (not information) of previous actors; and [2] a limited action space.
2. *Reputational herding*, like cascades, takes place when an agent chooses to ignore his or her private information and mimics the action of another agent who has acted previously. However, reputational herding models have an additional layer of mimicking, resulting from positive reputational properties that can be obtained by acting as part of a group or choosing a certain project. Evidence has been found that a forecaster's age is positively related to the absolute first difference between his forecast and the group mean. This has been interpreted as evidence that as a forecaster ages, evaluators develop tighter prior beliefs about the forecasters ability, and hence the forecaster has less incentive to herd with the group. On the other hand, the incentive for a second-mover to discard his private information and instead mimick the market leader increases with his initial reputation, as he strives to protect his current status and level of pay (Graham, 1999).
3. *Investigative herding* occurs when an analyst chooses to investigate a piece of information he or she believes others also will examine. The analyst would like to be the first to discover the information but can only profit from an investment if other investors follow suit and push the price of the asset in the direction anticipated by the first analyst. Otherwise, the first analyst may be stuck holding an asset that he or she cannot profitably sell.
4. *Empirical herding* refers to observations by many researchers of "herding" without reference to a specific model or explanation. There is indeed evidence of herding and clustering among pension funds, mutual funds, and institutional investors when a disproportionate share of investors engage in buying, or at other times selling, the same stock. These works suggest that clustering can result from momentum-following also called "positive feedback investment", e.g., buying past winners or perhaps from repeating the predominant buy or sell pattern from the previous period.

There are many reported case of herding. One of the most dramatic and clearest in recent times is the observation (Huberman and Regev, 2001) of a contagious speculation associated with a nonevent in the following sense. A Sunday New York Times article on a potential development of a new cancer-curing drugs caused the biotech company EntreMed's stock to rise from 12.063 at the Friday May 1, 1998 close to open at 85 on Monday May 4, close near 52 on the same day and remain above 39 in the three following weeks. The enthusiasm spilled over to other biotechnology stocks. It turns out that the potential breakthrough in cancer research already had been reported in one of the leading scientific journal 'Nature' and in various popular newspaper (including the Times) more than five months earlier. At that time, market reactions were essentially nil. Thus the enthusiastic public attention induced a long-term rise in share prices, even though no genuinely new information had been presented. The very prominent and exceptionally optimistic Sunday New York Times article of May 3, 1998 led to a rush on EntreMed's stock and other biotechnology companies' stocks, which is reminiscent of similar rushes leading to bubbles in historical times previously discussed. It is to be expected that information technology, the internet and biotechnology are among the leading new frontiers on which sensational stories will lead to enthusiasm, contagion, herding and speculative bubbles.

4.2. It is optimal to imitate when lacking information

All the traders in the world are organized into a network of family, friends, colleagues, contacts, and so on, which are sources of opinion and they influence each other *locally* through this network (Boissevain and Mitchell, 1973). We call "neighbors" of agent Anne on this world-wide graph the set of people in direct contact with Anne. Other sources of influence also involve newspapers, web sites, TV stations, and so on. Specifically, if Anne is directly connected with k "neighbors" in the worldwide graph of connections, then there are only two forces that influence Anne's opinion: (a) the opinions of these k people together with the influence of the media; and (b) an idiosyncratic signal that she alone receives (or generates). According to the concept of herding and imitation, the assumption is that agents tend to *imitate* the opinions of their "neighbors", not contradict them. It is easy to see that force (a) will tend to create order, while force (b) will tend to create disorder, or in other words, heterogeneity. The main story here is the fight between order and disorder and the question we are now going to investigate is: what behavior can result from this fight? Can the system go through unstable regimes, such as crashes? Are crashes predictable? We show that the science of self-organizing systems (sometimes also referred to as "complex systems") bears very significantly on these questions: the stock market and the web of traders' connections can be understood in large part from the science of critical phenomena, in a sense that we are going to examine in some depth in the following sections, from which important consequences can be derived.

To make progress, we formalize a bit the problem and consider a network of investors: each one can be named by an integer $i = 1, \ldots, I$, and $N(i)$ denotes the set of the agents who are directly connected to agent i according to the world-wide graph of acquaintances. If we isolate one trader, Anne, $N(\text{Anne})$ is the number of traders in direct contact with her and who can exchange direct information with her and exert a direct influence on her. For simplicity, we assume that any investor such as Anne can be in only one of several possible states. In the simplest version, we can consider only two possible states: $s_{\text{Anne}} = -1$ or $s_{\text{Anne}} = +1$. We could interpret these states as "buy" and "sell", "bullish" and "bearish", "optimistic" and "pessimistic", and so on. In the next paragraph, we show that, based only on the information of the actions $s_j(t-1)$ performed yesterday (at time $t-1$) by her $N(\text{Anne})$ "neighbors", Anne maximizes her return by having taken yesterday the decision $s_{\text{Anne}}(t-1)$ given by the sign of the sum of the actions of all her "neighbors". In other words, the optimal decision of Anne, based on the local polling of her "neighbors" who she hopes represents a sufficiently faithful representation of the market mood, is to imitate the majority of her neighbors. This is of course up to some possible deviations when she decides to follow her own idiosyncratic "intuition" rather than being influenced by her "neighbors". Such an idiosyncratic move can be captured in this model by a stochastic component independent of the decisions of the neighbors or of any other agent. Intuitively, the reason why it is in general optimal for Anne to follow the opinion of the majority is simply because prices move in that direction, forced by the law of supply and demand. This apparently innocuous evolution law produces remarkable self-organizing patterns.

Consider N traders in a network, whose links represent the communication channels through which the traders exchange information. The graph describes the chain of intermediate acquaintances between any two people in the world. We denote $N(i)$ the number of traders directly connected to a given trader i on the graph. The traders buy or sell one asset at price $p(t)$ which evolves as a function of time assumed to be discrete and measured in units of the time step Δt. In the simplest version of the model, each agent can either buy or sell only one unit of the asset. This is quantified by the buy state $s_i = +1$ or the sell state $s_i = -1$. Each agent can trade at time $t-1$ at the price $p(t-1)$ based on all previous information including that at $t-1$. The asset price variation is taken simply proportional to the aggregate sum $\sum_{i=1}^{N} s_i(t-1)$ of all traders' actions: indeed, if this sum is zero, there are as many buyers as there are sellers and the price does not change since there is a perfect balance between supply and demand. If, on the other hand, the sum is positive, there are more buy orders than sell orders, the price has to increase to balance the supply and the demand, as the asset is too rare to satisfy all the demand. There are many other influences impacting the price change from one day to the other, and this can usually be accounted for in a simple way by adding a stochastic component to the price variation. This term alone would give the usual log-normal random walk process (Cootner, 1967) while the balance between supply and demand together with imitation leads to some organization as we show below.

At time $t-1$, just when the price $p(t-1)$ has been announced, the trader i defines her strategy $s_i(t-1)$ that she will hold from $t-1$ to t, thus realizing the profit (or loss) equal to the price difference $(p(t) - p(t-1))$ times her position $s_i(t-1)$. To define her optimal strategy $s_i(t-1)$, the trader should calculate her expected profit P_{E}, given the past information and her position, and then choose $s_i(t-1)$ such that P_{E} is maximum. Since the price moves with the general opinion $\sum_{i=1}^{N} s_i(t-1)$, the best strategy is to buy if it is positive and sell if it is negative. The difficulty is that a given trader cannot poll the positions s_j that will take all other traders which will determine the price drift according to the balance between supply and demand. The next best thing that trader i can do is to poll her $N(i)$ "neighbors" and construct her prediction for the price drift from this information. The trader needs an additional information, namely the a priori probability P_+ and P_- for each trader to buy or sell. The probabilities P_+ and P_- are the only information that she can use for all the traders that she does not poll directly. From this, she can form her expectation of the price change. The simplest case corresponds to a market without drift where $P_+ = P_- = 1/2$.

Based on the previously stated rule that the price variation is proportional to the sum of actions of traders, the best guess of trader i is that the future price change will be proportional to the sum of the actions of her neighbors that she has been able to poll, hoping that this provides a sufficiently reliable sample of the total population. Traders are indeed constantly sharing information, calling each other to "take the temperature", effectively polling each other before taking actions. It is then clear that the strategy that maximizes her expected profit is such that her position is of the sign given by the sum of the actions of all her "neighbors". This is exactly the meaning of expression (3)

$$s_i(t-1) = \mathrm{sign}\left(K \sum_{j \in N_i} s_j + \epsilon_i \right) \tag{3}$$

such that this position $s_i(t-1)$ gives her the maximum payoff based on her best prediction of the price variation $p(t) - p(t-1)$ from yesterday to today. The function sign(x) is defined by being equal to $+1$ (to -1) for positive (negative) argument x, K is a positive constant of proportionality between the price change and the aggregate buy-sell orders. It is inversely proportional to the "market depth": the larger the market, the smaller is the relative impact of a given unbalance between buy and sell orders, hence the smaller is the price change. ϵ_i is a noise and $N(i)$ is the number of neighbors with whom trader i interacts significantly. In simple terms, this law (3) states that the best investment decision for a given trader is to take that of the majority of her neighbors, up to some uncertainly (noise) capturing the possibility that the majority of her neighbors might give an incorrect prediction of the behavior of the total market.

Expression (3) can be thought of as a mathematical formulation of Keynes' beauty contest. Keynes (1936) argued that stock prices are not only determined by the firm's fundamental value, but, in addition, mass psychology and investors' expectations influence financial markets significantly. It was his opinion that professional investors prefer to devote their energy, not to estimating fundamental values but rather, to analyzing how the crowd of investors is likely to behave in the future. As a result, he said, most persons are largely concerned, not with making superior long-term forecasts of the probable yield of an investment over its whole life but, with foreseeing changes in the conventional basis of valuation a short time ahead of the general public. Keynes uses his famous beauty contest as a parable for stock markets. In order to predict the winner of beauty contest, objective beauty is not much important, but knowledge or prediction of others'prediction of beauty is much more relevant. In Keynes'view, the optimal strategy is not to pick those faces the player thinks the prettiest, but those the other players are likely to think the average opinion will be, or those the other players will think the others will think the average opinion will be, or even further along this iterative loop. Expression (3) precisely captures this concept: the opinion s_i at time t of an agent i is a function of all the opinions of the other "neighboring" agents at the previous time $t-1$, which themselves depend on the opinion of the agent i at time $t-2$, and so on. In the stationary equilibrium situation in which all agents finally form an opinion after many such iterative feedbacks have had time to develop, the solution of (3) is precisely the one taking into account all the opinions in a completely self-consistent way compatible with the infinitely iterative loop. Similarly, Orléan (1984, 1986, 1989a, b, 1991, 1995) has captured the paradox of combining rational and imitative behavior under the name "mimetic rationality" (*rationalité mimétique*). He has developed models of mimetic contagion of investors in the stock markets that are based on irreversible processes of opinion forming. See also Krawiecki et al. (2002) for a recent generalization with time-varying coupling strength K leading to on-off intermittency and attractor bubbling.

4.3. Cooperative behaviors resulting from imitation

The imitative behavior discussed in Section 4.2 and captured by the expression (3) belongs to a very general class of stochastic dynamical models developed to describe interacting elements, particles, agents in a large variety of contexts, in particular in physics and biology (Liggett, 1985, 1997). The tendency or force towards imitation is governed by the coupling strength K; the tendency towards idiosyncratic (or noisy) behavior is governed by the amplitude σ of the noise term. Thus the value of K relative to σ determines the outcome of the battle between order and disorder, and eventually the structure of the market prices. More generally, the coupling strength K could be heterogeneous

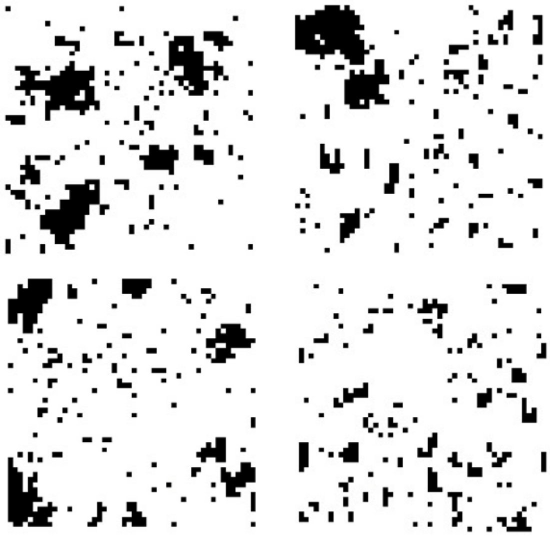

Fig. 7. Four snapshots at four successive times of the state of a planar system of 64×64 agents put on a regular square lattice. Each agent placed within a small square interacts with her four nearest neighbors according to the imitative rule (3). White (resp. black) squares correspond to "bull" (resp. "bear"). The four cases shown here correspond to the existence of a majority of buy orders as white is the predominant color.

across pairs of neighbors, and it would not substantially affect the properties of the model. Some of the K_{ij}'s could even be negative, as long as the average of all K_{ij}'s was strictly positive.

Expression (3) only describes the state of an agent at a given time. In the next instant, new ε_i's are realized, new influences propagate themselves to neighbors, and agents can change their decision. The system is thus constantly changing and reorganizing as shown in Fig. 7. The model does *not* assume instantaneous opinion interactions between neighbours. In real markets, opinions tend indeed not to be instantaneous but are formed over a period of time by a process involving family, friends, colleagues, newspapers, web sites, TV stations, and so on. Decisions about trading activity of a given agent may occur when the consensus from all these sources reaches a trigger level. This is precisely this feature of a threshold reached by a consensus that expression (3) captures: the consensus is quantified by the sum over the $N(i)$ agents connected to agent i and the threshold is provided by the sign function. The delay in the formation of the opinion of a given trader as a function of other traders' opinion is captured by the progressive spreading of information during successive updating steps (see for instance Liggett, 1985, 1997).

The simplest possible network is a two-dimensional grid in the Euclidean plane. Each agent has four nearest neighbors: one to the North, one to the South, the East and the West. The tendency K towards imitation is balanced by the tendency σ towards idiosyncratic behavior. In the context of the alignment of atomic spins to create magnetisation (magnets), this model is identical to the two-dimensional Ising model which has been solved explicitly by Onsager (1944). Only its formulation is different from what is usually found in textbooks (Goldenfeld, 1992), as we emphasize a dynamical view point.

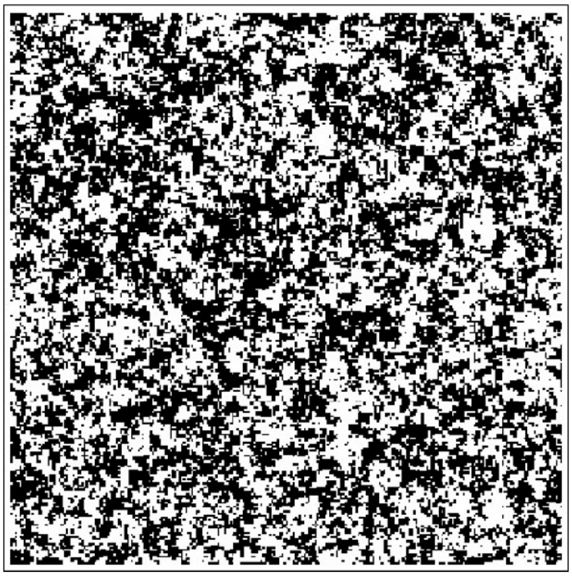

Fig. 8. $K < K_c$: buy (white squares) and sell (black squares) configuration in a two-dimensional Manhattan-like planar network of 256×256 agents interacting with their four nearest neighbors. There are approximately the same number of white and black sells, i.e., the market has no consensus. The size of largest local clusters quantifies the correlation length, i.e., the distance over which the local imitations between neighbors propagate before being significantly distorted by the "noise" in the transmission process resulting from the idiosyncratic signals of each agent.

In the Ising model, there exists a critical point K_c that determines the properties of the system. When $K < K_c$ (see Fig. 8), disorder reigns: the sensitivity to a small global influence is small, the clusters of agents who are in agreement remain of small size, and imitation only propagates between close neighbors. In this case, the susceptibility χ of the system to external news is small as many clusters of different opinion react incoherently, thus more or less cancelling out their response.

When the imitation strength K increases and gets close to K_c (see Fig. 9), order starts to appear: the system becomes extremely sensitive to a small global perturbation, agents who agree with each other form large clusters, and imitation propagates over long distances. In the Natural Sciences, these are the characteristics of *critical* phenomena. Formally, in this case the susceptibility χ of the system goes to infinity. The hallmark of criticality is the *power law*, and indeed the susceptibility goes to infinity according to a power law $\chi \approx A(K_c - K)^{-\gamma}$, where A is a positive constant and $\gamma > 0$ is called the *critical exponent* of the susceptibility (equal to 7/4 for the 2-d Ising model). This kind of critical behavior is found in many other models of interacting elements (Liggett, 1985, 1997) (see also Moss de Oliveira et al. (1999) for applications to finance among others). The large susceptibility means that the system is unstable: a small external perturbation may lead to a large collective reaction of the traders who may revise drastically their decision, which may abruptly produce a sudden unbalance between supply and demand, thus triggering a crash or a rally. This specific mechanism will be shown to lead to crashes in the model described in the next section.

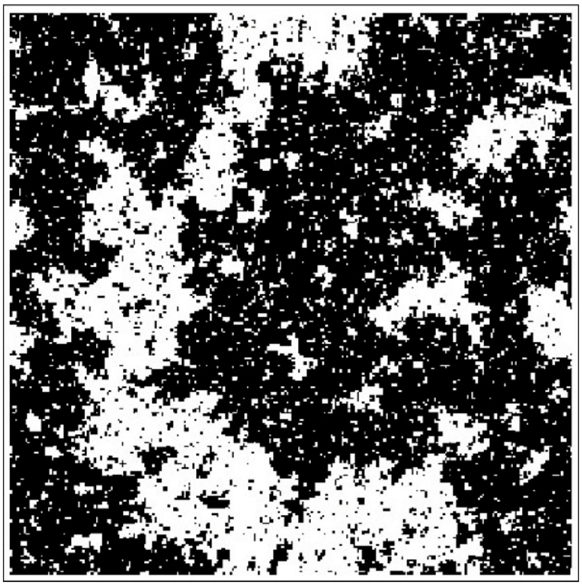

Fig. 9. Same as Fig. 8 for K close to K_c. There are still approximately the same number of white and black cells, i.e., the market has no consensus. However, the size of the largest local clusters has grown to become comparable to the total system size. In addition, holes and clusters of all sizes can be observed. The "scale-invariance" or "fractal" looking structure is the hallmark of a "critical state" for which the correlation length and the susceptibility become infinite (or simply bounded by the size of the system).

For even stronger imitation strength $K > K_c$, the imitation is so strong that the idiosyncratic signals become negligible and the traders self-organize into a strong imitative behavior as shown in Fig. 10. The selection of one of the two possible states is determined from small and subtle initial biases as well as from the fluctuations during the evolutionary dynamics.

These behaviors apply more generically to other network topologies. Indeed, the stock market constitutes an ensemble of interacting investors who differ in size by many orders of magnitudes ranging from individuals to gigantic professional investors, such as pension funds. Furthermore, structures at even higher levels, such as currency influence spheres (U.S.$, DM, YEN ...), exist and with the current globalization and de-regulation of the market one may argue that structures on the largest possible scale, i.e., the world economy, are beginning to form. This observation and the network of connections between traders show that the two-dimensional lattice representation used in the Figs. 7, 8, 9 and 10 is too naive. A better representation of the structure of the financial markets is that of hierarchical systems with "traders" on all levels of the market. Of course, this does not imply that any strict hierarchical structure of the stock market exists, but there are numerous examples of qualitatively hierarchical structures in society. In fact, one may say that horizontal organizations of individuals are rather rare. This means that the plane network used in our previous discussion may very well represent a gross over-simplification.

Even though the predictions of these models are quite detailed, they are very robust to model misspecification. We indeed claim that models that combine the following features would display the

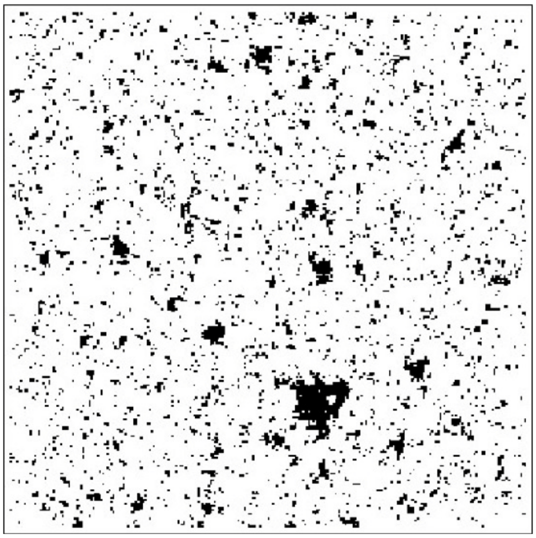

Fig. 10. Same as Fig. 8 for $K > K_c$. The imitation is so strong that the network of agents spontaneously break the symmetry between the two decisions and one of them predominates. Here, we show the case where the "buy" state has been selected. Interestingly, the collapse onto one of the two states is essentially random and results from the combined effect of a slight initial bias and of fluctuations during the imitation process. Only small and isolated islands of "bears" remain in an ocean of buyers. This state would correspond to a bubble, a strong bullish market.

same characteristics, in particular apparent coordinate buying and selling periods, leading eventually to several financial crashes. These features are:

1. A system of traders who are influenced by their "neighbors".
2. Local imitation propagating spontaneously into global cooperation.
3. Global cooperation among noise traders causing collective behavior.
4. Prices related to the properties of this system.
5. System parameters evolving slowly through time.

As we shall show in the following sections, a crash is most likely when the locally imitative system goes through a *critical* point.

In Physics, critical points are widely considered to be one of the most interesting properties of complex systems. A system goes critical when local influences propagate over long distances and the average state of the system becomes exquisitely sensitive to a small perturbation, i.e. different parts of the system become highly correlated. Another characteristic is that critical systems are self-similar across scales: in Fig. 9, at the critical point, an ocean of traders who are mostly bearish may have within it several continents of traders who are mostly bullish, each of which in turns surrounds seas of bearish traders with islands of bullish traders; the progression continues all the way down to the smallest possible scale: a single trader (Wilson, 1979). Intuitively speaking, critical self-similarity is why local imitation cascades through the scales into global coordination. Critical points are described in mathematical parlance as singularities associated with bifurcation and catastrophe theory.

The previous Ising model is one of the simplest possible description of cooperative behaviors resulting from repetitive interactions between agents. Many other models have recently been developed in order to capture more realistic properties of people and of their economic interactions. These multi-agent models, often explored by computer simulations, support the hypothesis that the observed characteristics of financial prices, such as nonGaussian "fat" tails of distributions of returns, mostly unpredictable returns, clustered and excess volatility, may result endogenously from the interaction between agents.

Several works have modelled the epidemics of opinion and speculative bubbles in financial markets from an adaptative agent point-of-view (Kirman, 1991; Lux, 1995, 1998; Lux and Marchesi, 1999, 2000). The main mechanism for bubbles is that above average returns are reflected in a generally more optimistic attitude that fosters the disposition to overtake others' bullish beliefs and vice versa. The adaptive nature of agents is reflected in the alternatives available to agents to choose between several class of strategies, for instance to invest according to fundamental economic valuation or by using technical analysis of past price trajectories. Other relevant works put more emphasis on the heterogeneity and threshold nature of decision making which lead in general to irregular cycles (Takayasu et al., 1992; Youssefmir et al., 1998; Levy et al., 1995; Sato and Takayasu, 1998; Levy et al., 2000; Gaunersdorfer, 2000).

5. Modelling financial bubbles and market crashes

In this section, we describe three complementary models that we have developed to describe bubbles and crashes. The first two models are extensions of the rational expectation model of bubbles and crashes of Blanchard (1979) and Blanchard and Watson (1982). They originally introduced the model of rational expectations (RE) bubbles to account for the possibility, often discussed in the empirical literature and by practitioners, that observed prices may deviate significantly and over extended time intervals from fundamental prices. While allowing for deviations from fundamental prices, rational bubbles keep a fundamental anchor point of economic modelling, namely that bubbles must obey the condition of rational expectations. In contrast, recent works stress that investors are not fully rational, or have at most bound rationality, and that behavioral and psychological mechanisms, such as herding, may be important in the shaping of market prices (Thaler, 1993; Shefrin, 2000; Shleifer, 2000). However, for fluid assets, dynamic investment strategies rarely perform over simple buy-and-hold strategies (Malkiel, 1999), in other words, the market is not far from being efficient and little arbitrage opportunities exist as a result of the constant search for gains by sophisticated investors. For the first two models, we shall work within the conditions of rational expectations and of no-arbitrage condition, taken as useful approximations. Indeed, the rationality of both expectations and behavior often does not imply that the price of an asset be equal to its fundamental value. In other words, there can be rational deviations of the price from this value, called rational bubbles. A rational bubble can arise when the actual market price depends positively on its own expected rate of change, as sometimes occurs in asset markets, which is the mechanism underlying the models of Blanchard (1979) and Blanchard and Watson (1982). The third model proposes to complement the modelling of bubbles and crashes by studying the effects of interactions between the two typical opposite attitudes of investors in stock markets, namely imitative and contrarian behaviors.

5.1. The risk-driven model

This first model contains the following ingredients (Johansen et al., 1999a, b, 2000a):

1. A system of traders who are influenced by their "neighbors".
2. Local imitation propagating spontaneously into global cooperation.
3. Global cooperation among traders causing crash.
4. Prices related to the properties of this system.

The interplay between the progressive strengthening of imitation controlled by the three first ingredients and the ubiquity of noise requires a stochastic description. A crash is not certain but can be characterized by its hazard rate $h(t)$, i.e., the probability per unit time that the crash will happen in the next instant if it has not happened yet.

The crash hazard rate $h(t)$ embodies subtle uncertainties of the market: when will the traders realize with sufficient clarity that the market is over-valued? When will a significant fraction of them believe that the bullish trend is not sustainable? When will they feel that other traders think that a crash is coming? Nowhere is Keynes's beauty contest analogy more relevant than in the characterization of the crash hazard rate, because the survival of the bubble rests on the overall confidence of investors in the market bullish trend.

A crash happens when a large group of agents place sell orders simultaneously. This group of agents must create enough of an imbalance in the order book for market makers to be unable to absorb the other side without lowering prices substantially. A notable fact is that the agents in this group typically do not know each other. They did not convene a meeting and decide to provoke a crash. Nor do they take orders from a leader. In fact, most of the time, these agents disagree with one another, and submit roughly as many buy orders as sell orders (these are all the times when a crash *does not* happen). The key question is to determine by what mechanism did they suddenly manage to organize a coordinated sell-off?

We propose the following answer (Johansen et al., 1999a, b) already outline above: all the traders in the world are organized into a network (of family, friends, colleagues, and so on) and they influence each other *locally* through this network: for instance, an active trader is constantly on the phone exchanging information and opinions with a set of selected colleagues. In addition, there are indirect interactions mediated for instance by the media. Specifically, if I am directly connected with k other traders, then there are only two forces that influence my opinion: (a) the opinions of these k people and of the global information network; and (b) an idiosyncratic signal that I alone generate. Our working assumption here is that agents tend to *imitate* the opinions of their connections. The force (a) will tend to create order, while force (b) will tend to create disorder. The main story here is a fight between order and disorder. As far as asset prices are concerned, a crash happens when order wins (everybody has the same opinion: selling), and normal times are when disorder wins (buyers and sellers disagree with each other and roughly balance each other out). We must stress that this is exactly the opposite of the popular characterization of crashes as times of chaos. Disorder, or a balanced and varied opinion spectrum, is what keeps the market liquid in normal times. This mechanism does not require an overarching coordination mechanism since macro-level coordination can arise from micro-level imitation and it relies on a realistic model of how agents form opinions by constantly interacting.

5.1.1. Finite-time singularity in the crash hazard rate

In the spirit of "mean field" theory of collective systems (Goldenfeld, 1992), the simplest way to describe an imitation process is to assume that the hazard rate $h(t)$ evolves according to the following equation:

$$\frac{\mathrm{d}h}{\mathrm{d}t} = Ch^{\delta} \quad \text{with } \delta > 1 \ , \tag{4}$$

where C is a positive constant. Mean field theory amounts to embody the diversity of trader actions by a single effective representative behavior determined from an average interaction between the traders. In this sense, $h(t)$ is the collective result of the interactions between traders. The term h^{δ} in the r.h.s. of (4) accounts for the fact that the hazard rate will increase or decrease due to the presence of *interactions* between the traders. The exponent $\delta > 1$ quantifies the effective number equal to $\delta - 1$ of interactions felt by a typical trader. The condition $\delta > 1$ is crucial to model interactions and is, as we now show, essential to obtain a singularity (critical point) in finite time. Indeed, integrating (4), we get

$$h(t) = \frac{B}{(t_c - t)^{\alpha}} \quad \text{with } \alpha \equiv \frac{1}{\delta - 1} \ . \tag{5}$$

The critical time t_c is determined by the initial conditions at some origin of time. The exponent α must lie between zero and one for an economic reason: otherwise, as we shall see, the price would go to infinity when approaching t_c (if the bubble has not crashed in the mean time). This condition translates into $2 < \delta < +\infty$: a typical trader must be connected to more than one other trader. There is a large body of literature in Physics, Biology and Mathematics on the microscopic modelling of systems of stochastic dynamical interacting agents that lead to critical behaviors of the type (5) (Liggett, 1985, 1997). The macroscopic model (4) can thus be substantiated by specific microscopic models (Johansen et al., 2000).

Before continuing, let us provide an intuitive explanation for the creation of a finite-time singularity at t_c. The faster-than-exponential growth of the return and of the crash hazard rate correspond to nonconstant growth rates, which increase with the return and with the hazard rate. The following reasoning allows us to understand intuitively the origin of the appearance of an infinite slope or infinite value in a finite time at t_c, called a finite-time singularity. Suppose for instance that the growth rate of the hazard rate doubles when the hazard rate doubles. For simplicity, we consider discrete time intervals as follows. Starting with a hazard rate of 1, we assume it grows at a constant rate of 1% per day until it doubles. We estimate the doubling time as proportional to the inverse of the growth rate, i.e., approximately $1/1\% = 1/0.01 =$ one hundred days. There is a multiplicative correction term equal to $\ln 2 = 0.69$ such that the doubling time is $\ln 2/1\% = 69$ days. But we factor out this proportionality factor $\ln 2 = 0.69$ for the sake of pedagogy and simplicity. Including it multiplies all time intervals below by 0.69 without changing the conclusions.

When the hazard rate turns 2, we assume that the growth rate doubles to 2% and stays fixed until the hazard rate doubles again to reach 4. This new doubling time is only approximately $1/0.02 = 50$ days at this 2% growth rate. When the hazard rate reaches 4, its growth rate is doubled to 4%. The doubling time of the hazard rate is therefore approximately halved to 25 days and the scenario continues with a doubling of the growth rate every time the hazard rate doubles. Since the doubling time is approximately halved at each step, we have the following sequence (time = 0, hazard rate = 1, growth rate = 1%), (time = 100, hazard rate = 2, growth rate = 2%), (time = 150, hazard rate = 4, growth rate = 4%), (time = 175, hazard rate = 8, growth rate = 8%) and so on. We observe that the time interval needed for the hazard rate to double is shrinking very rapidly by a factor of two at each step. In the same way that

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = 1 \ , \tag{6}$$

which was immortalized by the Ancient Greeks as Zeno's paradox, the infinite sequence of doubling thus takes a finite time and the hazard rate reaches infinity at a finite "critical time" approximately equal to $100 + 50 + 25 + \cdots = 200$ (a rigorous mathematical treatment requires a continuous time formulation, which does not change the qualitative content of the example). A spontaneous singularity has been created by the increasing growth rate! This process is quite general and applies as soon as the growth rate possesses the property of being multiplied by some factor larger than 1 when the hazard rate or any other observable is multiplied by some constant larger than 1.

5.1.2. Derivation from the microscoping Ising model

The phenomenological equations (4) and (5) can be derived from the microscopic model of agent interactions described by Eq. (3). For this, let us assume that the imitation strength K changes smoothly with time, as a result for instance of the varying confidence level of investors, the economic outlook, and so on. The simplest assumption, which does not change the nature of the argument, is that K is proportional to time. Initially, K is small and only small clusters of investors self-organize, as shown in Fig. 8. As K increases, the typical size of the clusters increases as shown in Fig. 9. These kinds of systems exhibiting cooperative behavior are characterized by a broad distribution of cluster sizes s (the size of the black islands for instance) up to a maximum s^* which itself increases in an accelerating fashion up to the critical value K_c. Right at $K = K_c$, the geography of clusters of a given kind becomes self-similar with a continuous hierarchy of sizes from the smallest (the individual investor) to the largest (the total system). Within this phenomenology, the probability for a crash to occur is constructed as follows.

First, a crash corresponds to a coordinated sell-off of a large number of investors. In our simple model, this will happen as soon as a single cluster of connected investors, which is sufficiently large to set the market off-balance, decides to sell-off. Recall indeed that "clusters" are defined by the condition that all investors in the same cluster move in concert. When a very large cluster of investors sells, this creates a sudden unbalance which triggers an abrupt drop of the price, hence a crash. To be concrete, we assume that a crash occurs when the size (number of investors) s of the active cluster is larger than some minimum value s_m. The specific value s_m is not important, only the fact that s_m is much larger than 1 so that a crash can only occur as a result of a cooperative action of many traders who destabilize the market. At this stage, we do not specify the amplitude of the crash, only its triggering as an instability. For this explanation to make sense, investors change opinion and send market orders only rarely. Therefore, we should expect only one or few large clusters to be simultaneously active and able to trigger a crash.

For a crash to occur, we thus need (1) to find at least one cluster of size larger than s_m and (2) to verify that this cluster is indeed actively selling-off. Since these two events are independent, the probability for a crash to occur is thus the product of the probability to find such a cluster of size larger than the threshold s_m by the probability that such a cluster begins to sell-off collectively. The probability to find a cluster of size s is a well-known characteristic of critical phenomena (Goldenfeld, 1992; Stauffer and Aharony, 1994): it is a power law distribution truncated smoothly at a maximum s^*; this maximum increases without bound (except for the total system size) on the approach to the critical value K_c of the imitation strength.

If the decision to sell off by an investor belonging to a given cluster of size s was independent of the decisions of all the other investors in the same cluster, then the probability per unit time that such a cluster of size s becomes active would be simply proportional to the number s of investors in that cluster. However, by the very definition of a cluster, investors belonging to a given cluster do interact with each other. Therefore, the decision of an investor to sell off is probably quite strongly coupled with those of the other investors in the same cluster. Hence, the probability per unit time that a specific cluster of s investors becomes active is a function of the number s of investors belonging to that cluster and of all the interactions between these investors. Clearly, the maximum number of interactions within a cluster is $s \times (s-1)/2$, that is, for large s, it becomes proportional to the square of the number of investors in that cluster. This occurs when each of the s investors speaks to each of his or her $s-1$ colleagues. The factor $1/2$ accounts for the fact that if investor Anne speaks to investor Paul then in general Paul also speaks to Anne and their two-ways interactions must be counted only once. Of course, one can imagine more complex situations in which Paul listen to Anne but Anne does not reciprocate but this does not change the results. Notwithstanding these complications, one sees that the probability $h(t)\Delta t$ per unit time Δt that a specific cluster of s investors becomes active must be a function growing with the cluster size s faster than s but probably slower than the maximum number of interactions (proportional to s^2). A simple parameterization is to take $h(t)\Delta t$ proportional to the cluster size s elevated to some power α larger than 1 but smaller than 2. This exponent α captures the collective organization within a cluster of size s due to the multiple interactions between its investors. It is related to the concept of fractal dimensions.

The probability for a crash to occur, which is the same as the probability of finding at least one active cluster of size larger than the minimum destabilizing size s_m, is therefore the sum over all sizes s larger than s_m of all the products of probabilities n_s to find a cluster of a specific size s by their probability per unit time to become active (itself proportional to s^α as we have argued). With mild technical conditions, it can then be shown that the crash hazard rate exhibits a power law acceleration with a singular behavior. Intuitively, this result stems from the interplay between the existence of larger and larger clusters as the interaction parameter K approached its critical value K_c and from the nonlinear accelerating probability per unit time for a cluster to become active as its typical size s^* grows with the approach of K to K_c.

The diverging acceleration of the crash probability implies a remarkable prediction for the crash hazard rate: indeed, the crash hazard rate is nothing but the rate of change of the probability of a crash as a function of time (conditioned on it not having happened yet). The crash hazard rate thus increases without bounds as K goes to K_c. The risk of a crash per unit time, knowing that the crash has not yet occurred, increases dramatically when the interaction between investors becomes strong enough so that the network of interactions between traders self-organized into a hierarchy containing a few large spontaneously formed groups acting collectively.

We stress that K_c is not the value of the imitation strength at which the crash occurs, because the crash could happen for any value before K_c, even though this is not very likely. K_c is the most probable value of the imitation strength for which the crash occurs. To translate these results as a function of time, it is natural to expect that the imitation strength K is changing slowly with time as a result of several factors influencing the tendency of investors to herd. A typical trajectory $K(t)$ of the imitation strength as a function of time t is erratic and smooth. The critical time t_c is defined as the time at which the critical imitation strength K_c is reached for the first time starting from some initial value. t_c is not the time of the crash, it is the end of the bubble. It is the most probable time of the crash because the hazard rate is largest at that time. Due to its probabilistic nature, the crash can occur at any other time, with a likelihood changing with time following the crash hazard rate.

The critical time t_c (or K_c) signals the death of the speculative bubble. We stress that t_c is not *the* time of the crash because the crash could happen at any time before t_c, even though this is not very likely. t_c is simply the most probable time of the crash. There exists a finite probability

$$1 - \int_{t_0}^{t_c} h(t)\, \mathrm{d}t > 0 \qquad (7)$$

of "landing" smoothly, i.e., of attaining the end of the bubble without crash. This residual probability is crucial for the coherence of the model, because otherwise agents would anticipate the crash and would exit from the market.

5.1.3. Dynamics of prices from the rational expectation condition

Assume for simplicity that, during a crash, the price drops by a fixed percentage $\kappa \in (0, 1)$, say between 20 and 30% of the price increase above a reference value p_1. Then, the dynamics of the asset price before the crash are given by

$$\mathrm{d}p = \mu(t)p(t)\,\mathrm{d}t - \kappa[p(t) - p_1]\,\mathrm{d}j \ , \qquad (8)$$

where j denotes a jump process whose value is zero before the crash and one afterwards. In this simplified model, we neglect interest rate, risk aversion, information asymmetry, and the market-clearing condition.

As a first-order approximation of the market organization, we assume that traders do their best and price the asset so that a fair game condition holds. Mathematically, this stylized rational expectation model is equivalent to the familiar martingale hypothesis:

$$\forall t' > t \qquad E_t[p(t')] = p(t) \ , \qquad (9)$$

where $p(t)$ denotes the price of the asset at time t and $E_t[\cdot]$ denotes the expectation conditional on information revealed up to time t. If we do not allow the asset price to fluctuate under the impact of noise, the solution to equation (9) is a constant: $p(t) = p(t_0)$, where t_0 denotes some initial time. $p(t)$ can be interpreted as the price in excess of the fundamental value of the asset. This rational expectation bubble model can be extended to general and arbitrary risk-aversion within the general stochastic discount factor theory (Sornette and Johansen, 2001).

Putting (8) in (9) leads to

$$\mu(t)p(t) = \kappa[p(t) - p_1]h(t) \qquad (10)$$

using $\mathrm{E}[\mathrm{d}j] = h(t)\,\mathrm{d}t$. In words, if the crash hazard rate $h(t)$ increases, the return μ increases to compensate the traders for the increasing risk. Plugging (10) into (8), we obtain a ordinary differential equation. For $p(t) - p(t_0) < p(t_0) - p_1$, its solution is

$$p(t) \approx p(t_0) + \kappa[p(t_0) - p_1] \int_{t_0}^{t} h(t')\,\mathrm{d}t' \quad \text{before the crash} . \tag{11}$$

If instead the price drops by a fixed percentage $\kappa \in (0, 1)$ of the price, the dynamics of the asset price before the crash is given by

$$\mathrm{d}p = \mu(t)p(t)\,\mathrm{d}t - \kappa p(t)\,\mathrm{d}j . \tag{12}$$

We then get

$$\mathrm{E}_t[\mathrm{d}p] = \mu(t)p(t)\,\mathrm{d}t - \kappa p(t)h(t)\,\mathrm{d}t = 0 , \tag{13}$$

which yields

$$\mu(t) = \kappa h(t) . \tag{14}$$

and the corresponding equation for the price is

$$\log\left[\frac{p(t)}{p(t_0)}\right] = \kappa \int_{t_0}^{t} h(t')\,\mathrm{d}t' \quad \text{before the crash} . \tag{15}$$

This gives the logarithm of the price as the relevant observable. These two different scenarios for the price drops raises a rather interesting question. If the first scenario is the correct one, then crashes are nothing but (a partial) depletion of preceding bubbles and hence signals the markets return towards equilibrium. Hence, it may as such be taken as a sign of economical health, as also suggested by Barro et al. (1989) in relation to the crash of October 1987. On the other hand, if the second scenario is true, this suggest that bubbles and crashes are instabilities which are built-in or inherent in the market structure and that they are signatures of a market constantly out-of-balance, signaling fundamental systemic instabilities. We will return to this question in the conclusion. Johansen and Sornette (2001b) have shown that the first scenario is slightly more warranted according to the data.

The higher the probability of a crash, the faster the price must increase (conditional on having no crash) in order to satisfy the martingale (no free lunch) condition. Intuitively, investors must be compensated by the chance of a higher return in order to be induced to hold an asset that might crash. This effect may go against the naive preconception that price is adversely affected by the probability of the crash, but our result is the only one consistent with rational expectations. Complementarily, from a behavioral and dynamical point of view of the financial market, a faster rising price decreases the probability that it can be sustained much longer and may announce an instable phase in the mind of investors. We thus face a kind of "chicken and egg" problem.

Plugging (5) into (11) gives the following price law:

$$p(t) \approx p_c - \frac{\kappa B}{z} \times (t_c - t)^z \quad \text{before the crash} . \tag{16}$$

where $z = 1 - \alpha \in (0, 1)$ and p_c is the price at the critical time (conditioned on no crash having been triggered). The price before the crash thus follows a power law with a finite upper bound p_c. The trend of the price becomes unbounded as we approach the critical date. This is to compensate for an unbounded crash rate in the next instant.

The last ingredient of the model is to recognize that the stock market is made of actors which differs in size by many orders of magnitudes ranging from individuals to gigantic professional investors, such as pension funds. Furthermore, structures at even higher levels, such as currency influence spheres (U.S.$, Euro, YEN ...), exist and with the current globalization and de-regulation of the market one may argue that structures on the largest possible scale, i.e., the world economy, are beginning to form. This means that the structure of the financial markets have features which resembles that of hierarchical systems with "traders" on all levels of the market. Of course, this does not imply that any strict hierarchical structure of the stock market exists, but there are numerous examples of qualitatively hierarchical structures in society. Models of imitative interactions on hierarchical structures recover the power law behavior (16) (Sornette and Johansen, 1998; Johansen et al., 2000). But in addition, they predict that the critical exponent α can be a complex number! The first order expansion of the general solution for the hazard rate is then

$$h(t) \approx B_0(t_c - t)^{-\alpha} + B_1(t_c - t)^{-\alpha}\cos[\omega \log(t_c - t) - \psi] \ . \tag{17}$$

Once again, the crash hazard rate explodes near the critical date. In addition, it now displays log-periodic oscillations. The evolution of the price before the crash and before the critical date is given by

$$p(t) \approx p_c - \frac{\kappa}{z}\{B_0(t_c - t)^z + B_1(t_c - t)^z \cos[\omega \log(t_c - t) - \phi]\} \ , \tag{18}$$

where ϕ is another phase constant. The key feature is that oscillations appear in the price of the asset before the critical date. This means that the local maxima of the function are separated by time intervals that tend to zero at the critical date, and do so in geometric progression, i.e., the ratio of consecutive time intervals between maxima is a constant

$$\lambda \equiv e^{2\pi/\omega} \ . \tag{19}$$

This is very useful from an empirical point of view because such oscillations are much more strikingly visible in actual data than a simple power law: a fit can "lock-in" on the oscillations which contain information about the critical date t_c. Note that complex exponents and log-periodic oscillations do not necessitate a pre-existing hierarchical structure as mentioned above, but may emerge spontaneously from the nonlinear complex dynamics of markets (Sornette, 1998).

To sum up, we have constructed a model in which the stock market price is driven by the risk of a crash, quantified by its hazard rate. In turn, imitation and herding forces drive the crash hazard rate. When the imitation strength becomes close to a critical value, the crash hazard rate diverges with a characteristic power law behavior. This leads to a specific power law acceleration of the market price, providing our first predictive precursory pattern anticipating a crash.

5.2. The price-driven model

The price-driven model inverts the logic of the previous risk-driven model: here, again as a result of the action of rational investors, the price is driving the crash hazard rate rather than the reverse. The price itself is driven up by the imitation and herding behavior of the "noisy" investors.

As before, a stochastic description is required to capture the interplay between the progressive strengthening of imitation controlled by the connections and interactions between traders and the ubiquity of idiosyncratic behavior as well as the influence of many other factors that are impossible to model in details. As a consequence, the price dynamics are stochastic and the occurrence of a crash is not certain but can be characterized by its hazard rate $h(t)$, defined as the probability per unit time that the crash will happen in the next instant if it has not happened yet.

Keeping a basic tenet of economic theory, rational expectations, the model developed in Sornette and Andersen (2002) captures the nonlinear positive feedback between agents in the stock market as an interplay between nonlinearity and multiplicative noise. The derived hyperbolic stochastic finite-time singularity formula transforms a Gaussian white noise into a rich time series possessing all the stylized facts of empirical prices, as well as accelerated speculative bubbles preceding crashes.

Let us give the premise of the model and some preliminary results. We start from the geometric Brownian model of the bubble price $B(t)$, $\mathrm{d}B = \mu B\,\mathrm{d}t + \sigma B\,\mathrm{d}W_t$, where μ is the instantaneous return rate, σ is the volatility and $\mathrm{d}W_t$ is the infinitesimal increment of the random walk with unit variance (Wiener process). We generalize this expression into

$$\mathrm{d}B(t) = \mu(B(t))B(t)\,\mathrm{d}t + \sigma(B(t))B(t)\,\mathrm{d}W_t - \kappa(t)B(t)\,\mathrm{d}j\ , \tag{20}$$

allowing $\mu(B(t))$ and $\sigma(B(t))$ to depend arbitrarily and nonlinearly on the instantaneous realization of the price. A jump term has been added to describe a correction or a crash of return amplitude κ, which can be a stochastic variable taken from an a priori arbitrary distribution. Immediately after the last crash which becomes the new origin of time 0, $\mathrm{d}j$ is reset to 0 and will eventually jump to 1 with a hazard rate $h(t)$, defined such that the probability that a crash occurs between t and $t + \mathrm{d}t$ conditioned on not having occurred since time 0 is $h(t)\,\mathrm{d}t$.

Following Blanchard (1979) and Blanchard and Watson (1982), $B(t)$ is a rational expectations bubble which accounts for the possibility, often discussed in the empirical literature and by practitioners, that observed prices may deviate significantly and over extended time intervals from fundamental prices. While allowing for deviations from fundamental prices, rational bubbles keep a fundamental anchor point of economic modelling, namely that bubbles must obey the condition of rational expectations. This translates essentially into the no-arbitrage condition with risk-neutrality, which states that the expectation of $\mathrm{d}B(t)$ conditioned on the past up to time t is zero. This allows us to determine the crash hazard rate $h(t)$ as a function of $B(t)$. Using the definition of the hazard rate $h(t)\,\mathrm{d}t = \langle \mathrm{d}j \rangle$, where the bracket denotes the expectation over all possible outcomes since the last crash, this leads to $\mu(B(t))B(t) - \langle\kappa\rangle B(t)h(t) = 0$, which provides the hazard rate as a function of price:

$$h(t) = \frac{\mu(B(t))}{\langle\kappa\rangle}\ . \tag{21}$$

Expression (21) quantifies the fact that the theory of rational expectations with risk-neutrality associates a risk to any price: for example, if the bubble price explodes, so will the crash hazard rate, so that the risk-return trade-off is always obeyed. We note that it is easy to incorporate risk-aversion by introducing a risk-premium rate or by amplifying the risk of a crash perceived by traders.

The dependence of $\mu(B(t))$ and $\sigma(B(t))$ is chosen so as to capture the possible appearance of positive feedbacks on prices. There are many mechanisms in the stock market and in the behavior of investors which may lead to positive feedbacks. First, investment strategies with “portfolio insurance” are such that sell orders are issued whenever a loss threshold (or stop loss) is passed. It is clear that by increasing the volume of sell order, this may lead to further price decreases. Some commentators have indeed attributed the crash of October 1987 to a cascade of sell orders. Second, there is a growing empirical evidence of the existence of herd or "crowd" behavior in speculative markets (Shiller, 2000), in fund behaviors (Scharfstein and Stein, 1990; Grinblatt et al., 1995) and in the forecasts made by financial analysts (Trueman, 1994). Although this behavior is inefficient from a social standpoint, it can be rational from the perspective of managers who are concerned about their reputations in the labor market. As we have already mentioned, such behavior can be rational and may occur as an information cascade, a situation in which every subsequent actor, based on the observations of others, makes the same choice independent of his/her private signal (Bikhchandani et al., 1992). Herding leads to positive nonlinear feedback. Another mechanism for positive feedbacks is the so-called "wealth" effect: a rise of the stock market increases the wealth of investors who spend more, adding to the earnings of companies, and thus increasing the value of their stock.

The evidence for nonlinearity has a strong empirical support: for instance, the coexistence of the absence of correlation of price changes and the strong autocorrelation of their absolute values can not be explained by any linear model (Hsieh, 1995). Comparing additively nonlinear processes and multiplicatively nonlinear models, the later class of models are found consistent with empirical price changes and with options' implied volatilities. With the additional insight that hedging strategies of general Black–Scholes option models lead to a positive feedback on the volatility (Sircar and Papanicolaou, 1998), we are led to propose the following simplistic nonlinear model with multiplicative noise in which the return rate and the volatility are nonlinear increasing power law of $B(t)$ (Sornette and Andersen, 2002):

$$\mu(B)B = \frac{m}{2B}[B\sigma(B)]^2 + \mu_0[B(t)/B_0]^m \ , \tag{22}$$

$$\sigma(B)B = \sigma_0[B(t)/B_0]^m \ , \tag{23}$$

where B_0, μ_0, $m > 0$ and σ_0 are four parameters of the model, setting respectively a reference scale, an effective drift and the strength of the nonlinear positive feedback. The first term in the r.h.s. (22) is added as a convenient device to simplify the Ito calculation of these stochastic differential equations. The model can be reformulated in the Stratonovich interpretation

$$\frac{\mathrm{d}B}{\mathrm{d}t} = (a\mu_0 + b\eta)B^m \ , \tag{24}$$

where a and b are two constants and η is a delta-correlated Gaussian white noise, in physicist's notation such that $\eta\,\mathrm{d}t \equiv \mathrm{d}W$. The form (24) exemplifies the fundamental ingredient of the theory developed in Sornette and Andersen (2002) based on the interplay between nonlinearity and multiplicative noise. The nonlinearity creates a singularity in finite time and the multiplicative noise makes it stochastic. The choice (22), (23) or (24) are the simplest generalization of the standard geometric Brownian model (20) recovered for the special case $m = 1$. The introduction of the exponent m is a straightforward mathematical trick to account in the simplest and most parsimonious way for the presence of nonlinearity. Note in particular that, in the limit where m becomes very large, the nonlinear function B^m tends to a threshold response. The power B^m can be decomposed as $B^m = B^{m-1} \times B$ stressing the fact that B^{m-1} plays the role of a growth rate, function of the price itself. The positive feedback effect is captured by the fact that a larger price B feeds a larger growth rate, which leads to a larger price and so no.

The solution of (20) with (22) and (23) is given by

$$B(t) = \alpha^{\alpha} \frac{1}{(\mu_0[t_c - t] - (\sigma_0/B_0^m)W(t))^{\alpha}} \quad \text{where } \alpha \equiv \frac{1}{m-1} \tag{25}$$

with $t_c = y_0/(m-1)\mu_0$ is a constant determined by the initial condition with $y_0 = 1/B(t=0)^{m-1}$. To grasp the meaning of (25), let us first consider the deterministic case $\sigma_0 = 0$, such that the return rate $\mu(B) \propto [B(t)]^{m-1}$ is the sole driving term. Then, (25) reduces to $B(t) \propto 1/[t_c - t]^{1/(m-1)}$, i.e., a positive feedback $m > 1$ of the price $B(t)$ on the return rate μ creates a finite-time singularity at a critical time t_c determined by the initial starting point. This power law acceleration of the price accounts for the effect of herding resulting from the positive feedback. It is in agreement with the empirical finding that price peaks have sharp concave upwards maxima (Roehner and Sornette, 1998). Reintroducing the stochastic component $\sigma_0 \neq 0$, we see from (25) that the finite-time singularity still exists but its visit is controlled by the first passage of a biased random walk at the position $\mu_0 t_c$ such that the denominator $\mu_0[t_c - t] - (\sigma_0/B_0^m)W(t)$ vanishes. In practice, a price trajectory will never sample the finite-time singularity as it is not allowed to approach too close to it due to the jump process $\mathrm{d}j$ defined in (20). Indeed, from the no-arbitrage condition, expression (21) for the crash hazard rate ensures that when the price explodes, so does $h(t)$ so that a crash will occur with larger and larger probability, ultimately screening the divergence which can never be reached. The endogeneous determination (21) of the crash probability also ensures that the denominator $\mu_0[t_c - t] - (\sigma_0/B_0^m)W(t)$ never becomes negative: when it approaches zero, $B(t)$ blows up and the crash hazard rate increases accordingly. A crash will occur with probability 1 before the denominator reaches zero. Hence, the price $B(t)$ remains always positive and real. We stress the remarkably simple and elegant constraint on the dynamics provided by the rational expectation condition that ensures the existence and stationarity of the dynamics at all times, notwithstanding the locally nonlinear stochastic explosive dynamics. When $\mu_0 > 0$, the random walk has a positive drift attracting the denominator in (25) to zero (i.e., attracting the bubble to infinity). However, by the mechanism explained above, as $B(t)$ increases, so does the crash hazard rate by relation (21). Eventually, a crash occurs that reset the bubble to a lower price. The random walk with drift goes on, eventually $B(t)$ increases again and reaches "dangerous waters", a crash occurs again, and so on. Note that a crash is not a certain event: an inflated bubble price can also deflate spontaneously by the random realization of the random walk $W(t)$ which brings back the denominator far from zero.

Fig. 11 shows a typical trajectory of the bubble component of the price generated by the nonlinear positive feedback model of Sornette and Andersen (2002), starting from some initial value up to the time just before the price starts to blow up. The simplest version of this model consists in a bubble price $B(t)$ being essentially a power of the inverse of a random walk $W(t)$ in the following sense. Starting from $B(0) = W(0) = 0$ at the origin of time, when the random walk approaches some value W_c here taken equal to 1, $B(t)$ increases and vice versa. In particular, when $W(t)$ approaches 1, $B(t)$ blows up and reaches a singularity at the time t_c when the random walk crosses 1. This process generalizes in the random domain the finite-time singularities described in Section 5.1.1, such that the monotonously increasing process culminating at a critical time t_c is replaced by the random walk that wanders up and down before eventually reaching the critical level. This nonlinear positive feedback bubble process $B(t)$ can thus be called a "singular inverse random walk". In absence of a crash, the process $B(t)$ can exist only up to a finite time: with probability one (i.e., with certainty),

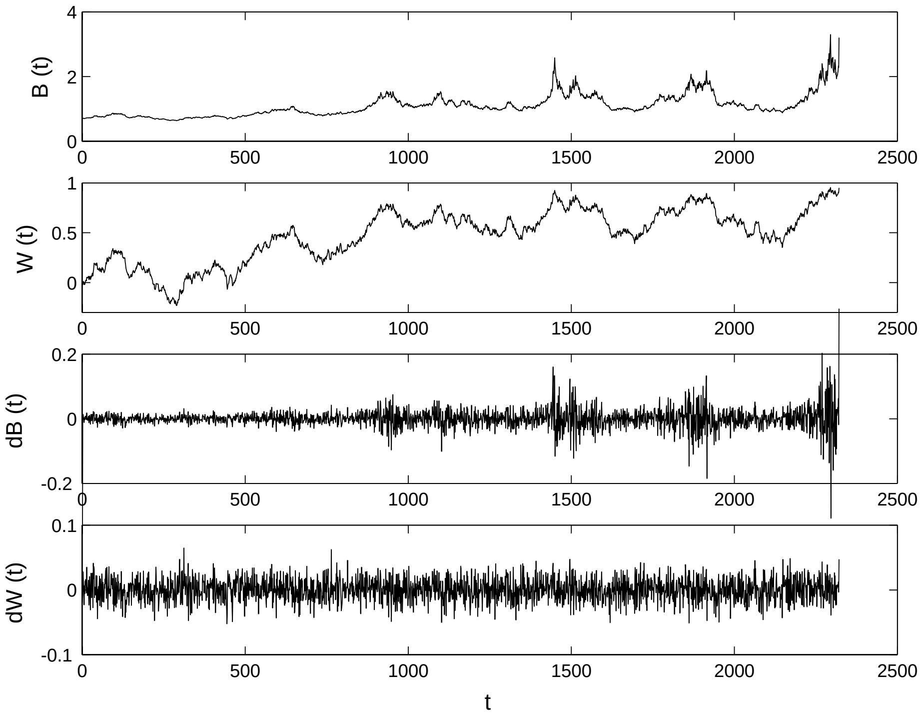

Fig. 11. Top panel: realization of a bubble price $B(t)$ as a function of time constructed from the "singular inverse random walk". This corresponds to a specific realization of the random numbers used in generating the random walks $W(t)$ represented in the second panel. The top panel is obtained by taking a power of the inverse of a constant W_c here taken equal to 1 minus the random walk shown in the second panel. In this case, when the random walk approach 1, the bubble diverges. Notice the similarity between the trajectories shown in the top ($B(t)$) and second ($W(t)$) panels as long as the random walk $W(t)$ does not approach too much the value $W_c = 1$. It is free to wander but when it approaches 1, the bubble price $B(t)$ shows much greater sensitivity and eventually diverges as $W(t)$ reaches 1. Before this happens, $B(t)$ can exhibit local peaks, i.e., local bubbles, which come back smoothly. This corresponds to a realization when the random walk approaches W_c without touching it and then spontaneously recedes away from it. The third (respectively fourth) panel shows the time series of the increments $dB(t) = B(t) - B(t-1)$ of the bubble (respectively $dW(t) = W(t) - W(t-1)$ of the random walk. Notice the intermittent bursts of strong volatility in the bubble compared to the featureless constant level of fluctuations of the random walk (reproduced from Sornette and Andersen (2002)).

we know from the study of random walks that $W(t)$ will eventually reach any level, in particular the value $W_c = 1$ in our example at which $B(t)$ diverges.

The second effect that tampers the possible divergence of the bubble price, by far the most important one in the regime of highly over-priced markets, is the impact of the price on the crash hazard rate discussed above: as the price blows up due to imitation, herding, speculation as well as randomness, the crash hazard rate increases even faster according to Eq. (21), so that a crash will occur and drive the price back closer to its fundamental value. The crashes are triggered in a random way governed by the crash hazard rate which is an increasing function of the bubble price. In the present formulation, the higher the bubble price is, the higher is the probability of a crash. In this model, a crash is similar to a purge administered to a patient.

This model (Sornette and Andersen, 2002) proposes two scenarios for the end of a bubble: either a spontaneous deflation or a crash. These two mechanisms are natural features of the model and have not been artificially added. These two scenarios are indeed observed in real markets, as will be described later.

This model has an interesting and far-reaching consequence in terms of the repetition and organization of crashes in time. Indeed, we see that each time the random walk approaches the chosen constant W_c, the bubble price blows up and, according to the no-arbitrage condition together with the rational expectations, this implies that the market enters "dangerous waters" with a crash looming ahead. The random walk model provides a very specific prediction on the waiting times between successive approaches to the critical value W_c, i.e., between successive bubbles. The distribution of these waiting times is found to be a very broad power law distribution, so broad that the average waiting time is mathematically infinite (Sornette, 2000a). In practice, this leads to two inter-related phenomena: clustering (bubbles tend to follow bubbles at short times) and long-term memory (there are very long waiting times between bubbles once a bubble has deflated for a sufficiently long time). The "singular inverse random walk" bubble model thus predicts very large intermittent fluctuations in the recurrence time of speculative bubbles.

Solution (25) can be used to invert real data during periods preceding financial crashes to obtain the relevant parameters. We present here some tests using an inversion method based on minimizing the Kolmogorov–Smirnov (KS) distance between the empirical distribution of returns and the synthetic one generated by the model, performed on the Hong Kong market prior to the crash which occurred in early 1994 and on the Nasdaq composite index prior to the crash of April 2000. To construct a meaningful distribution, we propose to add a constant fundamental price F to the bubble price $B(t)$ as only their sum is observable in real life:

$$P(t) = e^{rt}[F + B(t)] \ . \qquad (26)$$

We can also include the possibility for a interest rate r or growth of the economy with rate r. We denote $M = \mu_0/\alpha$ and $\sqrt{V} = \sigma_0/\alpha B_0^m$. For the Hang Seng index, the best fit is with $\alpha = 2.5, V = 1.1 \times 10^{-7}, M = 4.23 \times 10^{-5}, r = 0.00032$ and $F = 2267.3$. corresponding to a KS confidence level of 96.3%. This should be compared with the best Gaussian fit to the empirical price returns giving a KS confidence level of 11%. Thus the model "gaussianizes" the data at a very high significance level: a white-Gaussian noise input is transformed by the nonlinear multiplicative process into a realistically looking financial time series. For the Nasdaq composite index, we obtain $\alpha = 2.0, V = 2.1 \times 10^{-7}, M = -9.29 \times 10^{-6}, r = 0.00496$ and $F = 641.5$, corresponding to a KS confidence level of 85.9%. The corresponding best Gaussian fit to the empirical price gives a KS confidence level of 73%. Here, the improvement is less impressive but nevertheless present.

With the parameters of the model that have been obtained by the inversion, we can use them to generate many scenarios that are statistically equivalent to the real history of the Hang Seng and Nasdaq composite index. Fig. 12 shows 10 synthetic evolutions of the process (26) generated with the best parameter values for both bubbles. By comparison, the empirical prices are shown as the thick lines (one time step corresponds approximately to one trading day). The smooth continuous line close to the horizontal axis is the fundamental price Fe^{rt}.

This model together with the inversion procedure provides a new direct tool for detecting bubbles, for identifying their starting times and the plausible ends. Changing the initial time of the time series,

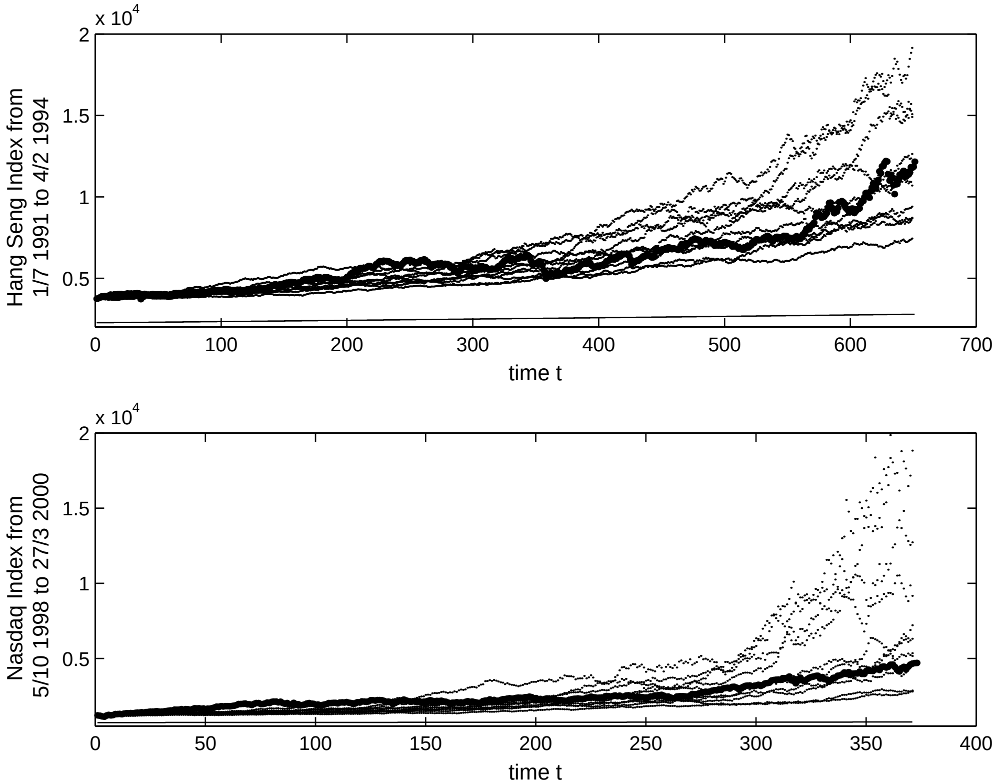

Fig. 12. Top panel: the Hang Seng index from July 1, 1991 to February 4, 1994 as well as 10 realizations of the "singular inverse random walk" bubble model generated by the nonlinear positive feedback model. Each realization corresponds to an arbitrary random walk whose drift and variance as been adjusted so as to fit best the distribution of the Heng Seng index returns. Bottom panel: the Nasdaq composite index bubble from October 5, 1998 to March 27, 2000 as well as 10 realizations of the "singular inverse random walk" bubble model generated by the nonlinear positive feedback model. Each realization corresponds to an arbitrary random walk whose drift and variance as been adjusted so as to fit best the distribution of the Nasdaq index returns (reproduced from Sornette and Andersen (2002)).

the KS probability of the resulting Gaussian fit of the transformed series $W(t)$ should allow us to determine the starting date beyond which the model becomes inadequate at a given statistical level. Furthermore, the exponent m (or equivalently α) provides a direct measure of the speculative mood. $m = 1$ is the normal regime, while $m > 1$ quantifies a positive self-reinforcing feedback. This opens the possibility for continuously monitoring it via the inversion procedure and using it as a "thermometer" of speculation. Furthermore, the variance V of the multiplicative noise is a measure of volatility, which is significantly more robust than standard estimators. This is due to the inversion of the nonlinear formula which removes a large part of the volatility clustering and of the heavy-tail nature of the distribution of returns. Its continuous monitoring via the inversion procedure suggests new ways of looking at dependence between assets. Preliminary analyses show that most of the stylized facts of financial time series are reproduced by this approach (Sornette and Andersen, 2002). These stylized facts concern the absence of two-point correlation between returns, the fat-tail structure of distributions of returns, the long-range dependence of the two-point correlation of volatility and their persistence, the multifractal structure of generalized moments of the absolute value of the returns, and so on. Application to shorter time scales covering quarters down to months should be explored to test whether this model and some of its variants may detect regime of abnormal behavior ($m \neq 1$) in financial time series.

We stress that the proposed class of nonlinear rational bubble model is fundamentally different from bubble models that have been tested previously: all previous models assumed exponentially growing bubbles and the results of statistical tests have not been convincing (Camerer, 1989; Adam and Szafarz, 1992). In contrast, bubbles may be super-exponential which make them different in principle from a fundamental price growing at a constant rate. By this work, we thus hope to rejuvenate the "old" theory of rational bubbles by extending its universe into the nonlinear stochastic regime.

An additional layer of refinement can easily be added. Indeed, following Hamilton (1989) which introduced the so-called Markov switching techniques for the analysis of price returns, many scholarly works have documented the empirical evidence of regime shifts in financial data sets (Van Norden and Schaller, 1993; Cai, 1994; Gray, 1996; Van Norden, 1996; Schaller and van Norden, 1997; Assoe, 1998; Chauvet, 1998; Driffill and Sola, 1998). For instance, Van Norden and Schaller (1997) have proposed a Markov regime switching model of speculative behavior whose key feature is similar to ours, namely over-valuation over the fundamental price increases the probability and expected size of a stock market crash.

This evidence taken together with the fact that bubbles are not expected to permeate the dynamics of the price all the time suggests the following natural extension of the model. In the simplest and most parsimonious extension, we can assume that only two regimes can occur: bubble and normal. The bubble regime follows the previous model definition and is punctuated by crashes occurring with the hazard rate governed by the price level. The normal regime can be for instance a standard random walk market model with constant small drift and volatility. The regime switches are assumed to be completely random. This very simple dynamical model recovers essentially all the stylized facts of empirical prices, i.e., no correlation of returns, long-range correlation of volatilities, fat-tail of return distributions, apparent fractality and multifractality and sharp peak-flat trough pattern of price peaks. In addition, the model predicts and we confirm by empirical data analysis that times of bubbles are associated with nonstationary increasing volatility correlations. According to this model, the apparent long-range correlation of volatility is proposed to result from random switching between normal and bubble regimes. In addition, and maybe most important, the visual appearance of price trajectories are very reminiscent of real ones, as shown in Fig. 12. The remarkably simple formulation of the price-driven "singular inverse random walk" bubble model is able to reproduce convincingly the salient properties and appearance of real price trajectories, with their randomness, bubbles and crashes.

5.3. Risk-driven versus price-driven models

In common, the risk-driven model of Section 5.1 and the price-driven model of Section 5.2 describe a system of two populations of traders, the "rational" and the "noisy" traders. Occasional imitative and herding behaviors of the "noisy" traders may cause global cooperation among traders causing a crash. The "rational" traders provide a direct link between the crash risks and the bubble price dynamics.

In the risk-driven model, the crash hazard rate determined from herding drives the bubble price. In the price-driven model, imitation and herding induce positive feedbacks on the price, which itself creates an increasing risk for a looming yet unrealized financial crash.

We believe that both models capture a part of reality. Studying them independently is the standard strategy of dividing-to-conquer the complexity of the world. The price-driven model appears maybe as the most natural and straightforward as it captures the intuition that sky-rocketing prices are unsustainable and announce endogeneously a significant correction or a crash. The risk-driven model captures a most subtle self-organization of stock markets, related to the ubiquitous balance between risk and returns. Both models embody the notion that the market anticipates the crash in a subtle self-organized and cooperative fashion, hence releasing precursory "fingerprints" observable in the stock market prices. In other words, this implies that market prices contain information on impending crashes. The next section explores the origin and nature of these precursory patterns and prepares the road for a full-fledge analysis of real stock market crashes and their precursors.

5.4. Imitation and contrarian behavior: hyperbolic bubbles, crashes and chaos

The model of bubbles and crashes that we now discuss complements the two previous models of rational expectation (RE) bubbles in that it describes a deterministic dynamics of prices embodying both the bubble phases and the crashes (Corcos et al., 2002). It is maybe the simplest analytically tractable model of the interplay between imitative and contrarian behavior in a stock market where agents can take at least two states, bullish or bearish. Each bullish (bearish) agent polls m "friends" and changes her opinion to bearish (bullish) (1) if at least $m\rho_{hb}$ ($m\rho_{bh}$) among the m agents inspected are bearish (bullish) or (2) if at least $m\rho_{hh} > m\rho_{hb}$ ($m\rho_{bb} > m\rho_{bh}$) among the m agents inspected are bullish (bearish). The condition (1) (resp. (2)) corresponds to imitative (antagonistic) behavior. In the limit where the number N of agents is infinite, by using combinatorial techniques, it can be shown that the dynamics of the fraction of bullish agents is deterministic and exhibits chaotic behavior in a significant domain of the parameter space $\{\rho_{hb}, \rho_{bh}, \rho_{hh}, \rho_{bb}, m\}$. The deterministic equation of the price trajectory is found to be of the form

$$p_{t+1} = F_m(p_t) , \tag{27}$$

where the function $F_m(x)$ is a sum of combinatorial factors. A typical chaotic trajectory can be shown to be characterized by intermittent phases of chaos, quasi-periodic behavior and super-exponentially growing bubbles followed by crashes. A typical bubble starts initially by growing at an exponential rate and then crosses over to a nonlinear power law growth rate leading to a finite-time singularity. The reinjection mechanism provided by the contrarian behavior introduces a nonlinear reinjection mechanism rounding off these singularity and leads to chaos. This model is one of the rare agent-based models that give rise to interesting nonperiodic complex dynamics in the limit of an infinite number N of agents. A finite number of agents introduces an endogeneous source of noise superimposed on the chaotic dynamics as shown in Fig. 13. One can observe burst of volatility, exploding bubbles and quiescent regimes.

The traditional concept of stock market dynamics envisions a stream of stochastic "news" that may move prices in random directions. This model, in contrast, demonstrates that certain types

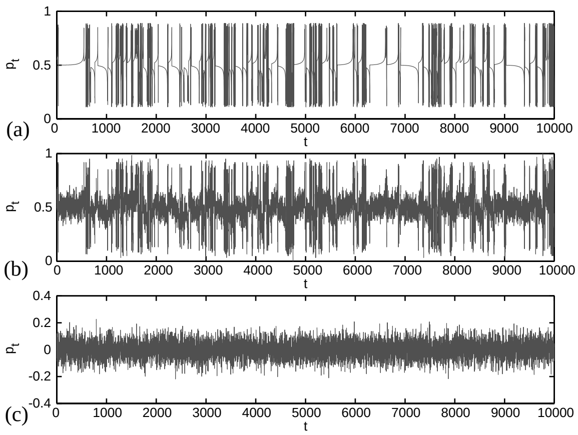

Fig. 13. Time evolution of the price p_t over 10 000 time steps for $m=60$ polled agents with (a) $N=\infty$, (b) $N=m+1=61$ agents and parameters $\rho_{hb}=\rho_{bh}=0.72$ and $\rho_{hh}=\rho_{bb}=0.85$. The panel (c) represents the noise due to the finite size of the system and is obtained by subtracting the time series in panel (a) from the time series in panel (b). Reproduced from Corcos et al. (2002).

of deterministic behavior—mimicry and contradictory behavior alone—can already lead to chaotic prices. While the traditional theory of rational anticipations exhibits and emphasizes self-re-inforcing mechanisms, without either predicting their inception nor their collapse, the strength of this model is to justify the occurrence of speculative bubbles. It allows for their collapse by taking into account the combination of mimetic and antagonistic behavior in the formation of expectations about prices. The specific feature of the model is to combine these two Keynesian aspects of speculation and enterprise and to derive from them behavioral rules based on collective opinion: the agents can adopt an imitative and gregarious behavior, or, on the contrary, anticipate a reversal of tendency, thereby detaching themselves from the current trend. It is this duality, the continuous coexistence of these two elements, which is at the origin of the properties of our model: chaotic behavior and the generation of bubbles. It is the common wisdom that deterministic chaos leads to a fundamental limit of predictability because the tiny inevitable fluctuations in those chaotic systems quickly snowball in unpredictable ways. This has been investigated in relation with for instance long-term weather patterns. In our model, the chaotic dynamics of the returns is not the limiting factor for predictability, as it contains too much residual correlations. Endogeneous fluctuations due to finite-size effects and external news (noise) seem to be needed to retrieve the observed randomness of stock market prices.

The model of imitative and contrarian behavior leads to accelerating bubble prices following finite-time singularity trajectories aborting into a crash. The accelerating phase is due to imitation. The crash is due to the contrarian behavior reinforced later by the imitation behavior. Quantitatively, the bubble-crash sequence can be described by studying the logarithm of $p-1/2$ (which is the deviation from equilibrium where the equilibrium is characterized by the equality between the fraction of bullish agents and the fraction of bearish agents) as a function of linear time. One observes first a linear trend which qualifies an exponential growth $p-1/2 \propto e^{\kappa t}$ (with the factor $\kappa > 0$), followed by a super-exponential growth accelerating so much as to give the impression of reaching a singularity in finite-time.

The understanding of this phenomenon comes from the behavior of the "elasticity" of $F_m(p) - p$ with respect to $p - 1/2$, i.e., the derivative of the logarithm of $F_m(p) - p$, where $F_m(p)$ is defined by (27), with respect to the logarithm of $p - 1/2$. Two regimes can be observed.

1. For small $p - 1/2$, the elasticity is 1, i.e.,

$$F_m(p) - p \simeq \alpha(m)\left(p - \frac{1}{2}\right) . \tag{28}$$

This expression (28) explains the exponential growth observed at early time.

2. For larger $p-1/2$, the elasticity increases above 1 and stabilizes to a value $\mu(m)$ before decreasing again due to the reinjection produced by the contrarian mechanism. The interval in $p - 1/2$ in which the slope is approximately stabilized at the value $\mu(m)$ enables us to write

$$F_m(p) - p \simeq \beta(m)\left(p - \frac{1}{2}\right)^{\mu(m)} \quad \text{with } \mu > 1 . \tag{29}$$

These two regimes can be collected in the following phenomenological expression for $F_m(p)$:

$$F_m(p) = \frac{1}{2} + (1 - 2g_m(1/2) - g'_m(1/2))\left(p - \frac{1}{2}\right) + \beta(m)\left(p - \frac{1}{2}\right)^{\mu(m)} , \tag{30}$$

$$= \frac{1}{2} + \left(p - \frac{1}{2}\right) + \alpha(m)\left(p - \frac{1}{2}\right) + \beta(m)\left(p - \frac{1}{2}\right)^{\mu(m)} \quad \text{with } \mu > 1 , \tag{31}$$

and

$$\alpha(m) = -2g_m(1/2) - g'_m(1/2) . \tag{32}$$

Introducing the notation $\varepsilon = p - 1/2$, the dynamics can be rewritten

$$\varepsilon' - \varepsilon = \alpha(m)\varepsilon + \beta(m)\varepsilon^{\mu(m)} , \tag{33}$$

which, in the continuous time limit, yields

$$\frac{d\varepsilon}{dt} = \alpha(m)\varepsilon + \beta(m)\varepsilon^{\mu(m)} . \tag{34}$$

Thus, for small ε, we obtain an exponential growth rate

$$\varepsilon_t \sim e^{\alpha(m)t} , \tag{35}$$

while for large enough ε

$$\varepsilon_t \sim (t_c - t)^{-(1/\mu(m)-1)} . \tag{36}$$

For example, for $m = 60$ with $\rho_{hb} = \rho_{bh} = 0.72$ and $\rho_{hh} = \rho_{bb} = 0.85$, $\mu(m) = 3$, which yields for large ε

$$p_t - \frac{1}{2} \sim \frac{1}{\sqrt{t_c - t}} . \tag{37}$$

The prediction (36) implies that the returns r_t should increase in an accelerating super-exponential fashion at the end of a bubble, leading to a price trajectory

$$\pi_t = \pi_c - C(t_c - t)^{(\mu(m)-2/\mu(m)-1)} , \tag{38}$$

where π_c is the culminating price of the bubble reached at $t=t_c$ when $\mu(m) > 2$, such the finite-time singularity in r_t gives rise only to an infinite slope of the price trajectory. This behavior (38) with an exponent $0 < (\mu(m) - 2/\mu(m) - 1) < 1$ has been documented in many bubbles (Sornette et al., 1996; Johansen et al., 1999, 2000; Johansen and Sornette, 1999a, b, 2000a; Sornette and Johansen, 2001; Sornette and Andersen, 2002; Sornette, 2002, 2003). The case $m = 60$ with $\rho_{hb} = \rho_{bh} = 0.72$ and $\rho_{hh} = \rho_{bb} = 0.85$ leads to $(\mu(m) - 2/\mu(m) - 1) = 1/2$, which is reasonable agreement with the values reported previously.

Interpreted within the present model, the exponent $(\mu(m) - 2/\mu(m) - 1)$ of the price singularity gives an estimation of the "connectivity" number m through the dependence of μ on m. Such a relationship has already been argued by Johansen et al. (2000) at a phenomenological level using a mean-field equation in which the exponent is directly related to the number of connections to a given agent.

This model developed recently has strong potential to provide a simple but powerful approach to modelling financial time series. It can be extended in many ways, which include (1) introducing at least a third state, called "neutral", in addition to the "bullish" and "bearish" states, (2) introducing a fundamental price, a population of value investors and assume that "noise traders" follow the imitative-contrarian strategy previously described, (3) considering the possibility for several stocks to be traded simultaneously, with in particular the introduction of a riskless asset.

6. Log-periodic oscillations decorating power laws

6.1. Status of log-periodicity

Log-periodicity is an observable signature of the symmetry of *discrete* scale invariance (DSI). DSI is a weaker symmetry than (continuous) scale invariance (Dubrulle et al., 1997). The latter is the symmetry of a system which manifests itself such that an observable $\mathcal{O}(x)$ as a function of the "control" parameter x is scale invariant under the change $x \to \lambda x$ for arbitrary λ, i.e., a number $\mu(\lambda)$ exists such that

$$\mathcal{O}(x) = \mu(\lambda)\mathcal{O}(\lambda x) . \tag{39}$$

The solution of (39) is simply a power law $\mathcal{O}(x)=x^{\alpha}$, with $\alpha=-(\log\mu/\log\lambda)$, which can be verified directly by insertion. In DSI, the system or the observable obeys scale invariance (39) only for *specific* choices of the magnification factor λ, which form in general an infinite but countable set of values $\lambda_1, \lambda_2, \ldots$ that can be written as $\lambda_n = \lambda^n$. λ is the fundamental scaling ratio determining the period of the resulting log-periodicity. This property can be qualitatively seen to encode a *lacunarity* of the fractal structure. The most general solution of (39) with λ (and therefore μ) is

$$\mathcal{O}(x) = x^{\alpha} P\left(\frac{\ln x}{\ln\lambda}\right) , \tag{40}$$

where $P(y)$ is an arbitrary periodic function of period 1 in the argument, hence the name log-periodicity. Expanding it in Fourier series $\sum_{n=-\infty}^{\infty} c_n \exp(2n\pi i(\ln x/\ln \lambda))$, we see that $\mathcal{O}(x)$ becomes a sum of power laws with the infinitely discrete spectrum of complex exponents $\alpha_n = \alpha + i2\pi n/\ln \lambda$, where n is an arbitrary integer. Thus, DSI leads to power laws with complex exponents, whose observable signature is log-periodicity. Specifically, for financial bubbles prior to large crashes, we shall see that a first order representation of Eq. (40)

$$I(t) = A + B(t_c - t)^{\beta} + C(t_c - t)^{\beta} \cos(\omega \ln(t_c - t) - \phi) \tag{41}$$

captures well the behavior of the market price $I(t)$ prior to a crash or large correction at a time $\approx t_c$.

There are many mechanisms known to generate log-periodicity (Sornette, 1998). The most obvious one is when the system possesses a pre-existing discrete hierarchical structure. There are however various dynamical mechanisms generating log-periodicity, without relying on a pre-existing discrete hierarchical structure. DSI may be produced dynamically and does not need to be pre-determined by e.g., a geometrical network. This is because there are many ways to break a symmetry, the subtlety here being to break it only partially.

6.2. Stock market price dynamics from the interplay between fundamental value investors and technical analysists

The importance of the interplay of two classes of investors, fundamental value investors and technical analysts (or trend followers), has been stressed by several recent works (see for instance Lux and Marchesi, 1999 and references therein) to be essential in order to retrieve the important stylized facts of stock market price statistics. We build on this insight and construct a simple model of price dynamics, whose innovation is to put emphasis on the fundamental *nonlinear* behavior of both classes of agents.

6.2.1. Nonlinear value and trend-following strategies

The price variation of an asset on the stock market is controlled by supply and demand, in other words by the net order size Ω through a market impact function (Farmer, 1998). Assuming that the ratio $\tilde{p}/p$ of the price $\tilde{p}$ at which the orders are executed over the previous quoted price p is solely a function of Ω and using the condition that it is impossible to make profits by repeatedly trading through a close circuit (i.e., buying and selling has to end up with a final net position equal to zero), Farmer (1998) has shown that the logarithm of the price is given by the following equation written in discrete form:

$$\ln p(t+1) - \ln p(t) = \frac{\Omega(t)}{L} \ . \tag{42}$$

The "market depth" L is the typical number of outstanding stocks traded per unit time and thus normalizes the impact of a given order size $\Omega(t)$ on the log-price variations. The net order size Ω summed over all traders is changing as a function of time so as to reflect the information flow in the market and the evolution of the traders' opinions and moods. A zero net order size $\Omega = 0$ corresponds to exact balance between supply and demand. Various derivations have established a connection between the price variation or the variation of the logarithm of the price to factors that control the net order size itself (Farmer, 1998; Bouchaud and Cont, 1998; Pandey and Stauffer, 2000).

Two basic ingredients of $\Omega(t)$ are thought to be important in determining the price dynamics: reversal to the fundamental value ($\Omega_{\rm fund}(t)$) and trend following ($\Omega_{\rm trend}(t)$). Other factors, such as risk aversion, may also play an important role.

Ide and Sornette (2002) propose to describe the reversal to estimated fundamental value by the contribution

$$\Omega_{\rm fund}(t) = -c[\ln p(t) - \ln p_{\rm f}]\,|\ln p(t) - \ln p_{\rm f}|^{n-1} , \qquad (43)$$

to the order size, where $p_{\rm f}$ is the estimated fundamental value and $n > 0$ is an exponent quantifying the nonlinear nature of reversion to $p_{\rm f}$. The strength of the reversion is measured by the coefficient $c > 0$, which reflects that the net order is negative (resp. positive) if the price is above (resp. below) $p_{\rm f}$. The nonlinear power law $[\ln p(t) - \ln p_{\rm f}]\,|\ln p(t) - \ln p_{\rm f}|^{n-1}$ of order n is chosen as the simplest function capturing the following effect. In principle, the fundamental value $p_{\rm f}$ is determined by the discounted expected future dividends and is thus dependent upon the forecast of their growth rate and of the risk-less interest rate, both variables being very difficult to predict. The fundamental value is thus extremely difficult to quantify with high precision and is often estimated within relatively large bounds: all of the methods of determining intrinsic value rely on assumptions that can turn out to be far off the mark. For instance, several academic studies have disputed the premise that a portfolio of sound, cheaply bought stocks will, over time, outperform a portfolio selected by any other method (see for instance, Lamont, 1988). As a consequence, a trader trying to track fundamental value has no incentive to react when she feels that the deviation is small since this deviation is more or less within the noise. Only when the departure of price from fundamental value becomes relatively large will the trader act. The relationship (43) with an exponent $n > 1$ precisely accounts for this effect: when n is significantly larger than 1, $|x|^n$ remains small for $|x| < 1$ and shoots up rapidly only when it becomes larger than 1, mimicking a smoothed threshold behavior. The nonlinear dependence of $\Omega_{\rm fund}(t)$ on $\ln[p(t)/p_{\rm f}] = \ln p(t) - \ln p_{\rm f}$ shown in (43) is the first novel element of our model. Usually, modellers reduce this term to the linear case $n = 1$ while, as we shall show, generalizing to larger values $n > 1$ will be a crucial feature of the price dynamics. In economic language, the exponent $n = {\rm d}\ln \Omega_{\rm fund}/{\rm d}\ln(\ln[p(t)/p_{\rm f}])$ is called the "elasticity" or "sensitivity" of the order size $\Omega_{\rm fund}$ with respect to the (normalized) log-price $\ln[p(t)/p_{\rm f}]$.

A related "sensitivity", that of the money demand to interest rate, has been recently documented to be larger than 1, similarly to the Ide–Sornette (2002) proposal of taking $n > 1$ in (43). Using a survey of roughly 2700 households, Mulligan and Sala-i-Martin (2000) estimated the interest elasticity of money demand (the sensitivity or log-derivative of money demand to interest rate) to be very small at low interest rates. This is due to the fact that few people decide to invest in interest-producing assets when rates are low, due to "shopping" costs. In contrast, for large interest rates or for those who own a significant bank account, the interest elasticity of money demand is significant. This is a clear-cut example of a threshold-like behavior characterized by a strong nonlinear response. This can be captured by $e \equiv {\rm d}\ln M/{\rm d}\ln r = (r/r_{\rm infl})^n$ with $n > 1$ such that the elasticity e of money demand M is negligible when the interest r is not significantly larger than the inflation rate $r_{\rm infl}$ and becomes large otherwise.

Trend following (in various elaborated forms) was (and probably is still) one of the major strategy used by technical analysts (see Andersen et al. (2000) for a review and references therein).

More generally, it results naturally when investment strategies are positively related to past price moves. Trend following can be captured by the following expression of the order size:

$$\Omega_{\mathrm{trend}}(t) = a_1[\ln p(t) - \ln p(t-1)] + a_2[\ln p(t) - \ln p(t-1)] \times |\ln p(t) - \ln p(t-1)|^{m-1} . \quad (44)$$

This expression corresponds to driving the price up if the preceding move was up ($a_1 > 0$ and $a_2 > 0$). The linear case ($a_1 > 0, a_2 = 0$) is usually chosen by modellers. Here, we generalize this model by adding the contribution proportional to $a_2 > 0$ from considerations similar to those leading to the nonlinear expression (43) for the reversal term with an exponent $n > 1$. We argue that the dependence of the order size at time t resulting from trend-following strategies is a nonlinear function with exponent $m > 1$ of the price change at previous time steps. Indeed, a small price change from time $t-1$ to time t may not be perceived as a significant and strong market signal. Since many of the investment strategies are nonlinear, it is natural to consider an average trend-following order size which increases in an accelerated manner as the price change increases in amplitude. Usually, trend-followers increase the size of their order faster than just proportionally to the last trend. This is reminiscent of the argument (Andersen et al., 2000) that traders's psychology is sensitive to a change of trend (acceleration or deceleration) and not simply to the trend (velocity). The fact that trend-following strategies have an impact on price proportional to the price change over the previous period raised to the power $m > 1$ means that trend-following strategies are not linear when averaged over all of them: they tend to under-react for small price changes and over-react for large ones. The second term of the right-hand-side of (44) with coefficient a_2 captures this phenomenology.

6.2.2. Nonlinear dynamical equation for stock market prices

Introducing the notation

$$x(t) = \ln[p(t)/p_{\mathrm{f}}] , \quad (45)$$

and the time scale δt corresponding to one time step, and putting all the contributions (43) and (44) into (42), with $\Omega(t) = \Omega_{\mathrm{fund}}(t) + \Omega_{\mathrm{trend}}(t)$, we get

$$x(t+\delta t) - x(t) = \frac{1}{L}(a_1[x(t) - x(t-\delta t)] + a_2[x(t) - x(t-\delta t)] \times |x(t) - x(t-\delta t)|^{m-1} - c x(t)|x(t)|^{n-1}) . \quad (46)$$

Expanding (46) as a Taylor series in powers of δt, we get

$$(\delta t)^2 \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\left[1 - \frac{a_1}{L}\right] \delta t \frac{\mathrm{d}x}{\mathrm{d}t} + \frac{a_2 (\delta t)^m}{L} \frac{\mathrm{d}x}{\mathrm{d}t} \left|\frac{\mathrm{d}x}{\mathrm{d}t}\right|^{m-1} - \frac{c}{L} x(t)|x(t)|^{n-1} + \mathcal{O}[(\delta t)^3] , \quad (47)$$

where $\mathcal{O}[(\delta t)^3]$ represents a term of the order of $(\delta t)^3$. Note the existence of the second order derivative, which results from the fact that the price variation from present to tomorrow is based on analysis of price change between yesterday and present. Hence the existence of the three time lags leading to inertia. A special case of expression (46) with a *linear* trend-following term ($a_2 = 0$) and a *linear* reversal term ($n = 1$) has been studied in Bouchaud and Cont (1998) and Farmer (1998), with the addition of a risk-aversion term and a noise term to account for all the other effects not accounted for by the two terms (43) and (44). We shall neglect risk-aversion as well as any other term and focus only on the reversal and trend-following terms previously discussed to explore the resulting price behaviors. Grassia (2000) has also studied a similar *linear* second-order differential equation derived from market delay, positive feedback and including a mechanism for quenching runaway markets.

Expression (46) is inspired by the continuous mean-field limit of the model of Pandey and Stauffer (2000), defined by starting from the percolation model of market price dynamics (Cont and Bouchaud, 2000; Chowdhury and Stauffer, 1999; Stauffer and Sornette, 1999) and developed to account for the dynamics of the Nikkei and Russian market recessions (Johansen and Sornette, 1999c, 2001b). The generalization assumes that trend-following and reversal to fundamental values are two forces that influence the probability that a trader buys or sells the market. In addition, Pandey and Stauffer (2000) consider as we do here that the dependence of the probability to enter the market is a nonlinear function with exponent $n > 1$ of the deviation between market price and fundamental price. However, they do not consider the possibility that $m > 1$ and stick to the linear trend-following case. We shall see that the analytical control offered by our continuous formulation allows us to get a clear understanding of the different dynamical phases.

Among the four terms of Eq. (47), the first term of its right-hand side is the least interesting. For $a_1 < L$, it corresponds to a damping term which becomes negligible compared to the second term in the terminal phase of the growth close to the singularity when $|\mathrm{d}x/\mathrm{d}t|$ becomes very large. For $a_1 > L$, it corresponds to a negative viscosity but the instability it provides is again subdominant for $m > 1$. The main ingredients here are the interplay between the inertia provided by the second derivative in the left-hand side, the destabilizing nonlinear trend-following term with coefficient $a_2 > 0$ and the nonlinear reversal term. In order to simplify the notation and to simplify the analysis of the different regimes, we shall neglect the first term of the right-hand side of (47), which amounts to take the special value $a_1 = L$. In a field theoretical sense, our theory is tuned right at the "critical point" with a vanishing "mass" term.

Eq. (47) can be viewed in two ways. It can be seen as a convenient short-hand notation for the intrinsically discrete equation (46), keeping the time step δt small but finite. In this interpretation, we pose

$$\alpha = a_2(\delta t)^{m-2}/L \ , \tag{48}$$

$$\gamma = c/L(\delta t)^2 \ , \tag{49}$$

which depend explicitly on δt, to get

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = \alpha \frac{\mathrm{d}x}{\mathrm{d}t} \left| \frac{\mathrm{d}x}{\mathrm{d}t} \right|^{m-1} - \gamma x(t)|x(t)|^{n-1} \ . \tag{50}$$

A second interpretation is to genuinely take the continuous limit $\delta t \to 0$ with the constraints $a_2/L \sim (\delta t)^{2-m}$ and $c/L \sim (\delta t)^2$. This allow us to define the now δt-independent coefficients α and γ according to (48) and (50) and obtain the truly continuous equation (50). This equation can also be

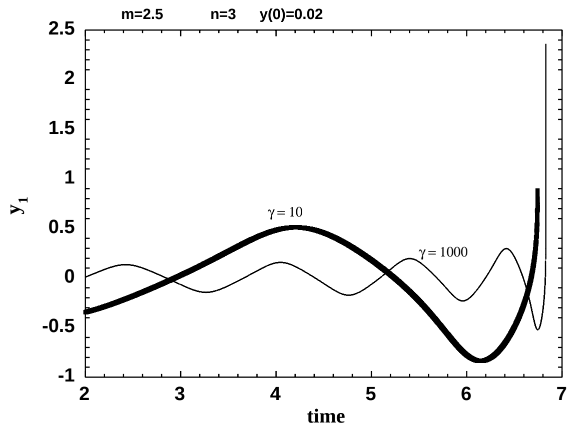

Fig. 14. "Reduced price" as a function of time for a trend-following exponent $m = 2.5$ with $n = 3$, $\alpha = 1$ and with two amplitudes $\gamma = 10$ and $\gamma = 1000$ of the fundamental reversal term. Reproduced from Ide and Sornette (2002).

written as

$$\frac{dy_1}{dt} = y_2 \ , \tag{51}$$

$$\frac{dy_2}{dt} = \alpha y_2|y_2|^{m-1} - \gamma y_1|y_1|^{n-1} \ . \tag{52}$$

This system leads to a finite-time singularity with accelerating oscillations for $m > 1$ and $n > 1$. The richness of behaviors results from the competition between these two terms.

6.2.3. Dynamical properties

The origin $(y_1 = 0, y_2 = 0)$ plays a special role as the unstable (for $m > 1$) fixed point around which spiral structures of trajectories are organized in phase space (y_1, y_2). It is particularly interesting that this point plays a special role since $y_1 = 0$ means that the observed price is equal to the fundamental price. If, in addition, $y_2 = 0$, there is no trend, i.e., the market "does not know" which direction to take. The fact that this is the point of instability around which the price trajectories organize themselves provides a fundamental understanding of the cause of the complexity of market price time series based on the instability of the fundamental price "equilibrium".

Fig. 14 shows the reduced price for the trend-following exponent $m = 2.5$. In this case, the reduced price goes to a constant at t_c with an infinite slope (the singularity is thus on its derivative, or "velocity"). We can also observe accelerating oscillations, reminiscent of log-periodicity. The novel feature is that the oscillations are only transient, leaving place to a pure final accelerating trend in the final approach to the critical time t_c.

Fig. 15 shows that the oscillations with varying frequency and amplitude seen in Fig. 14 are nothing but the projection on one axis of a spiraling structure in the plane. Actually, Fig. 15 shows more than that: in the plane of the reduced price y_1 and its "velocity" y_2, it shows two special trajectories that connect exactly the origin $y_1 = 0, y_2 = 0$ to infinity. From general mathematical

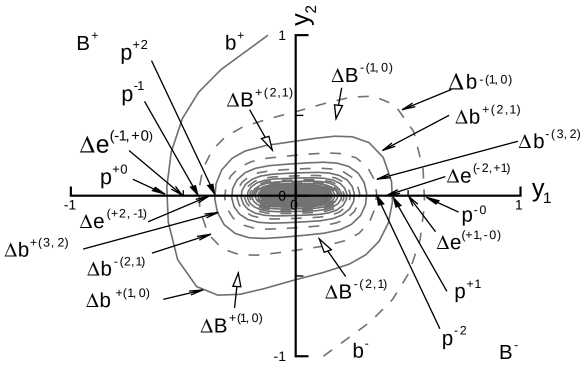

Fig. 15. Geometrical spiral showing two special trajectories (the continuous and dashed lines) in the “reduced price”–“velocity” plane (y_1, y_2) that connect exactly the origin $y_1 = 0, y_2 = 0$ to infinity. This spiraling structure, which exhibits scaling or fractal properties, is at the origin of the accelerating oscillations decorating the power law behavior close to the finite-time singularity. Reproduced from Ide and Sornette (2002).

theorems of dynamical systems, one can then show that any trajectory starting close to the origin will never be able to cross any of these two orbits. As a consequence, any real trajectory will be guided within the spiraling channel, winding around the central point 0 many times before exiting towards the finite-time singularities. The approximately log-periodic oscillations result from the oscillatory structure of the fundamental reversal term associated with the acceleration driven by the trend-following term. The conjunction of the two leads to the beautiful spiral, governing a hierarchical organization of the spiralling trajectories around the origin in the price-velocity space. See Ide and Sornette (2002) for a detailed mathematical study of this system.

In sum, the simple two-dimensional dynamical system (51,52) embodies two nonlinear terms, exerting respectively positive feedback and reversal, which compete to create a singularity in finite time decorated by accelerating oscillations. The power law singularity results from the increasing growth rate. The oscillations result from the restoring mechanism. As a function of the order of the nonlinearity of the growth rate and of the restoring term, a rich variety of behavior is observed. The dynamical behavior is traced back fundamentally to the self-similar spiral structure of trajectories in phase space unfolding around an unstable spiral point at the origin. The interplay between the restoring mechanism and the nonlinear growth rate leads to approximately log-periodic oscillations with remarkable scaling properties.

7. Autopsy of major crashes: universal exponents and log-periodicity

7.1. The crash of October 1987

As discussed in Section 2, the crash of October 1987 and its black Monday on October 19 remains one of the most striking drops ever seen on stock markets, both by its overwhelming amplitude and its encompassing sweep over most markets worldwide. It was preceded by a remarkably strong “bull” regime epitomized by the following quote from Wall Street Journal, on August 26, 1987, the day after the 1987 market peak: “In a market like this, every story is a positive one. Any news is

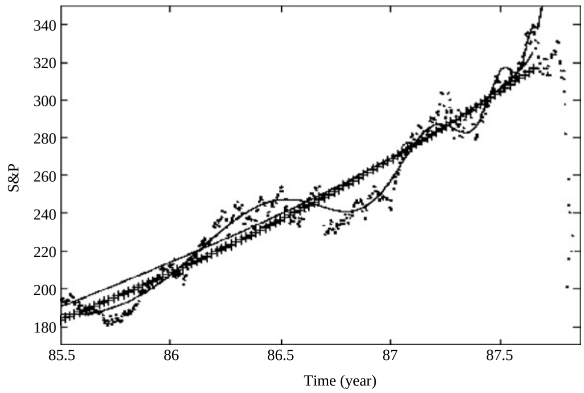

Fig. 16. Evolution as a function of time of the New York stock exchange index S&P500 from July 1985 to the end of October 1987 (557 trading days). The + represent a constant return increase of ≈ 30%/year and gives $var(F_{exp}) \approx 113$ (see text for definition). The best fit to the power-law (53) gives $A_1 \approx 327$, $B_1 \approx -79$, $t_c \approx 87.65$, $m_1 \approx 0.7$ and $\text{var}_{pow} \approx 107$. The best fit to expression (54) gives $A_2 \approx 412$, $B_2 \approx -165$, $t_c \approx 87.74$, $C \approx 12$, $\omega \approx 7.4$, $T = 2.0$, $m_2 \approx 0.33$ and $\text{var}_{lp} \approx 36$. One can observe four well-defined oscillations fitted by the expression (54), before finite size effects limit the theoretical divergence of the acceleration, at which point the bubble ends in the crash. All the fits are carried over the whole time interval shown, up to 87.6. The fit with Eq. (54) turns out to be very robust with respect to this upper bound which can be varied significantly. Reproduced from Sornette et al. (1996).

good news. It's pretty much taken for granted now that the market is going to go up". Investors were thus largely unaware of the forthcoming risk happenings (Grant, 1990).

7.1.1. Precursory pattern

Time is often converted into decimal year units: for nonleap years, 365 days = 1.00 year which leads to 1 day = 0.00274 years. Thus 0.01 year = 3.65 days and 0.1 year = 36.5 days or 5 weeks. For example, October 19, 1987 corresponds to 87.800.

Fig. 16 shows the evolution of the New York stock exchange index S&P500 from July 1985 to the end of October 1987 after the crash. The plusses (+) represent the best fit to an exponential growth obtained by assuming that the market is given an average return of about 30% per year. This first representation does not describe the apparent overall acceleration before the crash, occurring already more than a year in advance. This acceleration (*cusp*-like shape) is better represented by using power law functions that Sections 5 and 6 showed to be signatures of a critical behavior of the market. The monotonic line corresponds to the following power law parameterization:

$$F_{pow}(t) = A_1 + B_1(t_c - t)^{m_1} , \tag{53}$$

where t_c denotes the time at which the powerlaw fit of the S&P500 presents a (theoretically) diverging slope, announcing an imminent crash. In order to qualify and compare the fits, the variances, denoted var equal to the mean of the squares of the errors between theory and data, or its square-root called the root-mean-square (r.m.s.) are calculated. The ratio of two variances corresponding to two different hypotheses is taken as a qualifying statistic. The ratio of the variance of the constant rate hypothesis to that of the power law is equal to $\text{var}_{exp}/\text{var}_{pow} \approx 1.1$ indicating only a slightly better performance of the power law in capturing the acceleration, the number of free variables being the same and equal to 2.

However, already to the naked eye, the most striking feature in this acceleration is the presence of systematic oscillatory-like deviations. Inspired by the insight given in Section 5 and especially Section 6, the oscillatory continuous line is obtained by fitting the data by the following mathematical expression:

$$F_{lp}(t) = A_2 + B_2(t_c - t)^{m_2}[1 + C\cos(\omega \log((t_c - t)/T))] \ . \tag{54}$$

This equation is the simplest example of a log-periodic correction to a pure power law for an observable exhibiting a singularity at the time t_c at which the crash has the highest probability to occur. The log-periodicity here stems from the cosine function of the logarithm of the distance $t_c - t$ to the critical time t_c. Due to log-periodicity, the evolution of the financial index becomes (discretely) scale-invariant close to the critical point.

The log-periodic correction to scaling implies the existence of a hierarchy of characteristic time intervals $t_c - t_n$, given by the expression

$$T_n = T_c - (T_c - T_0)\lambda^{-n} \ , \tag{55}$$

with a preferred scaling ratio denoted λ. For the October 1987 crash, we find $\lambda \simeq 1.5 - 1.7$ (this value is remarkably universal and is found approximately the same for other crashes as we shall see). We expect a cut-off at short time scales (i.e. above $n \sim$ a few units) and also at large time scales due to the existence of finite size effects. These time scales $t_c - t_n$ are not universal but depend upon the specific market. What is expected to be universal are the ratios $(t_c - t_{n+1})/(t_c - t_n) = \lambda$. For details on the fitting procedure, we refer to Sornette et al. (1996).

It is possible to generalize the simple log-periodic power law formula used in Fig. 16 by using a mathematical tool, called bifurcation theory, to obtain its generic nonlinear correction, that allows one to account quantitatively for the behavior of the Dow Jones and S&P500 indices up to 8 years prior to the October 1987. The result of this theory presented in Sornette and Johansen (1997) is used to generate the fit shown in Fig. 17. One sees clearly that the new formula accounts remarkably well for almost eight years of market price behavior compared to only a little more than two years for the simple log-periodic formula shown in Fig. 16. The nonlinear theory developed in Sornette and Johansen (1997) leads to "log-frequency modulation", an effect first noticed empirically in Feigenbaum and Freund (1996). The remarkable quality of the fits shown in Figs. 16 and 17 have been assessed in Johansen and Sornette (1999b).

In a recent reanalysis, Feigenbaum (2001) examined the data in a new way by taking the first differences for the logarithm of the S&P500 from 1980 to 1987. The rational for taking the price variation rather than the price itself is that the fluctuations, noises or deviations are expected to be more random and thus more innocuous than for the price which is a cumulative quantity. By rigorous hypothesis testing, Feigenbaum found that the log-periodic component cannot be rejected at the 95%-confidence level: in plain words, this means that the probability that the log-periodic component results from chance is about or less than 0.05.

7.1.2. Aftershock patterns

If the concept of a crash as a kind of critical point has any value, we should be able to identify post-crash signatures of the underlying cooperativity. In fact, we should expect an at least qualitative

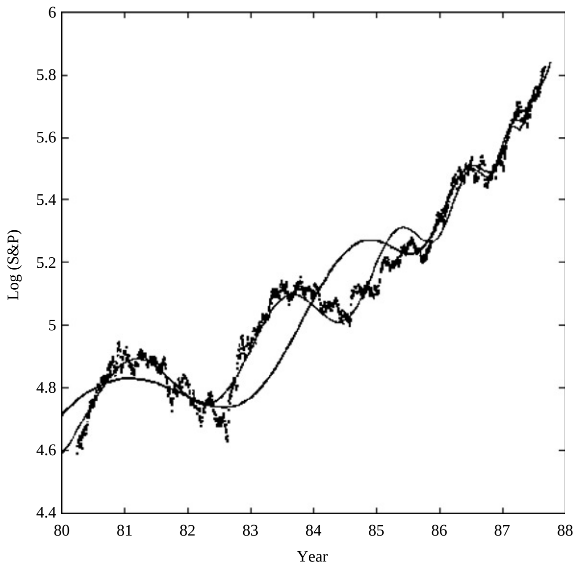

Fig. 17. Time dependence of the logarithm of the New York stock exchange index S&P500 from January 1980 to September 1987 and best fit by the improved nonlinear log-periodic formula developed in Sornette and Johansen (1997) (thin line). The exponent and log-periodic angular frequency are $m_2 = 0.33$ and $\omega^{1987} = 7.4$. The crash of October 19, 1987 corresponds to 1987.78 decimal years. The thick line is the fit by (54) on the subinterval from July 1985 to the end of 1987 and is represented on the full time interval starting in 1980. The comparison with the thin line allows one to visualize the frequency shift described by the nonlinear theory. Reproduced from Sornette and Johansen (1997).

symmetry between patterns before and after the crash. In other words, we should be able to document the existence of a critical exponent as well as log-periodic oscillations on relevant quantities after the crash. Such a signature in the volatility of the S&P500 index (a measure of the market risk perceived by investors), implied from the price of S&P500 options, can indeed be seen in Fig. 18.

Fig. 18 presents the time evolution of the implied volatility of the S&P500, taken from Chen et al. (1995). The perceived market risk is small prior to the crash, jumps up abruptly at the time of the crash and then decays slowly over several months. This decay to "normal times" of perceived risks is compatible with a slow power law decay decorated by log-periodic oscillations, which can be fitted by expression (54) with $t_c - t$ (before the crash) replaced by $t - t_c$ (after the crash). Our analysis with (54) with $t_c - t$ replaced by $t - t_c$ gives again an estimation of the position of the critical time t_c, which is found correctly within a few days. Note the long time scale covering a period of the order of a year involved in the relaxation of the volatility after the crash to a level comparable to the one before the crash. This implies the existence of a "memory effect": market participants remain nervous for quite a long time after the crash, after being burned out by the dramatic event.

It is also noteworthy that the S&P500 index as well as other markets worldwide have remained close to the after-crash level for a long time. For instance, by February 29, 1988, the world index stood at 72.7 (reference 100 on September 30, 1987). Thus, the price level established in the October crash seems to have been a virtually unbiased estimate of the average price level over the subsequent months (see also Fig. 19). This is in support of the idea of a critical point, according to which the event is an intrinsic signature of a self-organization of the markets worldwide.

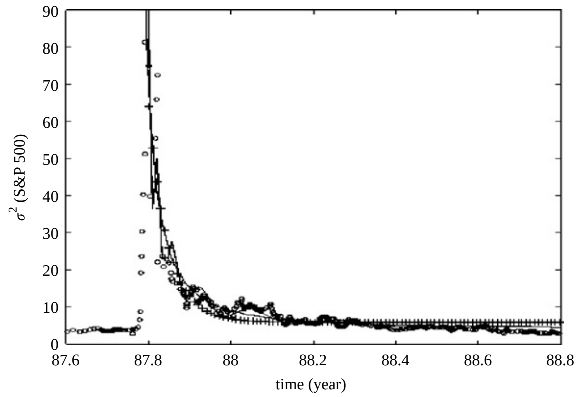

Fig. 18. Time evolution of the implied volatility of the S&P500 index (in logarithmic scale) after the October 1987 crash, taken from Chen et al. (1995). The $+$ represent an exponential decrease with $var(F_{\mathrm{exp}}) \approx 15$. The best fit to a power-law, represented by the monotonic line, gives $A_1 \approx 3.9$, $B_1 \approx 0.6$, $t_c = 87.75$, $m_1 \approx -1.5$ and $\mathrm{var}_{\mathrm{pow}} \approx 12$. The best fit to expression (54) with $t_c - t$ replaced by $t - t_c$ gives $A_2 \approx 3.4$, $B_2 \approx 0.9$, $t_c \approx 87.77$, $C \approx 0.3$, $\omega \approx 11$, $m_2 \approx -1.2$ and $\mathrm{var}_{lp} \approx 7$. One can observe six well-defined oscillations fitted by (54). Reproduced from Sornette et al. (1996).

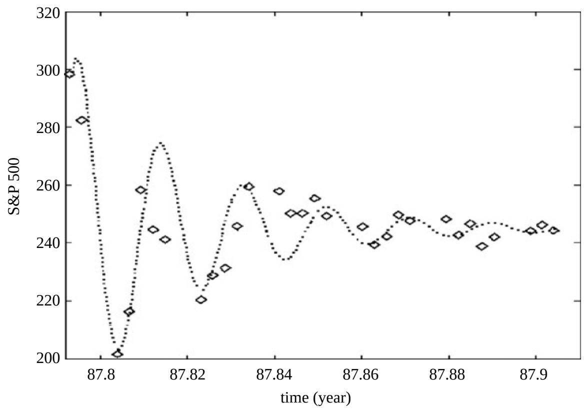

Fig. 19. Time evolution of the S&P500 index over a time window of a few weeks after the October 19, 1987 crash. The fit with an exponentially decaying sinusoidal function shown in dashed line suggests that a good model for the short-time response of the U.S. market is a *single* dissipative harmonic oscillator or damped pendulum. Reproduced from Sornette et al. (1996).

There is another striking signature of the cooperative behavior of the U.S. market, found by analyzing the time evolution of the S&P500 index over a time window of a few weeks after the October 19, 1987 crash. A fit shown in Fig. 19 with an exponentially decaying sinusoidal function suggests that the U.S. market behaved, for a few weeks after the crash, as a *single* dissipative harmonic oscillator, with a characteristic decay time of about one week equal to the period of the oscillations. In other words, the price followed the trajectory of a pendulum moving back and forth with damped oscillations around an equilibrium position.

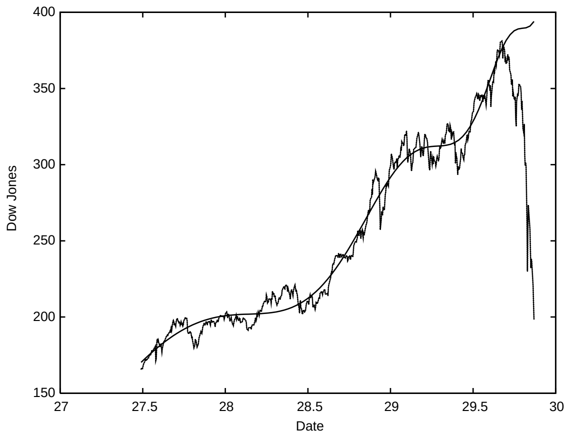

Fig. 20. The Dow Jones index prior to the October 1929 crash on Wall Street. The fit shown as a continuous line is Eq. (54) with $A_2 \approx 571$, $B_2 \approx -267$, $B_2C \approx 14.3, m_2 \approx 0.45, t_c \approx 1930.22$, $\omega \approx 7.9$ and $\phi \approx 1.0$. Reproduced from Johansen and Sornette (1999a).

This signature strengthens the view of a market as a cooperative self-organizing system. The basic story suggested by these figures is the following. Before the crash, imitation and speculation were rampant and led to a progressive "aggregation" of the multitude of agents into a large effective "super-agent", as illustrated in Figs. 16 and 17; right after the crash, the market behaved as a single "super-agent" finding rapidly the equilibrium price through a return to "equilibrium", as shown in Fig. 19. On longer time scales, the "super-agent" progressively was fragmented and the diversity of behaviors was rejuvenated as seen from Fig. 18.

7.2. The crash of October 1929

The crash of October 1929 is the other major historical market event of the twentieth century. Notwithstanding the differences in technologies and the absence of computers and other modern means of information transfer, the October 1929 crash exhibits many similarities with the October 1987 crash, so much so as shown in Figs. 20 and 21, that one can wonder about the similitudes: what has not changed over the history of mankind is the interplay between human's crave for exchanges and profits, and their fear of uncertainty and losses. The similarity between the two situations in 1929 and 1987 was in fact noticed at a qualitative level in an article in the *Wall Street Journal* on October 19, 1987, the very morning of the day of the stock market crash (with a plot of stock prices in the 1920s and the 1980s). See the discussion in Shiller (1989).

The similarity between the two crashes can be made quantitative by comparing the fit of the Dow Jones index with formula (54) from June 1927 till the maximum before the crash in October 1929, as shown in Fig. 20, to the corresponding fit for the October 1987 crash shown in Fig. 16. Notice the similar widths of the two time windows, the similar acceleration and oscillatory structures, quantified by similar exponents m_2 and log-periodic angular frequency ω: $m_2^{1987} = 0.33$ compared to $m_2^{1929} = 0.45$; $\omega^{1987} = 7.4$ compared to $\omega^{1987} = 7.9$. These numerical values are remarkably close and can be considered equal to within their uncertainties.

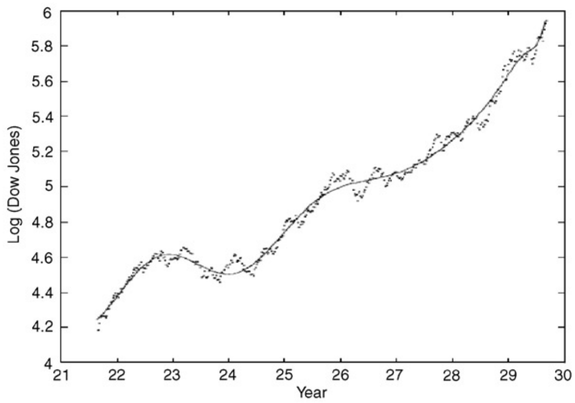

Fig. 21. Time dependence of the logarithm of the Dow Jones stock exchange index from June 1921 to September 1929 and best fit by the improved nonlinear log-periodic formula developed in Sornette and Johansen (1997). The crash of October 23, 1929 corresponds to 1929.81 decimal years. The parameters of the fit are: r.m.s. = 0.041, $t_c = 1929.84$ year, $m_2 = 0.63$, $\omega = 5.0$, $\Delta\omega = -70$, $\Delta t = 14$ years, $A_2 = 61$, $B_2 = -0.56$, $C = 0.08$. $\Delta\omega$ and Δt are two new parameters introduced in Sornette and Johansen (1997). Reproduced from Sornette and Johansen (1997).

Fig. 21 for the October 1929 crash is the analog of Fig. 17 for the October 1987 crash. It uses the improved nonlinear log-periodic formula developed in Sornette and Johansen (1997) over a much larger time window starting in June 1921. Also according to this improved theoretical formulation, the values of the exponent m_2 and of the log-periodic angular frequency ω for the two great crashes are quite close to each other: $m_2^{1929} = 0.63$ and $m_2^{1987} = 0.68$. This is in agreement with the universality of the exponent m_2 predicted from the renormalization group theory for log-periodicity (Saleur and Sornette, 1996; Sornette, 1998). A similar universality is also expected for the log-frequency, albeit with a weaker strength as it has been shown (Saleur and Sornette, 1996) that fluctuations and noise will modify ω differently depending on their nature. The fits indicate that $\omega_{1929} = 5.0$ and $\omega_{1987} = 8.9$. These values are not unexpected and fall within the range found for other crashes (see below). They correspond to a preferred scaling ratio equal respectively to $\lambda_{1929} = 3.5$ compared to $\lambda_{1987} = 2.0$.

The October 1929 and October 1987 thus exhibit two similar precursory patterns on the Dow Jones index, starting respectively 2.5 and 8 years before them. It is thus a striking observation that essentially similar crashes have punctuated this century, notwithstanding tremendous changes in all imaginable ways of life and work. The only thing that has probably changed little are the way humans think and behave. The concept that emerges here is that the organization of traders in financial markets leads intrinsically to “systemic instabilities”, that probably result in a very robust way from the fundamental nature of human beings, including our gregarious behavior, our greediness, our instinctive psychology during panics and crowd behavior and our risk aversion. The global behavior of the market, with its log-periodic structures that emerge as a result of the cooperative behavior of traders, is reminiscent of the process of the emergence of intelligent behavior at a macroscopic scale that individuals at the microscopic scale cannot perceive. This process has been discussed in biology for instance in animal populations such as ant colonies or in connection with the emergence of consciousness (Anderson et al., 1988).

There are however some differences between the two crashes. An important quantitative difference between the great crash of 1929 and the collapse of stock prices in October 1987 was that stock price variability in the year following the crash was much higher in 1929 than in 1987 (Romer, 1990). This has led economists to argue that the collapse of stock prices in October 1929 generated significant temporary increased uncertainty about future income that led consumers to forgo purchases of durable goods. Forecasters were then much more uncertain about the course of future income following the stock market crash than was typical even for unsettled times. Contemporary observers believed that consumer uncertainty was an important force depressing consumption, that may have been an important factor in the strengthening of the great depression. The increase of uncertainty after the October 1987 crash has led to a smaller effect, as no depression ensued. However, Fig. 18 clearly quantifies an increased uncertainty and risk, lasting months after the crash.

7.3. The three Hong Kong crashes of 1987, 1994 and 1997

Hong Kong has a strong free-market attitude, characterized by very few restrictions on both residents and nonresidents, private persons or companies, to operate, borrow, repatriate profit and capital. This goes on even after Hong Kong reverted to Chinese sovereignty on July 1st, 1997 as a Special Administrative Region (SAR) of the People's Republic of China, as it was promised a "high degree of autonomy" for at least 50 years from that date according to the terms of the Sino-British Joint Declaration. The SAR is ruled according to a mini-constitution, the Basic Law of the Hong Kong SAR. Hong Kong has no exchange controls and crossborder remittances are readily permitted. These rules have not changed since July 1st, 1997 when China took over sovereignty from the UK. Capital can thus flow in and out of the Hong Kong stock market in a very fluid manner. There are no restrictions on the conversion and remittance of dividends and interest. Investors bring their capital into Hong Kong through the open exchange market and remit it the same way.

Accordingly, we may expect speculative behavior and crowd effects to be free to express themselves in their full force. Indeed, the Hong Kong stock market provides maybe the best textbook-like examples of speculative bubbles decorated by log-periodic power law accelerations followed by crashes. Over the last 15 years only, one can identify three major bubbles and crashes. They are indicated as I, II and III in Fig. 22.

1. The first bubble and crash are shown in Fig. 23 and are synchronous to the worldwide October 1987 crash already discussed. On October 19, 1987, the Hang Seng index closed at 3362.4. On October 26, it closed at 2241.7, corresponding to a cumulative loss of 33.3%.
2. The second bubble ends in early 1994 and is shown in Fig. 24. The bubble ends by what we could call a "slow crash": on February 4, 1994, the Hang Seng index topped at 12157.6 and, a month later on March 3, 1994, it closed at 9802, corresponding to a cumulative loss of 19.4%. It went even further down over the next two months, with a close at 8421.7 on May, 9, 1994, corresponding to a cumulative loss since the high on February 4 of 30.7%.
3. The third bubble, shown in Fig. 25 ended in mid-august 1997 by a slow and regular decay until October 17, 1997, followed by an abrupt crash: the drop from 13601 on October 17 to 9059.9 on October 28 corresponds to a 33.4% loss. The worst daily plunge of 10% was the third biggest percentage fall following the 33.3% crash in October 1987 and 21.75% fall after the Tiananmen Square crackdown in June 1989.

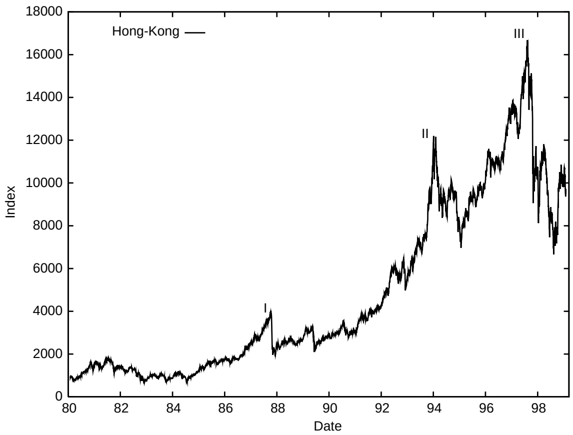

Fig. 22. The Hong Kong stock market index as a function of time. Three extended bubbles followed by large crashes can be identified. The approximate dates of the crashes are October 87 (I), January 94 (II) and October 97 (III). Reproduced from Johansen and Sornette (2001b).

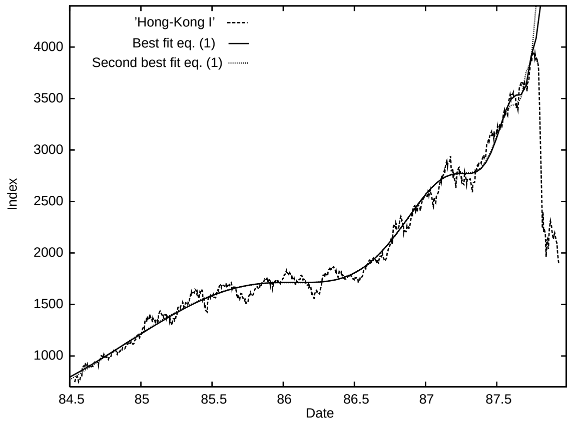

Fig. 23. Hong Kong stock market bubble ending with the crash of October 87. On October 19, 1987, the Hang Seng index closed at 3362.4. On October 26, it closed at 2241.7, corresponding to a loss of 33.3%. See Table 2 for the parameter values of the fit with Eq. (54). Reproduced from Johansen and Sornette (2001b).

Table 2 gives the parameters of the fits with Eq. (54) of the bubble phases of the three events I, II and III shown in Figs. 23–25. It is quite remarkable that the three bubbles on the Hong Kong stock market have essentially the same log-periodic angular frequency ω within $\pm 15\%$. These values are also quite similar to what has been found for bubbles on the USA market and for the FOREX (see below). In particular, for the October 1997 crash on the Hong Kong market, we have $m_2^{1987} = 0.33 < m_2^{\mathrm{HK}1997} = 0.34 < m_2^{1929} = 0.45$ and $\omega^{1987} = 7.4 < \omega^{\mathrm{HK}1997} = 7.5 < \omega^{1929} = 7.9$; the exponent m_2 and the log-periodic angular frequency ω for the October 1997 crash on the

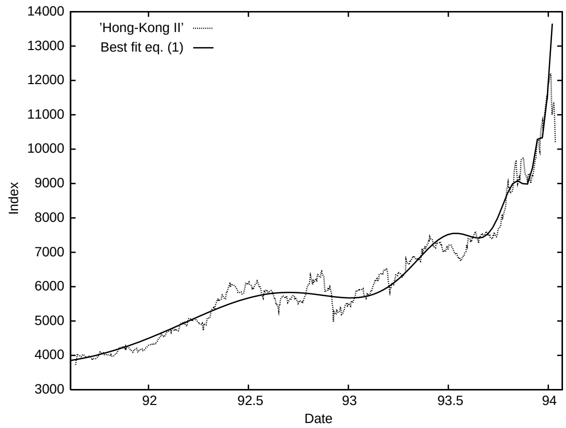

Fig. 24. Hong Kong stock market bubble ending with the crash of early 1994. On February 4, 1994, the Hang Seng index topped at 12157.6. A month later, on March 3, 1994, it closed at 9802, corresponding to a cumulative loss of 19.4%. It went even further down two months later, with a close at 8421.7 on May, 9, 1994, corresponding to a cumulative loss since the high on February 4 of 30.7%. See Table 2 for the parameter values of the fit with equation (54). Reproduced from Johansen and Sornette (2001b).

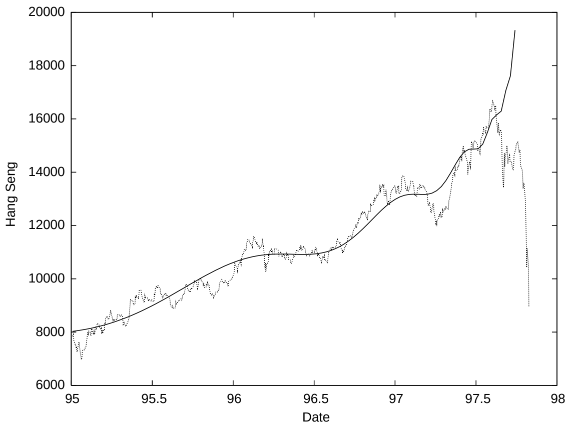

Fig. 25. The Hang Seng index prior to the October 1997 crash on the Hong Kong Stock Exchange. The index topped at 16460.5 on August 11, 1997. It then regularly decayed to 13601 reached on October 17, 1997. It then crashed abruptly reaching a close of 9059.9 on October 28, 1997, with an intra-day low of 8775.9. The amplitude of the total cumulative loss since the high on August 11 is 45%. The amplitude of the crash from October 17 to October 28 is 33.4%. The fit is Eq. (54) with $A_2 \approx 20077$, $B_2 \approx -8241$, $C \approx -397$, $m_2 \approx 0.34$, $t_c \approx 1997.74$, $\omega \approx 7.5$ and $\phi \approx 0.78$. Reproduced from Johansen and Sornette (1999a, 2001b).

Hong Kong Stock Exchange are perfectly bracketed by the two main crashes on Wall Street! Fig. 26 demonstrates the "universality" of the log-periodic component of the signals in the three bubbles preceding the three crashes on the Hong Kong market.

Table 2
Fit parameters of the three speculative bubbles on the Hong Kong stock market shown in Figs. 23–25 leading to a large crash. Multiple entries correspond to the two best fits. Reproduced from Johansen and Sornette (2001b)

Stock market	A_2	B_2	B_2C	m_2	t_c	ω	ϕ
Hong Kong I	5523; 4533	−3247; −2304	171; −174	0.29; 0.39	87.84; 87.78	5.6; 5.2	−1.6; 1.1
Hong Kong II	21121	−15113	−429	0.12	94.02	6.3	−0.6
Hong Kong III	20077	−8241	−397	0.34	97.74	7.5	0.8

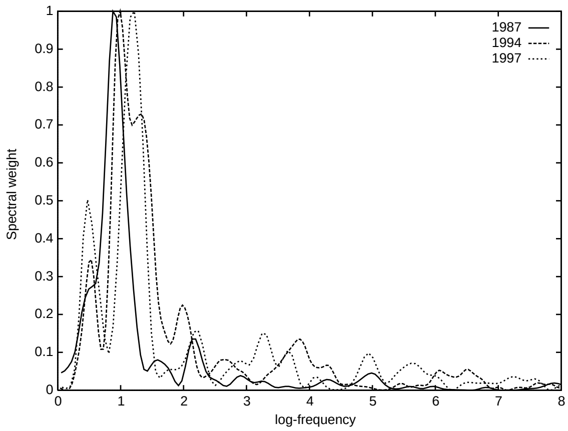

Fig. 26. Lomb spectral analysis of the three bubbles preceding the three crashes on the Hong Kong market is shown in Figs. 23–25. See Press et al. (1992) for explanations on the Lomb spectral analysis. All three bubbles are characterized by almost the same "universal" log-frequency $f \approx 1$ corresponding to a preferred scaling ratio of the discrete scale invariance equal to $\lambda = \exp(1/f) \approx 2.7$. Courtesy A. Johansen.

7.4. The crash of October 1997 and its resonance on the U.S. market

The Hong Kong market crash of October 1997 has been presented as a textbook example where contagion and speculation took a course of their own. When Malaysian Prime Minister Dr Mahathir Mohamad made his now famous address to the World Bank-International Monetary Fund seminar in Hong Kong in September 1997, many critics pooh-poohed his proposal to ban currency speculation as an attempt to hide the fact that Malaysia's economic fundamentals were weak. They pointed to the fact that the currency turmoil had not affected Hong Kong, whose economy was basically sound. Thus, if Malaysia and other countries were affected, that's because their economies were weak. At that time, it was easy to point out the deficits in the then current account of Thailand, Malaysia and Indonesia. In contrast, Hong Kong had a good current account situation and moreover had solid foreign reserves worth U.S.$88 billion. This theory of the strong-won't-be-affected already suffered a setback when the Taiwan currency's peg to the U.S. dollar had to be removed after the Taiwan authorities spent U.S.$5 billion to defend their currency from speculative attacks, and then gave up. The "coup de grace" came with the meltdown in Hong Kong in October 1997 which shocked the analysts and the media as this high-flying market was considered the safest haven in Asia.

In contrast to the meltdown in Asia's lesser markets as country after country, led by Thailand in July 1997, succumbed to economic and currency problems, Hong Kong was supposed to be different. With its Western-style markets, the second largest in Asia after Japan, it was thought to be immune to the financial flu that had swept through the rest of the continent. It is clear from our analysis of Section 5 and from the lessons of the two previous bubbles ending in October 1987 and in early 1994 that those assumptions naively overlooked the contagion leading to over-investments in the build-up period preceding the crash and the resulting instability, which left the Hong Kong market vulnerable to speculative attacks. Actually, hedge funds in particular are known to have taken positions consistent with a possible crisis on the currency and on the stock market, by "shorting" (selling) the currency to drive it down, forcing the Hong Kong government to raise interest rates to defend it by increasing the currency liquidity but as a consequence having equities suffer, making the stock market more unstable.

As we have already stressed, one should not mix the "local" cause from the fundamental cause of the instability. As the late George Stigler once put it, to blame 'the markets' for an outcome we don't like is like blaming the waiters in restaurants for obesity. Within the framework defended here (see also Sornette, 2003), crashes occur as possible (but not necessary) outcomes of a long preparation, that we refer for short as "herding", which makes the market enter into a more and more unstable regime. When in this state, there are many possible "local" causes that may cause it to stumble. Pushing the argument to the extreme to make it crystal clear, it is as if the responsibility for the collapse of the infamous Tacoma Narrows Bridge that once connected mainland Washington with the Olympic peninsula was attributed to strong wind. It is true that, on November 7, 1940, at approximately 11:00 AM, it suddenly collapsed after developing a remarkably "ordered" sway in response to a strong wind after it had been open to traffic for only a few months (see Tacoma Narrows Bridge historical film footage showing in 250 frames (10 s) the maximum torsional motion shortly before failure of this immense structure: http://cee.carleton.ca/Exhibits/Tacoma_Narrows/). However, the strong wind of that day is only the "local" cause while there is a more fundamental cause: the bridge, like most objects, has a small number of characteristic vibration frequencies, and one day the wind was exactly of the strength needed to excite one of them. The bridge responded by vibrating at this characteristic frequency so strongly, i.e., by "resonating", that it fractured the supports holding it together. The fundamental cause of the collapse of the Tacoma Narrows Bridge thus lies in an error of conception that enhanced the role of one specific mode of resonance. We propose that, analogously to the collapse of the Tacoma Narrows Bridge, many stock markets crash as the results of built-in or acquired instabilities. These instabilities may in turn be revealed by "small" perturbations that lead to the collapse.

The speculative attacks in periods of market instabilities are sometimes pointed at as possible causes of serious potential hazards for developing countries when allowing the global financial markets to have free play, especially when these countries come under pressure to open up their financial sectors to large foreign banks, insurance companies, stock broking firms and other institutions, under the World Trade Organization's financial services negotiations. We argue that the problem comes in fact fundamentally from the over-enthusiastic initial in-flux of capital as a result of herding, that initially profits the country, but at the risk of future instabilities: developing countries as well as investors "cannot have the cake and eat it too!" From an efficient market view point, the speculative attacks are nothing but the revelation of the instability and the means by which the markets are forced back to a more stable dynamical state.

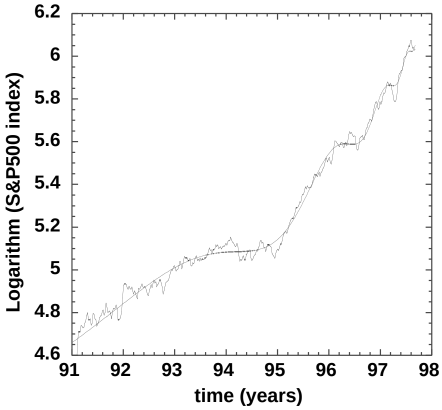

Fig. 27. The best fit shown as the smooth continuous line of the logarithm of the S&P500 index from January 1991 till September 4, 1997 (1997.678) by the improved nonlinear log-periodic formula developed in Sornette and Johansen (1997), already used in Figs. 17 and 21. The exponent m_2 and log-periodic angular frequency ω are respectively $m_2 = 0.73$ (compared to 0.63 for October 1929 and 0.33 for October 1987) and $\omega = 8.93$ (compared to 5.0 for October 1929 and 7.4 for October 1987). The critical time predicted by this fit is $t_c = 1997.948$, i.e., mid-December 1997. Courtesy A. Johansen.

Interestingly, the October 1997 crash on the Hong Kong market had important echos in other markets worldwide and in particular in the U.S. markets. The story is often told as if a "wave of selling", starting in Hong Kong, has spread first to other southeast Asian markets based on negative sentiment—which served to reaffirm the deep financial problems of the Asian tiger nations—then to the European markets, and finally to the U.S. market. The shares that were hardest hit in Western markets were the multinational companies, which receive part of their earnings from the southeast Asian region. The reason for their devaluation is that the region's economic slowdown would lower corporate profits. It is estimated that the 25 companies which make up one third of Wall Street's S&P500 index market capitalisation earn roughly half of their income from non-U.S. sources. Lower growth in southeast Asia heightened one of the biggest concerns of Wall Street investors. To carry on the then present "bull" run, the market needed sustained corporate earnings—if they were not forthcoming, the cycle of rising share prices would whither into one of falling share prices. Concern over earnings might have proved to be the straw that broke Wall Street's six-year bull run.

Fingerprints of herding and of an incoming instability were detected by several groups independently and announced publicly. According to our theory, the turmoil on the financial U.S. market in October 1997 should not be seen only as a passive reaction to the Hong Kong crash. The log-periodic power law signature observed on the U.S. market over several years before October 1997 (see Fig. 27) indicates that a similar "herding" instability was also developing simultaneously. In fact, the detection of log-periodic structures and a prediction of a stock market correction or a crash at the end of October 1997 was formally issued jointly ex-ante on September 17, 1997 by A. Johansen and the author, to the French office for the protection of proprietary softwares and inventions with registration number 94781. In addition, a trading strategy has been devised using put options in order to provide an experimental test of the theory. A 400% profit has been obtained in a two week period covering the mini-crash of October 28, 1997. The proof of this profit is available from a Merrill Lynch client cash management account released in November 1997. Using a variation of our theory which turns out to be slightly less reliable (see the comparative tests in Johansen and Sornette, 1999b), a group of physicists and economists (Vandewalle et al., 1998a) also made a public announcement published on September 18, 1997 in a Belgium journal (Dupuis, 1997) and communicated afterwards their methodology in a scientific publication (Vandewalle et al., 1998b). Two other groups have also analyzed, after the fact, the possibility to predict this event. Feigenbaum and Freund (1998) analyzed the log-periodic oscillations in the S&P500 and the NYSE in relation to the October 27'th "correction" seen on Wall Street. Gluzman and Yukalov (1998) proposed a new approach based on the algebraic self-similar renormalization group to analyze the time series corresponding to the October 1929 and 1987 crashes and the October 1997 correction of the New York Stock Exchange (NYSE) (Gluzman and Yukalov, 1998).

The best fit of the logarithm of the S&P500 index from January 1991 till September 4, 1997 by the improved nonlinear log-periodic formula developed in Sornette and Johansen (1997), already used in Figs. 17 and 21 is shown in Fig. 27. This result and many other analyses led to the prediction alluded to above. It turned out that the crash did not really occur: what happened was that the Dow plunged 554.26 points, finishing the day down 7.2%, and NASDAQ posted its biggest-ever (up to that time) one-day point loss. In accordance with a new rule passed after October 1987 "Black Monday", trading was halted on all major U.S. exchanges. Private communications from professional traders to the author indicate that many believed that a crash was coming but this turns out to be incorrect. This sentiment has also to be put in the perspective of the earlier sell-off at the beginning of the month triggered by Greenspan's statement that the boom in the U.S. economy was unsustainable and that the current rate of gains in the stock market was unrealistic.

It is actually interesting that the critical time t_c identified around this data indicated a change of regime rather than a real crash: after this turbulence, the U.S. market remained more or less flat, thus breaking the previous "bullish" regime, with large volatility until the end of January 1998, and then started again a new "bull" phase stopped in its course in August 1998, that we shall analyze below. The observation of a change of regime after t_c is in full agreement with the rational expectation model of a bubble and crash described in Section 5: the bubble expands, the market believes that a crash may be more and more probable, the prices develop characteristic structures of speculation and herding but the critical time passes without the crash happening. This can be interpreted as the nonzero probability scenario also predicted by the rational expectation model of a bubble and crash described in Section 5, that it is possible that no crash occurs over the whole lifetime of the bubble including t_c.

The simultaneity of the critical times t_c of the Hong Kong crash and of the end of the U.S. and European speculative bubble phases at the end of October 1997 may be neither a lucky occurrence nor a signature of a causal impact of one market (Hong Kong) onto others, as has been often discussed too naively. This simultaneity can actually be predicted in a model of rational expectation bubbles allowing the coupling and interactions between stock markets. For general interactions, if a critical time appears in one market, it should also be present in other markets as a result of the nonlinear interactions existing between the markets (Johansen and Sornette, 2001a).

7.5. Currency crashes

Currencies can also develop bubbles and crashes. The bubble on the dollar starting in the early 1980s and ending in 1985 is a remarkable example shown in Fig. 28.

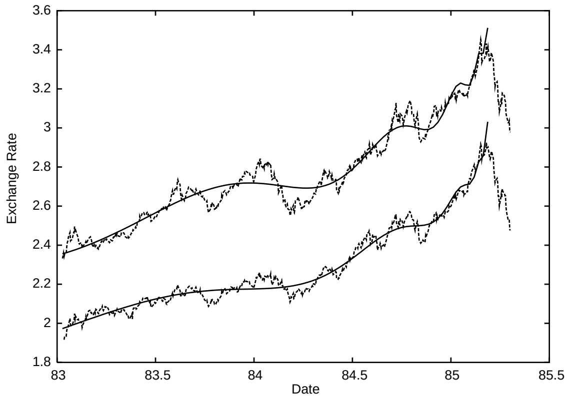

Fig. 28. The U.S.$ expressed in German Mark DEM (top curve) and in Swiss franc CHF (bottom curve) prior to its collapse on mid-1985. The fit to the DEM currency against the U.S. dollar with Eq. (54) is shown as the continuous and smooth line and give $A_2 \approx 3.88$, $B_2 \approx -1.2$, $B_2C \approx 0.08$, $m_2 \approx 0.28$, $t_c \approx 1985.20$, $\omega \approx 6.0$ and $\phi \approx -1.2$. The fit to the Swiss franc against the U.S. dollar with Eq. (54) gives $A_2 \approx 3.1$, $B_2 \approx -0.86$, $B_2C \approx 0.05$, $m_2 \approx 0.36$, $t_c \approx 1985.19$, $\omega \approx 5.2$ and $\phi \approx -0.59$. Note the small fluctuations in the value of the scaling ratio $2.2 \leqslant \lambda \leqslant 2.7$, which constitutes one of the key test of our "critical herding" theory. Reproduced from Johansen and Sornette (1999a).

The U.S. dollar experienced an unprecedented cumulative appreciation against the currencies of the major industrial countries starting around 1980, with several consequences: loss of competitiveness with important implications for domestic industries, increase of the U.S. merchandise trade deficit by as much as $45 billion by the end of 1983, with export sales about $35 billion lower and the import bill $10 billion higher. For instance, in 1982, it was already expected that, through its effects on export and import volume, the appreciation would reduce real gross national product by the end of 1983 to a level 1–1.5% lower than the 3rd quarter 1980 pre-appreciation level (Feldman, 1982). The appreciation of the U.S. dollar from 1980 to 1984 was accompanied by substantial decline in prices for the majority of manufactured imports from Canada, Germany, and Japan. However, for a substantial minority of prices, the imported items' dollar prices rose absolutely and in relation to the general U.S. price level. The median change was a price decline of 8% for imports from Canada and Japan, and a decrease of 28% for goods from Germany (Fieleke, 1985). As a positive effect, the impact on the U.S. inflation outlook was to improve it very significantly. There is also evidence that the strong dollar in the first half of the 1980s forced increased competition in U.S. product markets, especially vis-a-vis continental Europe (Knetter, 1994).

As we explained in Section 5, according to the rational expectation theory of speculative bubbles, prices can be driven up by an underlying looming risk of a strong correction or crash. Such a possibility has been advocated as an explanation for the strong appreciation of the U.S. dollar from 1980 to early 1985 (Kaminsky and Peruga, 1991). If the market believes that a discrete event may occur when the event does not materialize for some time, this may have two consequences: drive price up and lead to an apparent inefficient predictive performance of forward exchange rates (forward and future contracts are financial instruments that track closely "spot" prices as they embody the best information on the expectation of market participants on near-term spot price in the future). Indeed, from October 1979 to February 1985, forward rates systematically underpredicted the strength of the

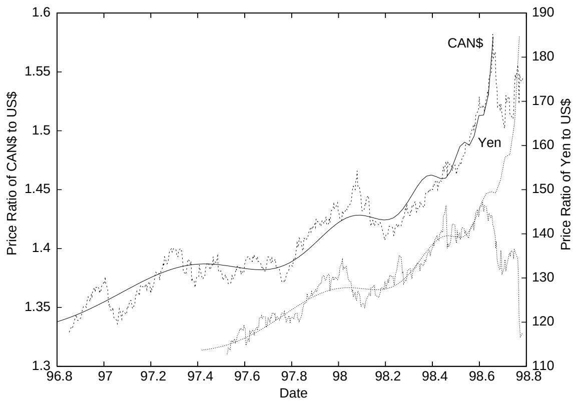

Fig. 29. The U.S. dollar expressed in CAN$ and YEN currencies prior to its drop starting in August 1998. The fit with Eq. (54) to the two exchange rates gives $A_2 \approx 1.62$, $B_2 \approx -0.22$, $B_2C \approx -0.011$, $m_2 \approx 0.26$, $t_c \approx 98.66$, $\phi \approx -0.79$, $\omega \approx 8.2$ and $A_2 \approx 207$, $B_2 \approx -85$, $B_2C \approx 2.8$, $m_2 \approx 0.19$, $t_c \approx 98.78$, $\phi \approx -1.4$, $\omega \approx 7.2$, respectively. Reproduced from Johansen et al. (1999).

U.S. dollar. Two discrete events could be identified as governing market expectations (Kaminsky and Peruga, 1991): (1) change in monetary regime in October 1979 and the resulting private sector doubts about the Federal Reserve's commitment to lower money growth and inflation; (2) private sector anticipation of the dollar's depreciation beginning in March 1985, i.e., anticipation of a strong correction, exactly as in the bubble-crash model of Section 5. The corresponding characteristic power law acceleration of bubbles decorated by log-periodic oscillations is shown in Fig. 28.

Expectations of future exchange rate have been shown to be excessive in the posterior period from 1985.2 to 1986.4, indicating bandwagon effects at work and the possibility of a rational speculative bubble (MacDonald and Torrance, 1988). As usual before a strong correction or a crash, analysts were showing over-confidence and there were many reassuring talks of the absence of a significant danger of collapse of the dollar, which has risen to unprecedented heights against foreign currencies (Holmes, 1985). On the long term however, it was clear that such a strong dollar was unsustainable and there were indications that the dollar was overvalued, in particular because foreign exchange markets generally hold that a nation's currency can remain strong over the longer term, only if the nation's current account is healthy: in contrast, for the first half of 1984, the U.S. current account suffered a seasonally adjusted deficit of around \$44.1 billion.

A similar but somewhat attenuated bubble of the U.S. dollar expressed respectively in Canadian dollar and Japanese Yen, extending over slightly less than a year and bursting in the summer of 1998, is shown in Fig. 29. Paul Krugman has suggested that this run-up on the Yen and Canadian dollar, as well as the near collapse of U.S. financial markets at the end of the summer of 1998, which is discussed in the next section, are the un-wanted "byproduct of a vast get-richer-quick scheme by a handful of shadowy financial operators" which backfired (Krugman, 1998). The remarkable quality of the fits of the data with our theory does indeed give credence to the role of speculation, imitation and herding, be them spontaneous, self-organized or manipulated in part. Actually, Frankel and Froot (1988, 1990) have found that, over the period 1981–1985, the market shifted away from the fundamentalists and toward the chartists or trend-followers.

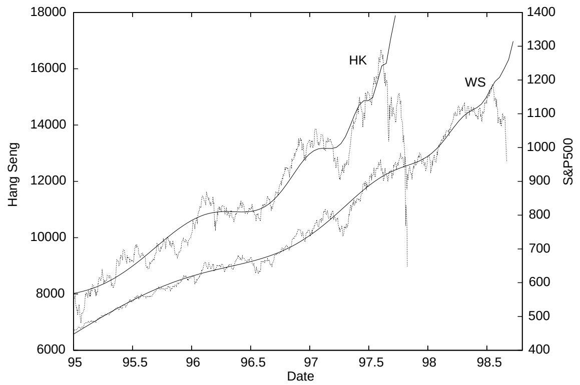

Fig. 30. The Hang Seng index prior to the October 1997 crash on the Hong-Kong Stock Exchange already shown in Fig. 25 and the S&P500 stock market index prior to the crash on Wall Street in August 1998. The fit to the S&P500 index is Eq. (54) with $A_2 \approx 1321$, $B_2 \approx -402$, $B_2C \approx 19.7$, $m_2 \approx 0.60$, $t_c \approx 98.72$, $\phi \approx 0.75$ and $\omega \approx 6.4$. Reproduced from Johansen et al. (1999).

7.6. The crash of August 1998

From its top on mid-June 1998 (1998.55) to its bottom on the first days of September 1998 (1998.67), the U.S. S&P500 stock market lost 19%. This "slow" crash and in particular the turbulent behavior of the stock markets worldwide starting mid-august are widely associated with and even attributed to the plunge of the Russian financial markets, the devaluation of its currency and the default of the government on its debts obligations.

The analysis presented in Fig. 30 suggests a different story: the Russian event may have been the triggering factor but not the fundamental cause! One can observe clear fingerprints of a kind of speculative herding, starting more than three years before, with its characteristic power law acceleration decorated by log-periodic oscillations. Table 3 gives a summary of the parameters of the log-periodic power law fit to the main bubbles and crashed discussed until now. The crash of August 1998 is seen to fit nicely in the family of crashes with "herding" signatures.

This indicates that the stock market was again developing an unstable bubble which would have culminated at some critical time $t_c \approx 1998.72$, close to the end of September 1998. According to the rational expectation bubble models of Section 5, the probability for a strong correction or a crash was increasing as t_c was approached, with a raising susceptibility to "external" perturbations, such as news or financial difficulties occurring somewhere in the "global village". The Russian meltdown was just such a perturbation. What is remarkable is that the U.S. market contained somehow the information of an upcoming instability through its unsustainable accelerated growth and structures! The financial world being an extremely complex system of interacting components, it is not farfetched to imagine that Russia was led to take actions against its unsustainable debt policy at the time of a strongly increasing concern by many about risks on investments made in developing countries.

The strong correction starting mid-august was not specific to the U.S. markets. Actually, it was much stronger in some other markets, such as the German market. Indeed, within the period of only 9 months preceding July 1998, the German DAX index went up from about 3700 to almost 6200 and then quickly declined over less than one month to below 4000. Precursory log-periodic structures

Table 3
Summary of the parameters of the log-periodic power law fit to the main bubbles and crashes discussed in this section (see Figs. 31, 32 and 33 below for the April 2000 crash on the Nasdaq and the two crashes on IBM and on Procter & Gamble)

Crash	t_c	t_{max}	t_{min}	drop	m_2	ω	λ	A_2	B_2	B_2C	Var
1929 (WS)	30.22	29.65	29.87	47%	0.45	7.9	2.2	571	−267	14.3	56
1985 (DEM)	85.20	85.15	85.30	14%	0.28	6.0	2.8	3.88	1.16	−0.77	0.0028
1985 (CHF)	85.19	85.18	85.30	15%	0.36	5.2	3.4	3.10	−0.86	−0.055	0.0012
1987 (WS)	87.74	87.65	87.80	30%	0.33	7.4	2.3	411	−165	12.2	36
1997 (H-K)	97.74	97.60	97.82	46%	0.34	7.5	2.3	20077	−8241	−397	190360
1998 (WS)	98.72	98.55	98.67	19%	0.60	6.4	2.7	1321	−402	19.7	375
1998 (YEN)	98.78	98.61	98.77	21%	0.19	7.2	2.4	207	−84.5	2.78	17
1998 (CAN$)	98.66	98.66	98.71	5.1%	0.26	8.2	2.2	1.62	−0.23	−0.011	0.00024
1999 (IBM)	99.56	99.53	99.81	34%	0.24	5.2	3.4				
2000 (P& G)	00.04	00.04	00.19	54%	0.35	6.6	2.6				
2000 (Nasdaq)	00.34	00.22	00.29	37%	0.27	7.0	2.4				

t_c is the critical time predicted from the fit of each financial time series to the Eq. (54). The other parameters of the fit are also shown. $\lambda = \exp[2\pi/\omega]$ is the preferred scaling ratio of the log-periodic oscillations. The error Var is the variance between the data and the fit and has units of price × price. Each fit is performed up to the time t_{max} at which the market index achieved its highest maximum before the crash. t_{min} is the time of the lowest point of the market disregarding smaller "plateaus". The percentage drop is calculated as the total loss from t_{max} to t_{min}. Reproduced from Johansen et al. (1999).

have been documented for this event over the nine months preceding July 1998 (Drozdz et al., 1999), with the addition that analogous log-periodic oscillations occurred also on smaller time scales as precursors of smaller intermediate decreases, with similar preferred scaling ratio λ at the various levels of resolution. However, the reliability of these observations at smaller time scales established by visual inspection in Drozdz et al. (1999) remain to be established with rigorous statistical tests.

7.7. The Nasdaq crash of April 2000

In the last few years of the second Millenium, there was a growing divergence in the stock market between "New Economy" and "Old Economy" stocks, between technology and almost everything else. Over 1998 and 1999, stocks in the Standard & Poor's technology sector have risen nearly four-fold, while the S&P500 index has gained just 50%. And without technology, the benchmark would be flat. In January 2000 alone, 30% of net inflows into mutual funds went to science and technology funds, versus just 8.7% into S&P500 index funds. As a consequence, the average price-over-earning ratio P/E for Nasdaq companies was above 200 (corresponding to a ridiculous earning yield of 0.5%), a stellar value above anything that serious economic valuation theory would consider reasonable. It is worth recalling that the very same concept and wording of a "New Economy" was hot in the minds and mouths of investors in the 1920s and in the early 1960s as already mentioned. In the 1920s, the new technologies of the time were General Electric, ATT and other electric and communication companies, and they also exhibited impressive price appreciations of the order of hundreds of percent in an 18 month time intervals before the 1929 crash. In the early 1960s, the growth stocks were in the new electronic industry like Texas Instruments and Varian Associates, which expected to exhibit a very fast rate of earning growth, were highly prized and far outdistanced the standard blue-chip stocks. Many companies associated with the esoteric high-tech of space travel and electronics sold in 1961 for over 200 times their previous year's earning. The "tronics boom", as it was called, has actually remarkably similar features to the "new economy" boom preceding the October 1929 crash or the "new economy" boom of the late 1990s, ending in the April 2000 crash on the Nasdaq index.

The Nasdaq Composite index dropped precipiteously with a low of 3227 on April 17, 2000, corresponding to a cumulative loss of 37% counted from its all-time high of 5133 reached on March 10, 2000. The Nasdaq Composite consists mainly of stock related to the so-called "New Economy", i.e., the Internet, software, computer hardware, telecommunication and so on. A main characteristic of these companies is that their price-earning-ratios (P/E's), and even more so their price-dividend-ratios, often came in three digits prior to the crash. Some companies, such as VA LINUX, actually had a *negative* Earning/Share of -1.68. Yet they were traded around \$40 per share which is close to the price of Ford in early March 2000. Opposed to this, "Old Economy" companies, such as Ford, General Motors and DaimlerChrysler, had P/E ≈ 10. The difference between "Old Economy" and "New Economy" stocks is thus the expectation of *future earnings* (Sornette, 2000b): investors, who expect an enormous increase in for example the sale of Internet and computer related products rather than in car sales, are hence more willing to invest in Cisco rather than in Ford notwithstanding the fact that the earning-per-share of the former is much smaller than for the later. For a similar price per share (approximately \$60 for Cisco and \$55 for Ford), the earning per share was \$0.37 for Cisco compared to \$6.0 for Ford (Cisco has a total market capitalisation of \$395 billions (close of April 14, 2000) compared to \$63 billions for Ford). In the standard fundamental valuation formula, in which the expected return of a company is the sum of the dividend return and of the growth rate, "New Economy" companies are supposed to compensate for their lack of present earnings by a fantastic potential growth. In essence, this means that the bull market observed in the Nasdaq in 1997–2000 has been fueled by expectations of increasing future earnings rather than economic fundamentals (and by the expectation that others will expect the same thing and will help increase the capital gains): the price-to-dividend ratio for a company such as Lucent Technologies (LU) with a capitalization of over \$300 billions prior to its crash on the 5 January 2000 is over 900 which means that you get a higher return on your checking account(!) unless the price of the stock increases. Opposed to this, an "Old Economy" company such as DaimlerChrysler gives a return which is more than 30 times higher. Nevertheless, the shares of Lucent Technologies rose by more than 40% during 1999 whereas the share of DaimlerChrysler declined by more than 40% in the same period. The recent crashes of IBM, LU and Procter & Gamble (P&G) correspond to a loss equivalent to many countries state budget. And this is usually attributed to a "business-as-usual" corporate statement of a slightly revised smaller-than-expected earnings!

These considerations make it credible that it is the *expectation* of future earnings and future capital gains rather than present economic reality that motivates the average investor, thus creating a speculative bubble. It has also been proposed (Mauboussin and Hiler, 1999) that better business models, the network effect, first-to-scale advantages and real options effect could account for the apparent over-valuation, providing a sound justification for the high prices of dot.com and other new-economy companies. These interesting views expounded in early 1999 were in synchrony with the bull market in 1999 and preceding years. They participated in the general optimistic view and

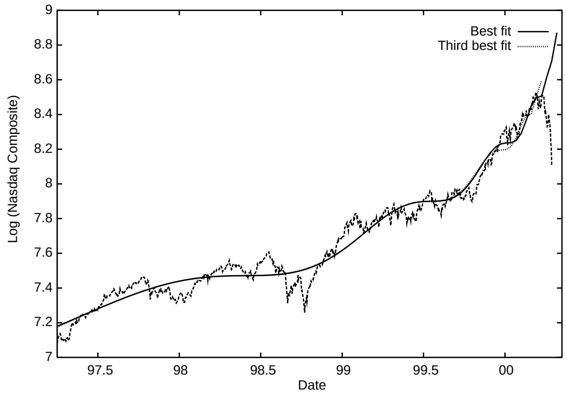

Fig. 31. Best (r.m.s. ≈ 0.061) and third best (r.m.s. ≈ 0.063) fits with Eq. (54) to the natural logarithm of the Nasdaq Composite. The parameter values of the fits are $A_2 \approx 9.5$, $B_2 \approx -1.7$, $B_2C \approx 0.06$, $m_2 \approx 0.27$, $t_c \approx 2000.33$, $\omega \approx 7.0$, $\phi \approx -0.1$ and $A_2 \approx 8.8$, $B_2 \approx -1.1$, $B_2C \approx 0.06$, $m_2 \approx 0.39$, $t_c \approx 2000.25$, $\omega \approx 6.5$, $\phi \approx -0.8$, respectively. Reproduced from Johansen and Sornette (2000a).

added to the herding strength. They seem less attractive in the context of the bearish phase of the Nasdaq market that has followed its crash in April 2000 and which is still running more than two years later: Koller and Zane (2001) argue that the traditional triumvirate, earnings growth, inflation, and interest rates, explains most of the growth and decay of U.S. indices (while not excluding the existence of a bubble of hugely capitalized new-technology companies).

Indeed, as already stressed, history provides many examples of bubbles, driven by unrealistic expectations of future earnings, followed by crashes (White, 1996; Kindleberger, 2000). The same basic ingredients are found repeatedly: fueled by initially well-founded economic fundamentals, investors develop a self-fulfilling enthusiasm by an imitative process or crowd behavior that leads to an unsustainable accelerating overvaluation. We propose that the fundamental origin of the crashes on the U.S. markets in 1929, 1962, 1987, 1998 and 2000 belongs to the same category, the difference being mainly in which sector the bubble was created: in 1929, it was utilities; in 1962, it was the electronic sector; in 1987, the bubble was supported by a general deregulation with new private investors with high expectations; in 1998, it was strong expectation on investment opportunities in Russia that collapsed; in 2000, it was the expectations on the Internet, telecommunication and so on that have fueled the bubble. However, sooner or later, investment values always revert to a fundamental level based on real cash flows.

Fig. 31 shows the logarithm of the Nasdaq Composite fitted with the log-periodic power law equation (54). The data interval to fit was identified using the same procedure as for the other crashes: the first point is the lowest value of the index prior to the onset of the bubble and the last point is that of the all-time high of the index. There exists some subtlety with respect to identifying the onset of the bubble, the end of the bubble being objectively defined as the date where the market reached its maximum. A bubble signifies an acceleration of the price. In the case of Nasdaq, it tripled from 1990 to 1997. However, the increase was a factor 4 in the 3 years preceding the current crash thus defining an "inflection point" in the index. In general, the identification of such an "inflection point" is quite straightforward on the most liquid markets, whereas this is not always

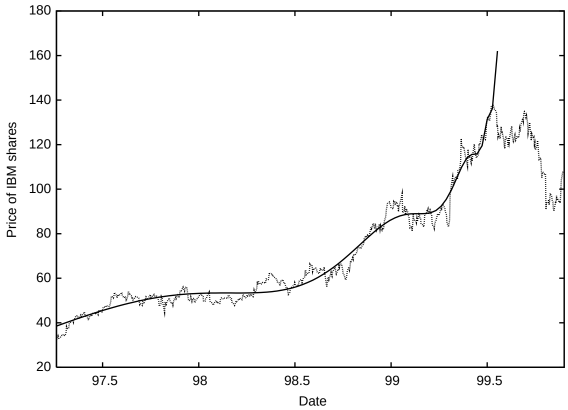

Fig. 32. Best (r.m.s. ≈ 3.7) fit with equation (54) to the price of IBM shares. The parameter values of the fits are $A_2 \approx 196$, $B_2 \approx -132$, $B_2C \approx -6.1$, $m_2 \approx 0.24$, $t_c \approx 99.56$, $\omega \approx 5.2$ and $\phi \approx 0.1$. Reproduced from Johansen and Sornette (2000a).

the case for the emergent markets (Johansen and Sornette, 2001b). With respect to details of the methodology of the fitting procedure, we refer the reader to Johansen et al. (1999).

Undoubtedly, observers and analysts have forged post-mortem stories linking the April 2000 crash in part with the effect of the crash of Microsoft Inc. resulting from the breaking of negotiations during the weekend of April 1st with the U.S. federal government on the antitrust issue, as well as from many other factors. Here, we interpret the Nasdaq crash as the natural death of a speculative bubble, anti-trust or not, the results presented here strongly suggesting that the bubble would have collapsed anyway. However, according to our analysis based on the probabilistic model of bubbles described in Sections 5 and 6, the exact timing of the death of the bubble is not fully deterministic and allows for stochastic influences, but within the remarkably tight bound of about one month (except for the slow 1962 crash).

Log-periodic critical signatures can also be detected on individual stocks as shown in Figs. 32 for IBM and 33 for Procter & Gamble. These two figures offer a quantification of the precursory signals. The signals are more noisy than for large indices but nevertheless clearly present. There is a weaker degree of generality for individual stocks as the valuation of a company is also a function of many other idiosyncratic factors associated with the specific course of the company. Dealing with broad market indices averages out all these specificities to mainly keep track of the overall market "sentiment" and direction. This is the main reason why the log-periodic power law precursors are stronger and more significant for aggregated financial series in comparison with individual assets. If speculation, imitation and herding become at some time the strongest force driving the price of an asset, we should then expect the log-periodic power law signatures to emerge again strongly above all the other idiosyncratic effects.

Generalization of this analysis to emergent markets, including six Latin-American stock market indices (Argentina, Brazil, Chile, Mexico, Peru and Venezuela) and six Asian stock market indices (Hong-Kong, Indonesia, Korea, Malaysia, Philippines and Thailand) has been performed in Johansen and Sornette (2001b). This work also discusses the existence of intermittent and strong correlation between markets following major international events.

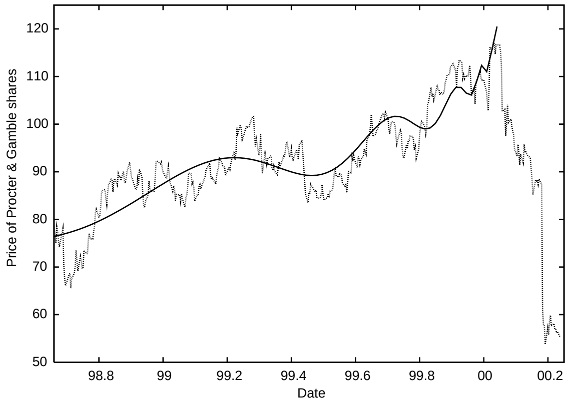

Fig. 33. Best (r.m.s. ≈ 4.3) fit with equation (54) to the price of Procter & Gamble shares. The parameter values of the fit are $A_2 \approx 124$, $B_2 \approx -38$, $B_2C \approx 4.8$, $m_2 \approx 0.35$, $t_c \approx 2000.04$, $\omega \approx 6.6$ and $\phi \approx -0.9$. Reproduced from Johansen and Sornette (2000a).

7.8. "Anti-bubbles"

We now summarize the evidence that imitation between traders and their herding behavior not only lead to speculative bubbles with accelerating over-valuations of financial markets possibly followed by crashes, but also to "anti-bubbles" with decelerating market devaluations following all-time highs (Johansen and Sornette, 1999c). There is thus a certain degree of symmetry of the speculative behavior between the "bull" and "bear" market regimes. This behavior is documented on the Japanese Nikkei stock index from 1 January 1990 until 31 December 1998, on the Gold future prices after 1980, and on the recent behavior of the U.S. S&P500 index from mid-2000 to August 2002, all of them after their all-time highs.

The question we ask is whether the cooperative herding behavior of traders might also produce market evolutions that are symmetric to the accelerating speculative bubbles often ending in crashes. This symmetry is performed with respect to a time inversion around a critical time t_c such that $t_c - t$ for $t < t_c$ is changed into $t - t_c$ for $t > t_c$. This symmetry suggests looking at *decelerating* devaluations instead of accelerating valuations. A related observation has been reported in Fig. 18 in relation to the October 1987 crash showing that the implied volatility of traded options has relaxed *after* the October 1987 crash to its long-term value, from a maximum at the time of the crash, according to a decaying power law with decelerating log-periodic oscillations. It is this type of behavior that we document now but for real prices.

The critical time t_c then corresponds to the culmination of the market, with either a power law increase with accelerating log-periodic oscillations preceding it or a power law decrease with decelerating log-periodic oscillations after it. In the Russian market, both structures appear simultaneously for the same t_c (Johansen and Sornette, 1999c). This is however a rather rare occurrence, probably because accelerating markets with log-periodicity almost inevitably end-up in a crash, a market rupture that thus breaks down the symmetry ($t_c - t$ for $t < t_c$ into $t - t_c$ for $t > t_c$). Herding behavior can occur and progressively weaken from a maximum in "bearish" (decreasing) market phases, even if the preceding "bullish" phase ending at t_c was not characterised by a strengthening imitation.

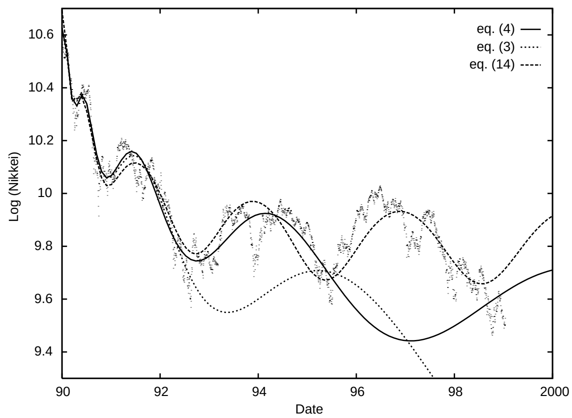

Fig. 34. Natural logarithm of the Nikkei stock market index after the start of the decline from January 1, 1990 until December 31, 1998. The dotted line is the simple log-periodic formula (54) used to fit adequately the interval of ≈ 2.6 years starting from January 1, 1990. The continuous line is the improved nonlinear log-periodic formula developed in Sornette and Johansen (1997) and already used for the 1929 and 1987 crashes over 8 years of data. It is used to fit adequately the interval of ≈ 5.5 years starting from January 1, 1990. The dashed line is an extension of the previous nonlinear log-periodic formula to the next-order of description which was developed in Johansen and Sornette (1999c) and is used to fit adequately the interval of ≈ 9 years starting from January 1, 1990 to December 1998. Reproduced from Johansen and Sornette (1999c).

The symmetry is thus statistical or global in general and holds in the ensemble rather than for each single case individually.

7.8.1. The "bearish" regime on the Nikkei starting from 1st January 1990

The most recent example of a genuine long-term depression comes from Japan, where the Nikkei has decreased by more than 60% in the 12 years following the all-time high of 31 December 1989. In Fig. 34, we see (the logarithm of) the Nikkei from 1 January 1990 until 31 December 1998. The three fits, shown as the undulating lines, use three mathematical expressions of increasing sophistication: the dotted line is the simple log-periodic formula (54); the continuous line is the improved nonlinear log-periodic formula developed in (Sornette and Johansen, 1997) and already used for the 1929 and 1987 crashes over 8 years of data; the dashed line is an extension of the previous nonlinear log-periodic formula to the next-order of description which was developed in Johansen and Sornette (1999c). This last most sophisticated mathematical formula predicts the transition from the log-frequency ω_1 close to t_c to $\omega_1+\omega_2$ for $T_1<\tau<T_2$ and to the log-frequency $\omega_1+\omega_2+\omega_3$ for $T_2<\tau$. Using indices 1, 2 and 3, respectively, for the simplest to the most sophisticated formulas, the parameter values of the first fit of the Nikkei are $A_1\approx 10.7$, $B_1\approx -0.54$, $B_1C_1\approx -0.11$, $m_1\approx 0.47$, $t_c\approx 89.99$, $\phi_1\approx -0.86$, $\omega_1\approx 4.9$ for equation (54). The parameter values of the second fit of the Nikkei are $A_2\approx 10.8, B_2\approx -0.70, B_2C_2\approx -0.11, m_2\approx 0.41, t_c\approx 89.97, \phi_2\approx 0.14, \omega_1\approx 4.8, T_1\approx 9.5$ years, $\omega_2\approx 4.9$. The third fit uses the entire time interval and is performed by adjusting only T_1, T_2, ω_2 and ω_3, while $m_3=m_2$, t_c and ω_1 are fixed at the values obtained from the previous fit. The values obtained for these four parameters are $T_1\approx 4.3$ years, $T_2\approx 7.8$ years, $\omega_2\approx -3.1$ and $T_2\approx 23$ years. Note that the values obtained for the two time scales T_1 and T_2

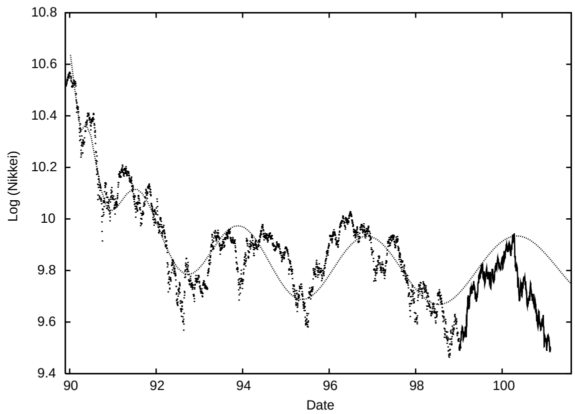

Fig. 35. Natural logarithm of the Nikkei stock market index after the start of the decline from January 1, 1990 until February 2001. The continuous smooth line is the extended nonlinear log-periodic formula which was developed in Johansen and Sornette (1999c) and is used to fit adequately the interval of ≈ 9 years starting from January 1, 1990. The Nikkei data is separated in two parts. The dotted line shows the data used to perform the fit with formula developed in Johansen and Sornette (1999c) and to issue the prediction in January 1999 (see Fig. 34). Its continuation as a continuous line gives the behavior of the Nikkei index after the prediction has been made. Reproduced from Johansen and Sornette (2000b).

confirms their ranking. This last fit predicts a change of regime and that the Nikkei should increase in 1999.

Not only do the first two equations agree remarkably well with respect to the parameter values produced by the fits, but they are also in good agreement with previous results obtained from stock market and Forex bubbles with respect to the values of exponent m_2. What lends credibility to the fit with the most sophisticated formula is that, despite its complex form, we get values for the two cross-over time scales T_1, T_2 which correspond to what is expected from the ranking and from the 9 year interval of the data. We refer to Johansen and Sornette (1999c) for a detailed and rather technical discussion.

The prediction summarized by Fig. 34 was made public on January 25, 1999 by posting a preprint on the Los Alamos www internet server, see http://xxx.lanl.gov/abs/cond-mat/9901268. The preprint was later published as Johansen and Sornette (1999c). The prediction stated that the Nikkei index should recover from its 14 year low (13232.74 on January 5, 1999) and reach ≈ 20500 a year later corresponding to an increase in the index of ≈ 50%. This prediction was mentioned in a wide-circulation journal in physical sciences which appeared in May 1999 (Stauffer, 1999).

In Fig. 35, the actual and predicted evolution of the Nikkei over 1999 and later are compared (Johansen and Sornette, 2000b). Not only did the Nikkei experience a trend reversal as predicted, but it has also followed the quantitative prediction with rather impressive precision. In particular, the prediction of the 50% increase at the end of 1999 is validated accurately. The prediction of another trend reversal is also accurately predicted, with the correct time for the reversal occurring beginning of 2000: the predicted maximum and observed one match closely. It is important to note that the error between the curve and the data has not grown after the last point used in the fit over 1999. This tells us that the prediction has performed well for more than one year. Furthermore, since the relative error between the fit and the data is within ±2% over a time period of 10 years, not only has the prediction performed well, but also the underlying model.

The fulfilling of this prediction is even more remarkable than the comparison between the curve and the data indicates, because it included a change of trend: at the time when the prediction was issued, the market was declining and showed no tendency to increase. Many economists were at that time very pessimistic and could not envision when Japan and its market would rebounce. For instance, P. Krugman wrote on July 14, 1998 in the Shizuoka Shimbun at the time of the banking scandal "the central problem with Japan right now is that there just is not enough demand to go around—that consumers and corporations are saving too much and borrowing too little... . So seizing these banks and putting them under more responsible management is, if anything, going to further reduce spending; it certainly will not in and of itself stimulate the economy... . But at best this will get the economy back to where it was a year or two ago—that is, depressed, but not actually plunging". Then, in the Financial Times, January 20, 1999, P. Krugman wrote in an article entitled "Japan heads for the edge" the following: "...the story is starting to look like a tragedy. A great economy, which does not deserve or need to be in a slump at all, is heading for the edge of the cliff—and its drivers refuse to turn the wheel". In a poll of 30 economists performed by Reuters (one of the major news and finance data provider in the world) in October 1998 reported in Indian Express on the 15 October (see http://www.indian-express.com/fe/daily/19981016/28955054.html), only two economists predicted growth for the fiscal year of 1998–1999. For the year 1999–2000 the prediction was a meager 0.1% growth. This majority of economists said that "a vicious cycle in the economy was unlikely to disappear any time soon as they expected little help from the government's economic stimulus measures... . Economists blamed moribund domestic demand, falling prices, weak capital spending and problems in the bad-loan laden banking sector for dragging down the economy".

It is in this context that we predicted an approximately 50% increase of the market in the 12 months following January 1999, assuming that the Nikkei would stay within the error-bars of the fit. Predictions of trend reversals is noteworthy difficult and unreliable, especially in the linear framework of auto-regressive models used in standard economic analyses. The present nonlinear framework is well-adapted to the forecasting of change of trends, which constitutes by far the most difficult challenge posed to forecasters. Here, we refer to our prediction of a trend reversal within the strict confine of our extended formula: trends are limited periods of times when the oscillatory behavior shown in Fig. 35 is monotonous. A change of trend thus corresponds to crossing a local maximum or minimum of the oscillations. Our formula seems to have predicted two changes of trends, bearish to bullish at the beginning of 1999 and bullish to bearish at the beginning of 2000.

7.8.2. The gold deflation price starting mid-1980

Another example of log-periodic decay is that of the Gold price after the burst of the bubble in 1980 as shown in Fig. 36. The bubble has an *average* power law acceleration as shown in the figure but *without* any visible log-periodic structure. A pure power law fit will however not "lock in" on the true date of the crash, but insists on an earlier date than the last data point. This suggests that the behavior of the price might be different in some sense in the last few weeks prior to the burst of the bubble. The continuous line before the peak is expression (54) fitted over an interval of ≈ 3 years. The parameter values of this fit are $A_2 \approx 8.5$, $B_2 \approx -111$, $B_2C \approx -110$, $m_2 \approx 0.41$,

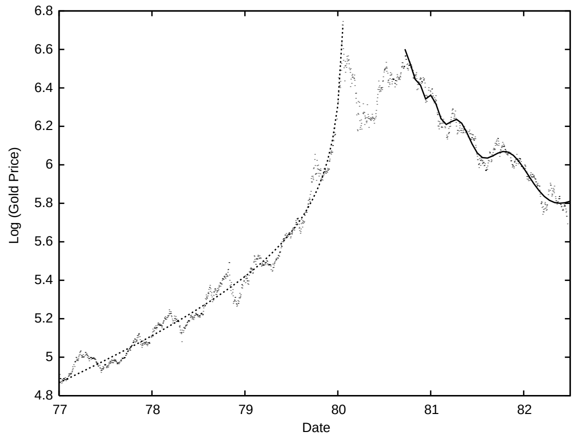

Fig. 36. Natural logarithm of the gold 100 Oz Future price in U.S.$ showing a power law acceleration followed by a decline of the price in the early eighties. The line after the peak is expression (54) fitted over an interval of ≈ 2 years. Reproduced from Johansen and Sornette (1999c).

$t_c \approx 80.08$, $\phi \approx -3.0$, $\omega \approx 0.05$. The price of gold after its peak is fitted by expression (54) and the result is shown as the undulating continuous line. Again, we obtain a reasonable agreement with previous results for the exponent m_2 with a good preferred scaling ratio $\lambda \approx 1.9$. The strength of the log-periodic oscillations compared to the leading behavior is ≈ 10%. The parameter values of the fit in this anti-bubble regime are $A_2 \approx 6.7$, $B_2 \approx -0.69$, $B_2 C \approx 0.06$, $m_2 \approx 0.45$, $t_c \approx 80.69$, $\phi \approx 1.4$, $\omega \approx 9.8$.

7.8.3. The U.S. 2000–2002 Market Descent: How Much Longer and Deeper?

Sornette and Zhou (2002) have recently analyzed the remarkable similarity in the behavior of the U.S. S&P500 index from 1996 to August 2002 and of the Japanese Nikkei index from 1985 to 1992, corresponding to an 11 years shift. In particular, the structure of the price trajectories in the bearish or anti-bubble phases are strikingly similar, as seen in Fig. 37.

Sornette and Zhou (2002) have performed a battery of tests, starting with parametric fits of the index with two log-periodic power law formulas, followed by the so-called Shank's transformation applied to characteristic times. They also carried out two spectral analysis, the Lomb periodogram applied to the parametrically detrended index and the nonparametric (H, q)-analysis of fractal signals (Zhou and Sornette, 2002b, c). These different approaches complement each other and confirm the presence of a very strong log-periodic structures. A rather novel feature is the detection of a significant second-order harmonic which provides a statistically significant improvement of the description of the data by the theory, as tested using the statistical theory of nested hypotheses. The description of the S&P500 index since mid-2000 to end of August 2002 based on the combination of the first and second log-periodic harmonics is shown in Fig. 38.

In the next two years, Sornette and Zhou (2002) predict an overall continuation of the bearish phase, punctuated by local rallies; specifically, they predict an overall increasing market until the end of the year 2002 or until the first quarter of 2003; they predict a severe following descent

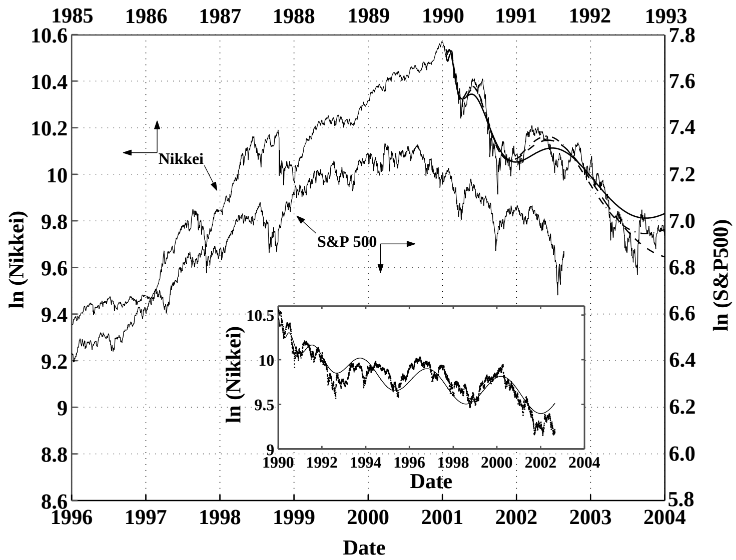

Fig. 37. Comparison between the evolutions of the U.S. S&P500 index from 1996 till August 24, 2002 (bottom and right axes) and the Japanese Nikkei index from 1985 to 1993 (top and left axes). The years are written on the horizontal axis (and marked by a tick on the axis) where January 1 of that year occurs. The dashed line is the simple log-periodic formula (54) fitted to the Nikkei index (with $t_c - t$ replaced by $t - t_c$). The data used in this fit goes from 01-Jan-1990 to 01-Jul-1992 (Johansen and Sornette, 1999c). The parameter values are t_c= 28-Dec-1989, $\alpha = 0.38$, $\omega = 5.0$, $\phi = 2.59$, $A = 10.76$, $B = -0.067$ and $C = -0.011$. The root-mean-square residue is $\chi = 0.0535$. The dash-dotted line is the improved nonlinear log-periodic formula developed in Sornette and Johansen (1997) fitted to the Nikkei index. The Nikkei index data used in this fit goes from 01-Jan-1990 to 01-Jul-1995 (Johansen and Sornette, 1999c). The parameter values are t_c= 27-Dec-1989, $\alpha = 0.38$, $\omega = 4.8$, $\phi = 6.27$, $\Delta_t = 6954$, $\Delta_\omega = 6.5$, $A = 10.77$, $B = -0.070$, $C = 0.012$. The root-mean-square residue is $\chi = 0.0603$. The continuous line is the fit of the Nikkei index with the third-order formula developed in Johansen and Sornette (1999c). The Nikkei index data used in the fit goes from 01-Jan-1990 to 31-Dec-2000. The fit is performed by fixing t_c, α and ω at the values obtained from the second-order fit and adjusting only Δ_t, Δ_t', Δ_ω, Δ_ω' and ϕ. The parameter values are $\Delta_t = 1696$, $\Delta_t' = 5146$, $\Delta_\omega = -1.7$, $\Delta_\omega' = 40$, $\phi = 6.27$, $A = 10.86$, $B = -0.090$, $C = -0.0095$. The root-mean-square residue of the fit is $\chi = 0.0867$. In the three fits, A, B and C are slaved to the other variables by the multiplier approach in each iteration of the optimization search. The inset shows the 13-year Nikkei anti-bubble with the fit with the third-order formula over these 13 years shown as the continuous line. The parameter values slightly different: $\Delta_t = 52414$, $\Delta_t' = 17425$, $\Delta_\omega = 23.7$, $\Delta_\omega' = 127.5$, $\phi = 5.57$, $A = 10.57$, $B = -0.045$, $C = 0.0087$. The root-mean-square residue of the fit is $\chi = 0.1101$. In all the fits, times are expressed in units of days, in contrast with the yearly unit used in Johansen and Sornette (1999c). Thus, the parameters B and C are different since they are unit-dependent, while all the other parameters are independent of the units. Reproduced from (Sornette and Zhou, 2002).

(with maybe one or two severe ups and downs in the middle) which stops during the first semester of 2004. Beyond this, they cannot be very certain due to the possible effect of additional nonlinear collective effects and of a real departure from the anti-bubble regime. The similarities between the two stock market indices may reflect deeper similarities between the fundamentals of two economies which both went through over-valuation with strong speculative phases preceding the transition to bearish phases characterized by a surprising number of bad surprises (bad loans for Japan and accounting frauds for the U.S.) sapping investors' confidence.

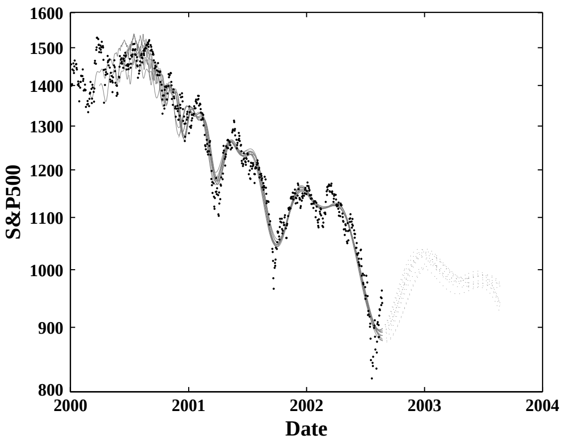

Fig. 38. Fitted trajectories using Eq. (54) (with $t_c - t$ replaced by $|t - t_c|$), each curve corresponding to a different starting time from Mar-01-2000 to Dec-01-2000 with one month interval. The different fits are obtained as a sensitivity test with respect to the starting time of the anti-bubble which is consistently found to start at $t_c \approx$ July 15–August 15, 2000. The dotted lines show the predicted future trajectories. One sees that the fits are quite robust with respect to different starting date t_{start} from Mar-01-2000 to Dec-01-2000. Reproduced from Sornette and Zhou (2002).

8. Synthesis

8.1. "Emergent" behavior of the stock market

In this paper, we have synthesized a large body of evidence in favor of the hypothesis that large stock market crashes are analogous to critical points studied in the statistical physics community in relation to magnetism, melting, and so on. Our main assumption is the existence of a cooperative behavior of traders imitating each other described in Sections 5 and 6. A general result of the theory is the existence of log-periodic structures decorating the time evolution of the system. The main point is that the market anticipates the crash in a subtle self-organized and cooperative fashion, hence releasing precursory "fingerprints" observable in the stock market prices. In other words, this implies that market prices contain information on impending crashes. If the traders were to learn how to decipher and use this information, they would act on it and on the knowledge that others act on it, nevertheless the crashes would still probably happen. Our results suggest a weaker form of the "weak efficient market hypothesis" (Fama, 1991), according to which the market prices contain, in addition to the information generally available to all, subtle information formed by the global market that most or all individual traders have not yet learned to decipher and use. Instead of the usual interpretation of the efficient market hypothesis in which traders extract and incorporate consciously (by their action) all information contained in the market prices, we propose that the market as a whole can exhibit an "emergent" behavior not shared by any of its constituents. In other words, we have in mind the process of the emergence of intelligent behaviors at a macroscopic scale that individuals at the microscopic scale cannot perceive. This process has been discussed in biology for instance in animal populations such as ant colonies (Wilson, 1971; Holldobler and Wilson, 1994) or in connection with the emergence of consciousness (Anderson et al., 1988; Holland, 1992).

Let us mention another realization of this concept, which is found in the information contained in option prices on the fluctuations of their underlying asset. Despite the fact that the prices do not follow geometrical brownian motion, whose existence is a prerequisite for most option pricing models, traders have apparently adapted to empirically incorporate subtle information in the correlation of price distributions with fat tails (Potters et al., 1998). In this case and in contrast to the crashes, the traders have had time to adapt. The reason is probably that traders have been exposed for decades to option trading in which the characteristic time scale for option lifetime is in the range of month to years at most. This is sufficient for an extensive learning process to occur. In contrast, only a few great crashes occur typically during a lifetime and this is certainly not enough to teach traders how to adapt to them. The situation may be compared to the ecology of biological species which constantly strive to adapt. By the forces of evolution, they generally succeed to survive by adaptation under slowly varying constraints. In contrast, life may exhibit successions of massive extinctions and booms probably associated with dramatically fast-occurring events, such as meteorite impacts and massive volcanic eruptions. The response of a complex system to such extreme events is a problem of outstanding importance that is just beginning to be studied (Commission on Physical Sciences, Mathematics, and Applications, 1990).

Most previous models proposed for crashes have pondered the possible mechanisms to explain the collapse of the price at very short time scales. Here in contrast, we propose that the underlying cause of the crash must be searched years before it in the progressive accelerating ascent of the market price, reflecting an increasing build-up of the market cooperativity. From that point of view, the specific manner by which prices collapsed is not of real importance since, according to the concept of the critical point, any small disturbance or process may have triggered the instability, once ripe. The intrinsic divergence of the sensitivity and the growing instability of the market close to a critical point might explain why attempts to unravel the local origin of the crash have been so diverse. Essentially all would work once the system is ripe. Our view is that the crash has an endogeneous origin and that exogeneous shocks only serve as triggering factors. We propose that the origin of the crash is much more subtle and is constructed progressively by the market as a whole. In this sense, this could be termed a systemic instability.

8.2. Implications for mitigations of crises

Economists, J.E. Stiglitz and recently P. Krugman in particular as well as financier Soros, have argued that markets should not be left completely alone. The mantra of the free-market purists requiring that markets should be totally free may not always be the best solution, because it overlooks two key problems: (1) the tendency of investors to develop strategies that may destabilize markets in a fundamental way and (2) the noninstantaneous adjustment of possible imbalance between countries. Financier George Soros has argued that real world international financial markets are inherently volatile and unstable since "market participants are trying to discount a future that is itself shaped by market expectations". This question is of course at the center of the debate on whether local and global markets are able to stabilize on their own after a crisis such as the Asian crisis which started in 1997. In this example, to justify the intervention of the IMF (international monetary fund), Treasury Secretary Rubin warned in January 1998 that global markets would not be able to stabilize in Asia on their own, and that a strong role on the part of the IMF and other international institutions, and governments, was necessary, least the crisis spread to other emerging markets in Latin America and Eastern Europe.

The following analogy with forest fires is useful to illustrate the nature of the problem. In many areas around the world, the dry season sees numerous large wildfires, sometimes with deaths of firefighters and other people, the destruction of many structures and of large forests. It is widely accepted that livestock grazing, timber harvesting, and fire suppression over the past century have led to unnatural conditions—excessive biomass (too many trees without sufficient biodiversity and dead woody material) and altered species mix—in the pine forests of the West of the U.S.A., in the Mediterranean countries and elsewhere. These conditions make the forests more susceptible to drought, insect and disease epidemics, and other forest-wide catastrophes and in particular large wildfires (Gorte, 1995). Interest in fuel management, to reduce fire control costs and damages, has thus been renewed with the numerous, destructive wildfires spread across the West of the U.S.A. The most-often used technique of fuel management is fire suppression. Recent reviews comparing Southern California on the one hand, where management is active since 1900, and Baja California (north of Mexico) on the other hand where management is essentially absent (a "let-burn" strategy) highlight a remarkable fact (Minnich and Chou, 1997; Moreno, 1998): only small and relatively moderate patches of fires occur in Baja California, compared to a wide distribution of fire sizes in Southern California including huge destructive fires. The selective elimination of small fires (those that can be controlled) in normal weather in Southern California restricts large fires to extreme weather episodes, a process that encourages broad-scale high spread rates and intensities. It is found that the danger of fire suppression is the inevitable development of coarse-scale bush fuel patchiness and large instance fires in contradistinction with the natural self-organization of small patchiness in left-burn areas. Taken at face value, the "let-burn" theory seems paradoxically the correct strategy which maximizes the protection of property and of resources, at minimal cost.

This conclusion seems to be correct when the fuel is left on its own to self-organize in a way consistent with the dynamics of fires. In other words, the fuel–fire constitutes a complex nonlinear system with negative and positive feedbacks that may be close to optimal: more fuel favors fire; fires decreases the instantaneous level of fuel but may accelerate its future production; many small fires create natural barriers for the development and extension of large fires; fires produce rich nutrients in the soil; fires have other benefits, for instance, a few species, notably lodgepole pine and jack pine, are serotinous—their cones will only open and spread their seeds when they have been exposed to the heat of a wildfire. The possibility for complex nonlinear systems to find the "optimal" or to be close to the optimal solution have been stressed before in several contexts (Crutchfield and Mitchell, 1995; Miltenberger et al., 1993; Sornette et al., 1994). Let us mention for instance a model of fault networks interacting through the elastic deformation of the crust and rupturing during earthquakes which finds that faults are the optimal geometrical structures accommodating the tectonic deformation: they result from a global mathematical optimization problem that the dynamics of the system solves in an analog computation, i.e., by following its self-organizing dynamics (as opposed to digital computation performed by digital computers). One of the notable levels of organization is called self-organized criticality (Bak, 1996; Sornette, 2000a) and has been applied in particular to explain forest fire distributions (Malamud et al., 1998).

Baja California could be a representative of this self-organized regime of the fuel–fire complex left to itself, leading to many small fires and few big ones. Southern California could illustrate the situation where interference both in the production of fuel and also in its combustion by fires (by trying to stop fires) leads to a very broad distribution with many small and moderate controlled fires and too many uncontrollable very large ones.

Where do stock markets stand in this picture? The proponents of the "left-alone" approach could get ammunition from the Baja-Southern California comparison, but they would forget an essential element: stock markets and economies are more like Southern California than Baja California. They are not isolated. Even if no government or regulation interfere, they are "forced" by many external economic, political, climatic influences that impact them and on which they may also have some impact. If the example of the wildland fires has something to teach us, it is that we must incorporate in our understanding both the self-organizing dynamics of the fuel-fire complex as well as the different exogeneous sources of randomness (weather and wind regimes, natural lightning strike distribution, and so on).

The question of whether some regulation could be useful is translated into whether Southern California fires would be better left alone. Since the management approach fails to function fully satisfactorily, one may wonder whether the let-burn scenario would not be better. This has in fact been implemented in Yellowstone park as the "let-burn" policy but was abandoned following the huge Yellowstone fires of 1988. Even the "leave-burn" strategy may turn out to be unrealistic from a societal point-of-view because allowing a specific fire to burn down may lead to socially unbearable risks or emotional sensitivity, often discounted over a very short time horizon (as opposed to the long-term view of land management implicit in the left-burn strategy).

We suggest that the most momentous events in stock markets, the large financial crashes, can indeed be seen as the response of a self-organized system forced by a multitude of external factors in the presence of regulations. The external forcing is an essential element to consider and it modifies the perspective on the "left-alone" scenario. For instance, during the recent Asian crises, the International Monetary Fund and the U.S. government considered that controls on the international flow of capital were counterproductive or impractical. J.E. Stiglitz, the chief economist of the IMF until 2000, has argued that in some cases it was justified to restrict short-term flows of money in and out of a developing economy and that industrialized countries sometimes pushed developing nations too fast to deregulate their financial systems. The challenge remains, as always, to encourage and work with countries that are ready and able to implement strong corrective actions and to cooperate toward finding the financial solutions best suited to the needs of the individual case and the broader functioning of the global financial system when difficulties arise (Checki and Stern, 2000).

Another important issue concerns the endogeneous versus exogeneous nature of shocks. Sornette et al. (2002) have shown that it is possible in some cases to distinguish the effects on the financial volatility of the September 11, 2001 attack or of the coup against Gorbachev on August, 19, 1991 (exogeneous shocks) from financial crashes such as October 1987 as well as smaller volatility bursts (endogeneous shocks). Using a parsimonious autoregressive process (the "multifractal random walk") with long-range memory defined on the logarithm of the volatility, they predict strikingly different response functions of the price volatility to great external shocks compared to endogeneous shocks, i.e., which result from the cooperative accumulation of many small shocks. This approach views the origin of endogeneous shocks as the coherent accumulations of tiny bad news, and thus provides a natural unification of previous explanations of large crashes including October 1987. Sornette and Helmstetter (2003) have suggested that these results are generally valid for systems with long-range persistence and memory, which can exhibit different precursory as well as recovery patterns in response to shocks of exogeneous versus endogeneous origins. By endogeneous, one can consider either fluctuations resulting from an underlying chaotic dynamics or from a stochastic forcing origin which may be external or be an effective coarse-grained description of the microscopic fluctuations. In this scenario, endogeneous shocks result from a kind of constructive interference of accumulated fluctuations whose impacts survive longer than the large shocks themselves. As a consequence, the recovery after an endogeneous shock is in general slower at early times and can be at long times either slower or faster than after an exogeneous perturbation. This offers the tantalizing possibility of distinguishing between an endogeneous versus exogeneous cause of a given shock, even when there is no "smoking gun". This could help in investigating the exogeneous versus self-organized origins in problems such as the causes of major biological extinctions, of change of weather regimes and of the climate, in tracing the source of social upheaval and wars, and so on.

8.3. Predictions

Ultimately, only forward predictions can demonstrate the usefulness of a theory, thus only time will tell. However, as we have suggested by the many examples reported in Section 7, the analysis points to an interesting predictive potential. However, a fundamental question concerns the use of a reliable crash prediction scheme, if any. Assume that a crash prediction is issued stating that a crash of an amplitude between 20% and 30% will occur between one and two months from now. At least three different scenarios are possible (Johansen and Sornette, 2000a):

- Nobody believes the prediction which was then futile and, assuming that the prediction was correct, the market crashes. One may consider this as a victory for the "predictors" but as we have experienced in relation to our quantitative prediction of the change in regime of the Nikkei index (Johansen and Sornette, 1999c, 2000b), this would only be considered by some critics just another "lucky one" without any statistical significance.
- Everybody believes the warning, which causes panic and the market crashes as consequence. The prediction hence seems self-fulfilling and the success is attributed more to the panic effect than to a real predictive power.
- Sufficiently many investors believe that the prediction *may* be correct, investors make reasonable adjustments and the steam goes off the bubble. The prediction hence disproves itself.

None of these scenarios are attractive. In the first two, the crash is not avoided and in the last scenario the prediction disproves itself and as a consequence the theory looks unreliable. This seems to be the inescapable lot of scientific investigations of systems with learning and reflective abilities, in contrast with the usual inanimate and unchanging physical laws of nature. Furthermore, this touches the key-problem of scientific responsibility. Naturally, scientists have a responsibility to publish their findings. However, when it comes to the practical implementation of those findings in society, the question becomes considerably more complex, as history has taught us. We believe however that increased awareness of the potential for market instabilities, offered in particular by our approach, will help in constructing a more stable and efficient stock market.

Specific guidelines for prediction and careful tests are presented in Sornette and Johansen (2001) and especially in Sornette (2003). In particular, Sornette (2003) explains how and to what degree crashes as well as other large market events, may be predicted. This work examines in details what are the forecasting skills of the proposed methodology and their limitations, in particular in terms of the horizon of visibility and expected precision. Several cases studies are presented in details, with a careful count of successes and failures. See also Johansen and Sornette (2001b) for applications to emergent markets, Johansen and Sornette for a systematic test of the correspondence between outliers and preceding log-periodic power law signatures and Sornette and Zhou (2002) for a live prediction on the future evolution of the U.S. stock market in the next two years, from August 2002 to the first semester of 2004.

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