The Emergence of Econophysics: A New Approach in Modern Financial Theory

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Financial economics and mathematical finance are the two traditional scientific disciplines that constitute modern financial theory. Both these "players" use models and theories from their original disciplines (i.e., economics and mathematics) to analyze financial markets and to develop financial tools. Both are recent developments—less than fifty years old. While some studies on what was to become modern financial theory were produced prior to the 1960s, they were marginal and did not yet constitute either an academic or a scientific discipline;[1] applied mathematics and empirical investigations into finance existed, but these were isolated contributions, and most of them did not have a solid theoretical underpinning.[2]

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[1] Examples are the works of Jules Regnault (1863), Louis Bachelier ([1900] 1995), Vincenz Bronzin (1908), Alfred Cowles (1933, 1944), and Holbrook Working (1934, 1935). For recent secondary sources on those works, see Poitras 2000, Preda 2001, Courtault and Kabanov 2002, Dimand 2004, Preda 2004, Jovanovic 2006b, Poitras 2006, and Poitras and Jovanovic 2007.

[2] An absence of theory characterizes all existing works written between the 1930s and the 1960s. Cowles (1933), Working (1934), and Maurice George Kendall (1953) were the first English and American authors to analyze the random character of stock prices, but none of them put forward a theory to explain the phenomenon. During the 1950s theoreticians pointed out the absence of theoretical explanations, an absence that was particularly striking after the Koopmans-Vining debate in the late 1940s, which set the NBER against the Cowles Commission over the lack of theoretical explanations and the need to link measurement with theory (Jovanovic 2008).

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Almost from their very beginnings, financial economics and mathematical finance progressively developed their own specificities and models. And while they use the same terms (among others, *efficient market* and *option pricing theory*), they sometimes define them differently or treat them in a quite different way. Moreover, each discipline is bound by its own theoretical foundations, which sometimes place limits on the introduction of new models or hypotheses. As a result, work in financial theory has gradually appeared outside these two traditional disciplines. Although financial economics and mathematical finance still largely dominate modern financial theory, in the past few years a new player has increasingly been making itself felt and could lead to a rethinking of some of the theoretical foundations of modern financial theory. This new player is econophysics.

Econophysics is a very recent movement that is beginning to interest increasing numbers of financial practitioners (Farmer, Shubik, and Smith 2005). To date, no history of econophysics has been produced.[3] This article aims at filling this gap and, more generally, makes three contributions to the history of modern financial theory: an analysis of the theoretical foundations of econophysics (and their connections with the history of financial economics); a study of the reasons underlying the emergence of econophysics; and a presentation of the manner in which econophysics has become the third component of modern financial theory.

The article is divided into three parts. The first introduces and defines econophysics. This part also deals with the institutional development of econophysics by presenting a snapshot of the field, by examining strategies econophysicists have developed to incorporate their new approach into modern financial theory, and by studying econophysics' major distinguishing feature, which is the use of stable Lévy processes. The second analyzes the evolution of mathematical tools used by econophysicists, considering that modern probability theory plays a key role in the history of financial economics.[4] We then explain why these tools, which were introduced into financial economics in the 1960s, were subsequently not used by financial economists. We also present the alternative paths being explored in financial economics. The third part studies the reasons underlying the emergence of econophysics during the 1990s.

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[3] The few papers that deal with econophysics (Yegorov 2007, Săvoiu and Iorga-Simăn 2008, and Daniel and Sornette 2010) provide no exhaustive historical analysis of the approach.

[4] As we explain in two earlier papers (Jovanovic 2008 and Jovanovic and Schinckus 2012), the major hypotheses, models, and results of financial economics find their origin in modern probability theory; the institutionalization of financial economics was also made possible by the link with modern probability theory.

Two crucial elements are identified: first, the evolution of statistical and probabilistic tools; second, the emergence of new empirical data.

1. The Emergence of a New Player in Modern Financial Theory

1.1. Definition of Econophysics: Statistical Physics Applied to Economics

Very broadly speaking, econophysics refers to the extension of physics to the study of problems generally considered as falling within the sphere of economics.

The influence of physics on economics is nothing new. A number of writers have studied the "physical attraction"[5] that economics has felt for the hard sciences. Philip Mirowski (1989) extensively highlighted the role of physics in the development of marginalist economics and mathematical economics. Bruna Ingrao and Giorgio Israel (1990) drew renewed attention to the influence of mechanics in the conceptualization of equilibrium in economics. And Claude Ménard (1981), Margaret Schabas (1990), and Harro Maas (2005) documented the role of physics in the economic works of Augustin Cournot and William Stanley Jevons.[6]

Financial economics, and more generally finance, has also been subject to the influence of physics. One of the first authors to bring physics closer to the financial domain was Jules Regnault, who did so in the second half of the nineteenth century.[7] In the twentieth century, a number of physics concepts played a part in the development of modern financial theory. The best-known application of physics to finance is the application of the heat-diffusion formula (applied by Louis Bachelier [(1900) 1995] and by Fischer Black and Myron Scholes [1973]),[8] and a number of studies implicitly or explicitly referred to a concept from the field of physics: Brownian motion.[9] But as Joseph L. McCauley (2004) points out, in spite

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[5] We have borrowed the phrase from Philippe Le Gall (2002, 43).

[6] For an excellent introduction to the analysis of methodology transfer between the physical sciences and economics, see Le Gall 2002.

[7] See Jovanovic 2000 and Jovanovic and Le Gall 2001 on this subject.

[8] Bachelier was trained in mathematical physics. For Bachelier's influence on modern financial theory, see Dimand and Ben-El-Mechaiekh 2006, Jovanovic 2010, or Taqqu 2001. Regarding the importance of Fischer Black's contribution, see Mehrling 2005.

[9] For example, Working 1934 and Osborne 1959. Note, however, that Brownian motion, as a mathematical object, was first modeled to represent stock market variations by Bachelier ([1900] 1995).

of these theoretical and historical links between physics and finance, econophysics represents a fundamentally new approach. Its practitioners are not economists taking their inspiration from the work of physicists to develop their discipline, as has been seen repeatedly in the history of economics. This time, it is physicists who are going beyond the boundaries of their discipline, using their methods to study various problems thrown up by the social sciences. Econophysicists do not contend that they are attempting to integrate physics concepts into financial economics as it exists today, but rather that they are seeking to replace the theoretical framework that currently dominates it with a new framework derived directly from statistical physics.[10]

This movement was initiated in the 1970s, when certain physicists began publishing articles devoted to the study of social phenomena. While some physicists (or sometimes mathematicians working on mathematics applied to physics) extended what is called "catastrophe theory"[11] to the social sciences, others created a new field called "sociophysics."[12] Although catastrophe theory has commanded respect among mathematicians, few applications of catastrophe theory in economics have been seen (as Rosser 2009 attests),[13] whereas sociophysics has been gaining in popularity. Indeed, the number of physicists publishing papers devoted to the analysis of social phenomena and the number of themes studied are increasing, examples being the formation of social groups (Weidlich 1971), social mimetism (Callen and Shapiro 1974), industrial strikes (Galam, Gefen,

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[10] This explicit desire for a methodological break echoes the Kuhnian idea of the need for theoretical discontinuity in order to develop a new paradigm.

[11] Catastrophe theory originated with the work of the French mathematician René Thom in the 1960s. It became popular in the 1970s through the efforts of another mathematician, Christopher Zeeman (1974, 1977), who proposed the term *catastrophe theory*. This theory is a special case of singularity theory, which is in turn the key element of bifurcation theory, part of the study of nonlinear dynamical systems. See Rosser 2009 for further information about catastrophe theory applied in economics.

[12] This term was proposed by Serge Galam, Yuval Gefen, and Yonathan Shapir in a 1982 article. In Galam's (2004, 50) view, one of the reasons why physicists attempt to explain social phenomena stems from a kind of mismatch between the theoretical power of physics and the inert nature of its subject matter: "During my research, I started to advocate the use of modern theory phase transitions to describe social, psychological, political and economical phenomena. My claim was motivated by an analysis of some epistemological contradictions within physics. On the one hand, the power of concepts and tools of statistical physics were enormous, and on the other hand, I was expecting that physics would soon reach the limits of investigating inert matter."

[13] The progressive rejection of catastrophe theory in economics was essentially the result of debates and critiques of the theory (Zahler and Sussmann 1977; Cobb, Koppstein, and Chen 1983).

and Shapir 1982), democratic structures (Galam 1986), and elections (Galam 2004; Ferreira and Dionísio 2008).

In the 1990s physicists turned their attention to economics, and particularly financial economics, giving rise to econophysics.[14] Although the movement's official birth announcement came in a 1996 article by H. Eugene Stanley et al. (1996),[15] econophysics was at that time still a young and ill-defined field. Rosario N. Mantegna and Stanley (1999, 2) defined econophysics as "a quantitative approach using ideas, models, concepts and computational methods of statistical physics."[16] This definition seemed to gain ground as a compromise and is found in a number of books and articles produced by the movement, for example Wang, Jinshan, and Di 2004, Rickles 2007, and Rosser 2007. However, an analysis of the themes studied by econophysics shows that research conducted in this field mainly concerns the study of financial phenomena, marginalizing other themes analyzed by economics.[17]

1.2. The Institutionalization of Econophysics

To gain recognition for their field of research, econophysicists have adopted various strategies for spreading their knowledge. Symposia have

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[14] The influence of physics on the study of financial markets is not new, as witnessed by the work of Bachelier ([1900] 1995) and Black and Scholes (1973). Nevertheless, we cannot yet refer to Black and Scholes's model as econophysics in the term's current meaning, since it was completely integrated into the dominant theoretical current of economics and finance (Kast 1991). Econophysics is not an "adapted import" of the methodology used in physics; rather, it is closer to a "methodological invasion." We return to this point in the next section.

[15] This article is also the origin of the term *econophysics*. We would point out, however, that Ryszard Kutner and Dariusz Grech (2008) trace the informal birth of the approach to a paper by Rosario N. Mantegna (1991) that studied the evolution of returns in financial markets in terms of stable Lévy processes.

[16] To present econophysics as an extension of statistical mechanics necessitates a better definition of this approach in physics. Statistical mechanics attempts mainly to explain in statistical terms the behavior and macroscopic evolution of a complex system on the basis of interactions of a large number of microscopic constituents (atoms, electrons, ions, etc.) that make it up (Ruelle 1991, 155). Applied to finance, this type of reasoning allows one to consider the market as the statistical and macroscopic results of a very large number of heterogeneous interactions at the microscopic level.

[17] Although the application of statistical physics to economics touches on a number of subjects, such as corporate revenue (Okuyama, Takayasu, and Takayasu 1999), the emergence of money (Shinohara and Gunji 2001), and global demand (Donangelo and Sneppen 2000), these fields are marginal to judge by the number of articles published by physicists on the subject of financial markets. It is no accident, then, that the characteristics of econophysics mentioned by Dean Rickles (2007, 952) all relate to finance.

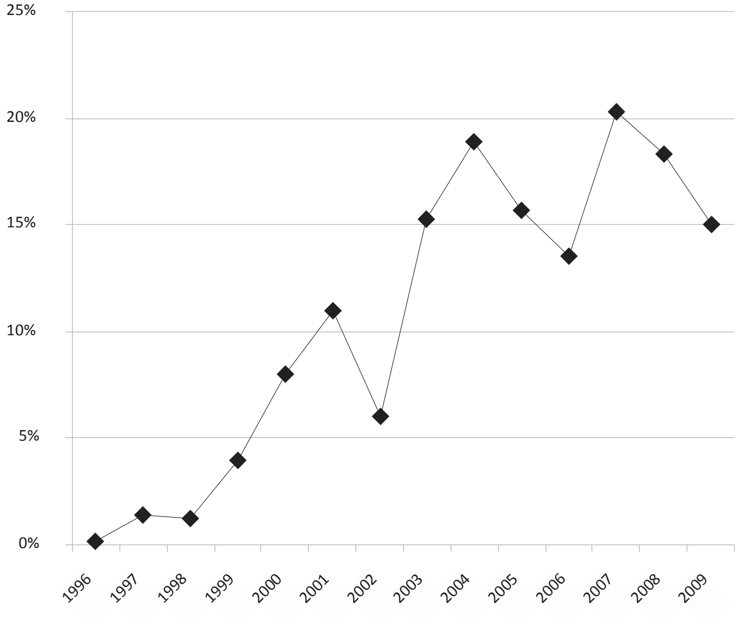

Figure 1 Percentage of articles on econophysics published in *Physica A* since 1996

been organized, several specialized journals have been created, and specific courses have been set up by physics departments in order to promote scientific recognition and institutionalization of the new approach. All these strategies have played a part not only in disseminating econophysics but in creating a shared scientific culture (Nadeau 1995).

The first publications date from the 1990s. The founding article by Stanley et al., published in 1996, strongly influenced physicists and mathematicians who developed a non-Gaussian approach to the study of financial returns (Kutner and Grech 2008). As figure 1 shows, the proportion of articles devoted to econophysics in one (*Physica A*) of the three journals that publish the great majority of articles on the subject—*Physica A,* the *International Journal of Modern Physics C*, and the *European Journal of Physics B*—has grown steadily.

The trend observed in figure 1 for *Physica A*[18] is also found in the *International Journal of Modern Physics C* and the *European Journal of*

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[18] The nonlinearity of this trend is mainly due to the special issues devoted to econophysics that *Physica A* publishes almost every year.

Physics B[19] (Gingras and Schinckus 2012). This sustained growth in the number of articles published each year earned econophysics official recognition as a subdiscipline of the physical sciences in 2003—fewer than ten years after its birth.[20]

The emerging editorial activity in econophysics has followed a relatively clear line: econophysicists have preferred to publish and gain acceptance in journals devoted to a preexisting theoretical field in physics (statistical physics) rather than create new journals outside a preexisting scientific space and structure. Moreover, these journals are among the most prestigious in physics. This editorial orientation results from the methodology used by econophysicists (derived from statistical physics) but also from the new community's hope, on the one hand, to quickly gain recognition from the existing scientific community and, on the other hand, to reach a larger audience.

The 1990s, then, stand as the decade in which econophysics emerged thanks to growth in the number of publications. Textbooks on econophysics were not far behind, the first being published in 1999 by Mantegna and Stanley. Textbooks do not have the same epistemological status as collections of articles. The latter are frequently aimed at spreading knowledge in an approach to the subject matter that remains exploratory and not unified. Textbooks, on the other hand, are more strongly grounded in a more unified analysis. They therefore require a period of homogenization of the discipline, and this is why they represent an additional step in the institutionalization process. Given that collections of articles are published before textbooks, the interval between the publication of the former and that of the latter gives an indication of the discipline's evolution (Jovanovic 2008): econophysics appears, therefore, as a theoretical approach that is evolving relatively rapidly. Barely a decade was enough to see the appearance of the first textbooks presenting econophysics as a unified and coherent field.[21]

This process of institutionalization was reinforced through the ability of econophysicists to connect with other research themes. In 2006, the Society for Economic Science with Heterogeneous Interacting Agents

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[19] These journals also publish a special annual edition devoted to papers presented at conferences on econophysics.

[20] Econophysics made its appearance in the PACS (Physics and Astrophysics Classification Scheme) in 2003 under heading *89.65 Gh*.

[21] As a point of comparison, consider that in behavioral finance the time that elapsed between the publication of collections of articles and the publication of the first textbooks was more than two decades. On this subject, see Schinckus 2009b.

(ESHIA) was created to promote interdisciplinary research combining economics, physics, and computer science (essentially, artificial intelligence). Of course, this project does not directly correspond with econophysics, since the analysis of the heterogeneity and interaction of agents is an approach that covers a larger field including experimental psychology and artificial intelligence. However, the new journal of the ESHIA (the *Journal of Economic Interaction & Coordination)* has been inviting authors to submit papers devoted to econophysics.

A further indicator of the emergence and institutionalization of the new scientific community is the organization of symposia and workshops. The first conference devoted to econophysics was organized in 1997 by the physics department of the University of Budapest. Two years later, the first conference recognized and supported by the European Association of Physicists was held in Dublin, resulting in the creation of an annual conference known as APFA (Application of Physics in Financial Analysis). Today, conferences and symposia dedicated to econophysics are quite numerous, notable among them being the Nikkei Econophysics Research Workshop and Symposium and the Econophysics Colloquium. In addition to the many publications on econophysics, all these regular events constitute institutional spaces that are helping to make econophysics a true scientific community.

The last major element in the institutionalization of econophysics is university education. Today, the physics departments of the Universities of Fribourg, Ulm, Münster, and Dublin offer courses in econophysics. Since 2002, the Universities of Warsaw and Wrołcaw have been offering a bachelor's and a master's degree in econophysics respectively (Kutner and Grech 2008). Finally, the University of Houston created the first doctoral program in econophysics in 2006.[22] All these programs are offered by physics departments, and courses are essentially oriented toward statistical physics and condensed-matter physics. In order to familiarize students with the economic reality they are supposed to describe, these programs do provide some courses on financial and macroeconomic conditions as they currently exist in the real world, but they are not based on the *theoretical* foundations of finance and macroeconomics.[23]

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[22] Information on the program may be found at http://phys.uh.edu/research/econophysics/index.php.

[23] For further information on these programs, see Kutner and Grech 2008, 644, and the websites of the universities mentioned in the paragraph. On the organization of the BSc and master's programs in econophysics at the University of Warsaw, see Kutner and Grech 2008, 637.

1.3. Econophysics' Major Distinguishing Feature: The Use of Stable Lévy Processes

Although econophysics and financial economics share the same topics (mainly the analysis of stock-price variations), they differ in the mathematics they use. Econophysics' distinctive feature is the use of stable Lévy processes (which follow an α-stable law of type $P(X > x) = x^{-\alpha}$ with a constant parameter α) for modeling stock-price variations.

In general, a Lévy process, named after the French mathematician Paul Lévy, is a time stochastic process with stationary[24] and independent increments known as *càdlàg paths*.[25] More precisely, Lévy worked on a generalization of the Gaussian statistical framework by developing a new class of distribution called Lévy α-stable. Lévy's α-stable movements are processes whose accretions are independent and stationary and follow an α-stable law of type $P(X > x) = x^{-\alpha}$ in which it is possible to observe constancy of the parameter α. Particular cases of Lévy processes are jump-diffusion processes and stable Lévy processes.

A jump-diffusion process is a process generally composed of a Poisson process and a Wiener process (Brownian motion), which is characterized by Gaussian distribution. Overall it is a Brownian motion with jumps at a specific gap dictated by the Poisson process.

A stable Lévy process is a jump process, characterized by a Lévy stable distribution whose power-law tail is described by $x^{-\alpha}$, with the α coefficient between 1 and 2. A Lévy stable distribution with $\alpha = 2$ is a Gaussian distribution; with $\alpha = 1$ it is a Cauchy distribution; and with $\alpha = 3/2$ it is a Pareto distribution.[26]

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[24] The "stationary" character means that the process that causes price variations remains the same over time, but it would be erroneous to associate this stationary character with continuity of the process. This is what Benoit Mandelbrot (1997, 138) pointed out in discussing the link between discontinuity and stationariness. "It is believed that stationariness excludes any major change and any non-banal configuration. But nothing limits the calculation of probabilities to the study of small fluctuations around a probable value." He continues to argue this point by adding that "the observation of long tails is intimately related to the symptom of discontinuity. . . . Each time a price undergoes strong discontinuity, the new point is added to the distribution tails of price changes" (143).

[25] In mathematics, a *càdlàg* (French "continu à droite, limite à gauche"), *RCLL* ("right continuous with left limits"), or *corlol* ("continuous on (the) right, limit on (the) left") function is a function defined on the real numbers (or a subset of them) that is everywhere right-continuous and has left limits everywhere. Càdlàg functions are important in the study of stochastic processes that admit (or even require) jumps, unlike Brownian motion, which has continuous sample paths.

[26] For a statistical presentation of these specific laws, see Schoutens 2003.

As we will explain in sections 2 and 3 of the present article, econophysicists are interested in modeling jumps in stock-price variations. To do that, they use jump stable Lévy processes. To understand the specificities of econophysics, it is crucial to make a distinction between jump stable Lévy processes and jump-diffusion processes.

Jump-diffusion processes are a combination of several classical processes that allow simulation of large price variations, with the distribution of each process having a finite variance and finite variation as well. A process that combines a normal law with a Poisson law is an example of such a process. By opposition, jump stable Lévy processes have a distribution with infinite variance.

One should note here the existence of other jump stable Lévy processes, such as the normal inverse Gaussian process, hyperbolic motion, the variance gamma process, and the CGMY model, all of which, unlike stable Lévy processes, have finite variance.

Of course the most important property of stable Lévy processes—the property that distinguishes them from other processes (e.g., a jump-diffusion process) and makes them unique—is stability. This means that the sum of many random variables that have stable distributions with the same α will have a stable distribution with the same tail coefficient α. The scaling property is also observed only in stable distributions: this means that distributions taken at different step lengths are identical upon normalization. This property has also been noticed in empirical price distributions.

Another difference between stable processes and jump-diffusion processes is that stable Lévy processes have infinite activity (an infinite number of jumps on each time interval) and infinite variation, while jump-diffusion processes have finite activity (a finite number of jumps on each time interval) and finite variation.

Such distinctions are necessary to understand the links between econophysics and financial economics. Indeed, the vocabulary used is misleading: financial economists use Lévy processes (such as Gaussian processes), while econophysicists use stable Lévy processes but employ the term *Lévy processes*. Although stable Lévy processes are constitutive of the emergence of econophysics, econophysicists are not alone in having attempted to apply them to the analysis of financial markets. Financial economists first tried to integrate them into their framework, as explained in the next section of our article.

2. The Origin of the Mathematical Tools Used in Econophysics and the Reason for Their Nonuse in Financial Economics

The history of financial economics is closely linked with the history of modern probability theory, to which it owes its major results, hypotheses, and models (Davis and Etheridge 2006; Jovanovic 2008). Moreover, one specific probability distribution plays a key role in the history of the discipline: Gaussian distribution (also known as normal distribution). This distribution underlies the creation of the majority of theories and models from the mainstream: the efficient market hypothesis, modern portfolio theory, CAPM, and the Black and Scholes model. We can therefore consider this distribution as a constituent of financial economics. But econophysics rejects the idea that financial distributions must be described only with a Gaussian distribution,[27] and, as we explain in the section 3, this rejection characterizes the emergence of econophysics. In view of this, the second section of our article will explain the origin of Gaussian distribution in financial economics, attempts to use stable Lévy processes, the reasons why financial economists stopped using them, and the alternative approaches they have developed.

2.1. The Origin of the Gaussian Approach in Financial Economics

Financial economics is mainly characterized by a high level of mathematization in the modeling of stock market returns. Modeling stock market returns or stock-market-price variations is the first step in the development of financial models. This is why financial economists have always focused their attention and research on such problems. Stock price variations and stock market returns have been successively modeled using a random walk, Brownian motion, and a martingale (Stabile 2005; Poitras 2006; Poitras and Jovanovic 2007; Jovanovic 2009). Because these mathematical models require a statistical characterization of changes in price or returns, the work of determining the statistical distribution of returns is a key problem in financial economics and, more generally, in modern financial theory.

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[27] In section 3 we explain that econophysics provides a generalization of the Gaussian framework, allowing greater flexibility in fitting models to describe empirical observations.

The first statistical representations of variations in the price of financial assets were made on the basis of a Gaussian framework.[28] Jules Regnault in 1863 was directly influenced by Adolphe Quételet's work on the application of normal distribution to social phenomena (Jovanovic 2001, 2006a). Bachelier ([1900] 1995), whose work was clearly influenced by Regnault's (Jovanovic 2000, 2009, 2012), retained a Gaussian description of the evolution of variation in asset prices.[29] Similarly, all the empirical work that emerged from the 1930s onward (Cowles 1933; Working 1934; Cowles and Jones 1937; Kendall 1953) used this Gaussian framework because at the time it was difficult to use other kinds of statistical distribution.[30] Indeed, all non-Gaussian observations and "white noise" were characterized through a Gaussian standardization.

This Gaussian description of financial reality progressively crystallized and was reinforced when Paul Samuelson (1965) introduced geometric Brownian motion to describe the continuity of trajectories.[31] Since then, Gaussian distribution of returns on assets has strongly contributed to the development of modern financial theory. From Harry Markowitz's modern portfolio theory (MPT) to the capital asset pricing model (CAPM) and the Black and Scholes model, through to the recent development of value at risk (VaR), Gaussian distribution of returns on assets has played a central role in the construction of financial economics (Geman 2002). However, from the time the first statistical databases of prices were constructed in the early twentieth century, some authors noted that the distributions were leptokurtic.[32] This characteristic of statistical distribution was incompatible with Gaussian distribution, and mathematical and statis-

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[28] A Gaussian perspective is the framework most used in science to describe random phenomena (Stewart 1992). Two arguments can explain this observation: the simplicity of Gaussian distribution (only two statistical moments are needed in order to describe a random phenomenon) and the statistical foundations of this Gaussian framework that are directly rooted in the central-limit theorem (Belkacem 1996).

[29] Bachelier needed normal law to demonstrate the equivalence between the results obtained in discrete time and in continuous time.

[30] Although some non-Gaussian distributions (Cauchy or Lévy distributions) existed, no author, except Luigi Amoroso, used them in a dynamic approach (Tusset 2010). And we had to wait for developments in modern probability theory in order to be able to use these statistical tools in finance.

[31] One of the principal characteristics of Brownian motion is precisely its normal distribution.

[32] Wesley Mitchell (1915) and Frederick Mills (1927, chap. 3), who were among the first to collect financial data, stressed this leptokurtic character. Later, starting with the initial work in econometrics, this character was frequently mentioned, as in Kendall 1953 and Cootner 1962. Obviously, none of these authors can be considered as an econophysicist.

tical work to model leptokurtic distribution appeared later.[33] At that time, while specialists were able to identify a non-Gaussian phenomenon, they had no statistical tools for dynamic analysis of observations of this kind. Non-Gaussian distribution was then only a matter of observation, and it was not modeled by a specific statistical framework.

2.2. The First Attempt to Generalize the Gaussian Framework

It was not until the 1960s that the leptokurtic nature of distributions was integrated into mathematical models used in finance, thanks to, among other things, access to the tools of modern probability theory.

In the 1960s, Benoit Mandelbrot (1962, 1963, 1965), Samuelson (1965), and Eugene F. Fama (1965) proposed studying financial markets using a non-Gaussian statistical framework directly inspired by Lévy's work (1924) on the stability of probability distributions and the generalization of the central-limit theorem proposed by Boris Vladimirovich Gnedenko and Andrei N. Kolmogorov (1954).[34] Mandelbrot was the first to attempt to use an extended Gaussian framework in finance. Using two models that he called M1963 and M1965, he opened two new research themes in the statistical modeling of financial uncertainty: one calls into question the *independence* of observations (between themselves) while the other examines the *stationary* character of these observations.[35] The first makes it possible to take into account observable and apparent cycles in the markets, and the second, the apparent discontinuity of the price of assets in the markets.

In his first model (M1963), Mandelbrot demonstrated how what Lévy called "α-stable" processes were entirely suitable for studying the discontinuity of price changes. To characterize this variability with respect

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[33] The leptokurtic nature of distribution tails was studied by Vilfredo Pareto (1848–1923) at the beginning of the twentieth century when he analyzed the distribution of wealth in Italy. His study informed subsequent research throughout the twentieth century (Barbut 2003). See also Tusset 2010.

[34] In accordance with this generalization, the sum of random variables according to Lévy laws, distributed independently and identically, converge toward a stable Lévy law having the same parameters. This generalization of the central-limit theorem is important because it represents a justification and a strong statistical foundation for the use of Lévy laws to characterize complex phenomena.

[35] *Stationary* means that variations in price remain the same over time; *independent* means that there is no link (no correlation) between variations in position.

to abrupt or discontinuous variations, Mandelbrot and James R. Wallis (1968) talked of a “Noah effect.”[36] Models that explicitly rejected the Gaussian framework and especially its continuity hypothesis needed to be integrated into a new probabilistic perception of uncertainty. Only these studies, as we shall see, are part of what we have termed econophysics.[37]

Mandelbrot worked with Fama on applications such as these in finance. In his article, Fama (1965) gave a mathematical reinterpretation of modern portfolio theory by Harry Markowitz (together with Sharpe’s diagonal model) in a Paretian statistical framework, but he was unable to provide a theoretical interpretation of his work because the parameter of risk (variance) was infinite (Fama 1965, 414). When Mandelbrot (1962, 1963, 1966) and Fama (1963, 1965) proposed characterizing the uncertainty of the evolution of quotations by using Pareto’s law, they were working directly within a probabilistic “stable Lévy” framework. They thus initiated a theoretical movement by proposing a generalization of the Gaussian framework to describe financial markets.

2.3. The Nonuse of Lévy’s “α-Stable” Processes in Financial Economics

Although Lévy processes, in their Paretian form, provide a better description of the evolution of financial markets, stable Lévy processes have not been used in financial economics.[38] To understand this point, we must go back to the 1960s and specifically to the writings of Mandelbrot and Fama on Paretian processes.

Laws that are α-stable present Paretian distribution tails that allow them to take into account price variations that are very large in relation to average variations. This is an essential property of α-stable laws, since it enables them to integrate the possibility of price “jumps.” But this char-

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[36] Mandelbrot and Wallis (1968) were referring indirectly to the biblical tale of Noah. When a “deluge” (stock market crash) is observed in financial markets, “even a big bank or brokerage house may resemble a small boat in a huge storm” (Mandelbrot 2005, 222).

[37] This origin of econophysics is explicitly recognized in the specialized econophysics literature (Mantegna and Stanley 1999; Roehner 2002; McCauley 2004) and claimed by Mandelbrot (2005) himself. It can be noted, however, that only a small number of physicists have proposed work based on an α-stable analysis defended in Mandelbrot’s first model—see, for example, Mantegna and Stanley 1999 and Sornette and Johansen 1997.

[38] Very recently, there have been some timid attempts. For further details, see Geman 2002.

acteristic, together with the stability of the distribution, means that variance can vary considerably depending on the size of the sample and the observation scale. Consequently, this variance does not tend toward a limit value. The variation is said, therefore, to be *infinite* because it does not tend toward a fixed value. This infinite variance appears to be one of the major reasons for the difficulties of using α-stable processes in financial economics.

Many researchers considered the infinite-variance hypothesis unacceptable because it is meaningless in the financial economics framework. Variance and the expected mean are the two main variables for theoretical interpretations. In the 1960s, the period in which financial economics was constituted as a scientific discipline, the relationship between risk and return was taken from Markowitz's work (1952, 1959). Markowitz associated risk with variance and return with the mean. In this perspective, if variance were infinite (as it is in a stable Lévy process), it became impossible to understand the notion of risk as Markowitz had defined it.

In addition to these difficulties, authors had to face the indeterminacy of variance on the one hand, and on the other the fact that no computational definition yet existed for evaluating parameters of stable Lévy processes. Fama (1965) himself regretted this point. He explained that the next step in the acceptability of stable Lévy processes in financial economics would be "to develop more adequate statistical tools for dealing with stable Paretian distributions" (Fama 1965, 419). A reminder of this statistical problem is found in papers dedicated to the estimation of parameters of stable distributions (Fama and Roll 1968, 1971). In addition, some authors expressed their skepticism about the opportunity to use stable Lévy processes. Robert Rupert Officer (1972, 811) explained that financial data "have some but not all properties of a stable process" and that several "inconsistencies with the stable hypothesis were also observed." He concluded that the evolution of financial markets could not be described through a stable Lévy process.

These difficulties explain why very few economists followed the path opened by Fama and Mandelbrot toward using stable Lévy processes. Fama and Richard Roll (1968, 1971), Robert Blattberg and Thomas Sargent (1971), and Peter K. Clark (1973) were exceptions. Even Fama (1976, 26) himself preferred to use normal distribution to describe monthly variations:

> Statistical tools for handling data from nonnormal stable distributions are primitive relative to the tools that are available to handle data

> from normal distributions. Moreover, although most of the models of the theory of finance can be developed from the assumption of stable nonnormal return distributions . . . , the exposition is simpler when the models are based on the assumption that return distributions are normal. Thus, the costs of rejecting normality for securities returns in favor of stable nonnormal distributions are substantial, and it behooves us to investigate the stable nonnormal hypothesis further.

In other words, the opportunity costs of using jump stable Lévy processes were too great at that time. However, despite such conclusions, research on integrating the leptokurtic character of distribution, or other characteristics from stable Lévy processes, was continued by financial economists.

2.4. Alternative Paths Explored for Using Stable Lévy Processes

While in the 1970s the use of stable Lévy processes seemed too complicated, financial economists explored alternative frameworks for α-stable distributions for the purpose of describing large price variations. The first path was the use of a combination of two (or more) different kinds of distribution, usually a normal distribution combined with a Poisson law. Poisson law allows the simulation of jumps in stock-price dynamics. This combination was first introduced by Robert C. Merton (1976).

In his 1976 article, Merton offered an extension of Black, Scholes, and Merton's 1973 option pricing model (Black and Scholes 1973; Merton 1973). This approach opened a new field of research called "jump processes." However, the mathematical foundations of Black, Scholes, and Merton's 1973 model were not sufficiently developed to allow Merton to see that his model loses many of its properties. One of the most interesting properties of the model is the completeness of the markets. This completeness is a condition for having a general equilibrium such as Kenneth Arrow and Gerard Debreu defined it. It was J. Michael Harrison and David M. Kreps (1979), Harrison and Stanley R. Pliska (1981), and Kreps (1981) who provided the mathematical foundations of Black, Scholes, and Merton's model. And it was only with Harrison and Pliska (1981) that we can show that Merton's 1976 model, like any jump-process model, does not permit a single solution, with the result that there are arbitrage opportunities (in other words, the market is not efficient).[39] Indeed, since the development of the mathematical framework by Harrison and Kreps (1979) and Harrison and Pliska (1981), we have known that jump processes create an incompleteness market (which means that arbitrages exist).[40] As Vasanttilak Naik and Moon Lee (1990) explained, with the jump-diffusion model proposed by Merton the market is not complete in the Harrison and Pliska (1981) sense, with the result that contingent claims in such a model cannot be priced simply by no-arbitrage arguments. In other words, modern financial theory's theoretical framework, like any other theoretical framework, creates some limits. One of these limits is the use of some stochastic processes, in particular, stable Lévy processes.

In other words, in the 1970s and the 1980s, mathematical finance emerged, providing a very technical interpretation of the arbitrage condition. However, despite this evolution of finance into a more mathematical field, technical tools did not exist to explore Merton's (1976) attempt to integrate jump stable Lévy processes into financial economics. Things started to change in the 1990s.

3. The New Context of the 1990s That Allowed Econophysics to Emerge

For an understanding of the "revolution" that began in the 1990s, two points deserve mention. First, as stated earlier, financial economics is closely linked with modern probability theory, which is the source of its major hypotheses, models, and results. Second, statistical physics is mainly concerned with providing the best possible representation of real phenomena. Econophysicists, then, are less concerned than are economists with theoretical explanation, focusing instead on simulating real phenomena.

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[39] The no-arbitrage condition and the idea of equilibrium are theoretically interconnected but the two concepts are different. The first is a consequence of the second, which is rarely used by financial economists; see Sharpe 1964, 434, for further information about the importance of equilibrium in financial economics. As Jérôme Detemple and Shashidar Murthy (1997) explained, the condition associated with no-arbitrage is less restrictive than the theoretical assumptions related to the idea of equilibrium. However, even if no-arbitrage is less restrictive than the assumption of complete equilibrium, this condition requires that the solution be unique, and the jump-process models used in the 1970s did not meet that condition.

[40] More precisely, Harrison and Kreps (1979) and Harrison and Pliska (1981) showed how a process must be continuous in order to have unicity of the equivalent martingale measure and consequently a unique price.

These two points are crucial for understanding how econophysics arose out of technical concerns, as this section will explain.

3.1. New Mathematics:
The Truncation of Lévy Laws

Econophysics can be thought of as the continuation of thermodynamics, and the use of Lévy processes in this field allowed more accurate modeling of the phenomenon of turbulence. The first studies on the subject were those of Kolmogorov on the scale invariance of turbulence in the 1930s. This theme was subsequently addressed by many physicists and mathematicians, particularly by Mandelbrot in the 1960s when he defined fractal mathematics[41] and applied it to the phenomenon of turbulence.

Despite the extension of probability theory to thermodynamics, physicists did not seem disposed to integrate stable Lévy processes into physics (Gupta and Campanha 2002, 382). This methodological position (like the abandonment of α-stable processes in financial economics) is explained by the fact that processes with infinite variance are not physically plausible from a theoretical viewpoint:

> Stochastic processes with infinite variance, although well defined mathematically, are extremely difficult to use and, moreover, raise fundamental questions when applied to real systems. For example, in physical systems, the second moment is often related to the system temperature, so infinite variance implies an infinite temperature. (Mantegna and Stanley 1999, 4)

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[41] Fractal mathematics was essentially developed by Mandelbrot, who attempted to develop a new geometry to describe a large number of complex, "irregular" phenomena encountered in nature (Mandelbrot 2005, 147). The main idea of fractal geometry is that certain aspects of reality have the same structure seen from afar or close up, at any scale, and that only the "details having no effect change when they are enlarged for a close-up view" (Mandelbrot 1995, 36). Mandelbrot uses the phrase "principle of scale" to illustrate this constancy of structure between two levels of enlargement, and he adds that "a phenomenon satisfies the principle of scale if all the quantities relating to this phenomenon are linked together by a law of scale" (53). Lévy processes will permit a statistical reformulation of fractal geometry by means of the notion of invariance. It then becomes possible to link two variables (X and Y) each characterizing the level of the zoom operated on the phenomenon under study. In this way, two levels, with each of which a variable is associated and which both comply with the principle of scale, can be linked by a scaling law (or a power law). Regarding the history of statistical physics and the importance of fractal mathematics in this discipline, see Ruelle 1991 or Barberousse 2002.

As Hari Gupta and José Campanha (1999, 234) point out, stable Lévy processes "have mathematical properties that discourage a physical approach because they have infinite variance." In their view, this property of physical systems is the direct result of the thermodynamic hypotheses set out by Ludwig Boltzmann in 1872 when he laid the foundations of contemporary statistical mechanics.[42] Physicists, then, seem to be facing the same theoretical impasse as Fama and Mandelbrot in the 1960s: the infinite character of variance.[43]

Nevertheless, in the 1970s, a very specialized literature dedicated to the parameterization of Lévy distributions was developed (Paulson, Holcomb, and Leitch 1975; Chambers, Mallows, and Stuck 1976; Koutrouvelis 1980). By offering different frameworks to compute parameters of Lévy distributions, this literature favored the increasing use of stable Lévy processes in physics (particularly in statistical physics) in the 1980s (Shlesinger, Zaslavsky, and Frisch 1994). While these works on the parameterization of Lévy distributions generated much technical debate in the statistical and mathematical literature,[44] several authors tried to overcome the problem of the infinite character of variance.

This mathematical difficulty was resolved by introducing truncated Lévy distribution during the 1990s. Physicists have chosen to characterize turbulence phenomena using Lévy processes while explicitly rejecting the idea of infinite variance. To achieve this, a number of writers have proposed statistical methods for the "standardization" of "α-stable" distributions so that variance is no longer infinite. The most widespread method consists in truncating Lévy distributions.[45] Generally, this truncation operation can be rendered as follows:

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[42] While statistical physics cannot be reduced to the use of stable Lévy processes, econophysics is a more specific field that focuses on the application of stable Lévy processes to the turbulence phenomenon. More particularly, the possibility of using these processes to characterize the statistical behaviors of particles has led some physicists to extend their application to the statistical description of financial distributions. Today, the literature of econophysics is mainly (but not solely) based on the application of stable Lévy processes to financial economics; see Gingras and Schinckus 2012 for a bibliometric study of this point.

[43] Note that a number of studies have been carried out on a new data dependency structure to replace the concept of variance with the notion of "covariation" (Föllmer, Protter, and Shiryayev 1995). However, these studies have not been unanimously accepted by physicists.

[44] See Nolan 2009 for further information on these debates.

[45] The truncation of a Lévy distribution consists in normalizing it using a particular function so that its variance is finite. For example, one can combine a non-truncated Lévy process for the distribution center and explain the tail ends using exponential distributions. On this topic, see Gupta and Campanha 2002.

$$P(X > x) = P_\alpha(x)\ \varphi(x),$$

where $P_\alpha(x)$ designates the probability distribution in its Lévy form and $\varphi(x)$ is a truncation function allowing finite variance to be obtained.

This truncation function can take a number of forms, the simplest being to integrate a standardization constant into a Lévy distribution. This is what Mantegna did in 1991 when he gave the first statistical answer to the problem of the infinite variance of stable Lévy processes. This article gave rise to (and is still giving rise to) a great deal of research on truncation functions. Today it is possible to find several types of truncation functions that can be used depending on the characteristics of the system under study.[46]

The operation of truncating Lévy processes allowed physicists to use these processes to characterize turbulence phenomena statistically without having a problem of indeterminate variance. The statistical response given by physicists to this indeterminate nature of variance also contributed to the emergence of econophysics, since it was now possible to apply this standardization operation in such a way that the evolution of financial markets could be described using stable Lévy processes.

3.2. New Empirical Data

A second reason explains the emergence of econophysics: the evolution of technology—specifically, computer science. Developments in computing have had a double influence: first, they have allowed better differentiation of the empirical distribution of stock-price variations; second, they have led to an increase in extreme stock-price variations.

Today, electronic markets rule the financial sphere through real-time data, allowing a more accurate study of how these data evolve. Accumulated data are stored in the form of time series. While time-series data have been studied by economists for several decades, the automation of markets has enabled “intraday” data, providing “three orders of magnitude more data” to be recorded (Stanley et al. 2000, 339). The quantity of data is an important factor at a statistical level because the larger the sample, the more reliable the identification of statistical patterns. These new data have led to an increasing number of statistical works about financial markets.

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[46] Some examples are the abruptly truncated Lévy distribution (Jaroszewicz, Mariani, and Ferraro 2005); the exponentially truncated Lévy distribution (Matsushita, Rathie, and Da Silva 2003); and the gradually truncated Lévy distribution (Gupta and Campanha 1999). On these normalization methods, see Vasconcelos 2004 or Gupta and Campanha 1999, 2002.

Econophysics and financial economics both use a common hypothesis: the stationary[47] ergodic hypothesis[48] (Schinckus 2009a), according to which future data will be a statistical reflection of past data. In this perspective, the bigger the sample, the more accurate the statistical analysis. Let us mention first that it is difficult to determine with certainty whether empirical data are distributed in accordance with a power law or another kind of law. Michael Mitzenmacher (2004) noted that these laws are close to the so-called exponential laws,[49] pointing out that only a large volume of data makes it possible to distinguish between the two types of law (power law and normal law). Consequently, the use of intraday data has made it possible to construct samples sufficiently broad to definitively confirm Mandelbrot's idea that the evolution of financial markets could be characterized using stable Lévy processes (Kou 2008). This accumulation of statistical data has also favored the application of stable Lévy processes in financial economics, specifically power laws, the principal tool of econophysics.

Another element that has favored the development of an econophysics approach is the economic consequences of the computerization of financial markets. The growing liquidity of markets following their computerization has strongly accentuated speculation and market volatility (Barber and Odean 2001, 47). This greater volatility has resulted in an increase in

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[47] Nonstationarity is a sufficient, but not a necessary, condition for nonergodicity. Some economists have claimed that the economy is a nonstationary process moving through historical time because societal actions have direct influences on this process. Because not only statistical factors are relevant in the economy, Keynes (1939) wrote a famous criticism of Tinbergen's econometric methodology claiming that economic time series are not stationary because "the economic environment is not homogeneous over a period of time." More recently Robert Solow (1985, 328) has written, "Much of what we observe cannot be treated as the realization of a stationary stochastic process without straining credulity."

[48] This hypothesis comes from thermodynamics and assumes implicitly reversible processes. Reversibility is often confused with the notion of recoverability, which means the retrieval of an initial state (Uffink 2006). From a statistical point of view, reversibility refers to the idea that the times series can be defined by the same process through time—see Ramsey and Rothman 1996 for a formal definition of reversibility in time series. The ergodic hypothesis was generalized in economics by Samuelson (1969, 184), who made the acceptance of the ergodic hypothesis "the *sine qua non* condition of the scientific method in economics." He indicated that he used the term *ergodic* "by analogy to the use of this term in [nineteenth-century] statistical mechanics" in order to remove economics from the "realm of genuine history" and keep it in the "realm of science" (184). For further information about the use of the ergodic hypothesis in economics, see Davidson 1991.

[49] Generally speaking, we can define an exponential law by the following relation: $P(X > x) = \lambda e^{-\lambda x}$, where λ is a positive parameter.

extreme variations of quotations. This more volatile reality needs new statistical tools suited to the analysis of extreme phenomena. Because Lévy's laws are one of these statistical tools, increased market volatility has been a parameter favorable to the development of approaches such as econophysics.

We observe, then, a double contribution of technology to the emergence of econophysics: the first contribution is direct, arising from the computerization of financial markets (better data analysis and storage); the second, more indirect, is the result of financial behavior that computerization has led to.

Mantegna and Stanley (1999, 6), Joseph L. McCauley (2004, 7), and Zdzislaw Burda, Jerzy Jurkiewicz, and Maciej A. Nowak (2003) also point to the role played by computerization in the emergence of econophysics, especially the fact that the process of computerization has expanded the statistical reproducibility of markets. Statistical regularities identified over large samples provide theoreticians with a more accurate picture of market evolution. Greater quantities of information are available, creating the illusion[50] that past behaviors of stock-exchange returns are reproducible statistically. Fabian Muniesa (2003, 391) called such hope for the future reproduction of financial markets based on past statistical information "statistical utopia." Jean-Philippe Bouchaud (2002) explained that the computerization of financial marketplaces has transformed financial market analysis into a true "empirical (rather than axiomatic)[51] science" that makes it, according to econophysicists, "a natural area for physicists" (Gallegati et al. 2006, 1).

3.3. From Outside to Inside

While these two elements were factors that triggered the emergence of econophysics, a third element to consider, and one that should not be underestimated, is the particular position of econophysics in relation to modern financial theory.

As we have explained, econophysics is characterized by the use of stable Lévy processes. In financial economics, use of these processes was difficult because they conflicted with the discipline's probabilistic frame-

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[50] This illusion refers to the implicit hypothesis of ergodicity of financial data and the idea that only statistical factors are relevant in the analysis of financial phenomena.

[51] Econophysicists explicitly reject a priori and axiomatic approaches. They prefer to describe reality as it is rather than as it should be (Schinckus 2010).

work as defined by the work of Harrison, Kreps, and Pliska. Econophysicists seem to have ignored these constraints imposed by the foundations of modern financial theory. For example, studies by econophysicists of option pricing ignore the fact that one of the strengths of the Black and Scholes model is that this pricing is made possible by a replicating portfolio. The use of stable Lévy processes poses serious problems for obtaining a replicating portfolio. In our view, this difficulty explains why econophysicists have positioned themselves in theoretical niches that mathematicians and economists have barely investigated, or not investigated at all, because of the constraints of the theoretical framework. For example, there is a fundamental difference in views about financial market equilibrium. While modern financial theory provides a less restrictive condition (a no-arbitrage condition) than the traditional economic equilibrium, econophysics has developed a technical framework without taking into account the theoretical assumptions related to economic equilibrium or to the no-arbitrage condition. In fact, these notions do not play a key role in econophysics;[52] they instead appear as an a priori belief[53] that provides a "standardized approach and a standardized language in which to explain each conclusion" (Farmer and Geanakoplos 2009, 17). Specifically, econophysicists do not reject the concept of equilibrium, but they consider that there is not necessarily a convergence toward such a state. Similarly, while they do not reject the condition of no-arbitrage, they are indifferent to this restriction.

Moreover, the outsider position of econophysicists explains why "econophysics generally produces a mathematically more robust explanation of the particular behavior being studied, but . . . rarely postulates new economic or financial theories, or finds contradictory evidence to existing theories" (Ray 2008, 175).[54]

The fact that stable Lévy processes conflicted with the probabilistic framework of modern financial theory also, in our view, provides the main explanation for the marginal use of stable Lévy processes in mathematical

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[52] For instance, equilibrium is considered as merely a *potential* state of the system because "there is no empirical evidence for equilibrium" seen as a final state of the system (McCauley 2004, 6).

[53] When econophysicists deal with equilibrium, they use rather a "statistical equilibrium" coming from a statistical mechanism (i.e., a reconciliation between a mechanism and thermodynamics). See Bouchaud 2002. See Schinckus 2011 for further information about the importance of equilibrium in econophysics.

[54] Note, however, that a small number of authors, for example, Jean-Philippe Bouchaud, have attempted to reconcile results produced by econophysics with the financial economics framework.

finance. And, while financial mathematicians could be attracted by the use of Lévy-stable classes,[55] the connections between mathematical finance and financial economics keep financial mathematicians from adopting these classes of processes. Despite this conflict, financial economists and financial mathematicians have developed a few models based on stable Lévy processes since the 1990s. Among the generalized Lévy processes developed in mathematical finance and financial economics are the normal inverse Gaussian process (Schoutens 2003), the variance gamma (Madan and Seneta 1990; Petroni 2007),[56] the generalized hyperbolic process (Eberlein and Keller 1995),[57] and the CGMY process (Carr et al. 2002).[58] Despite these exceptions, we must conclude that it is precisely because econophysicists have developed their work outside the theoretical framework of modern financial theory that they can apply such processes more freely.

Econophysicists have taken advantage of a very specific opportunity in that they use stable Lévy processes to model stock-exchange variations independently of the traditional framework of modern financial theory and more specifically the theoretical framework of financial economics.

Two recent developments should, however, be noted. First, the *Encyclopedia of Quantitative Finance,* published in 2010, which provides an exhaustive presentation of the state of knowledge in its field, contains several entries devoted to econophysics. Second, econophysicists are gradually succeeding in taking control of recognized economics and finance journals. Since the appointment of J. Barkley Rosser[59] as editor-in-chief in 2002, the *Journal of Economic Behavior & Organization* has begun publishing regular articles on the issue of complexity in economics,

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[55] This attraction is all the greater in that, in line with the works of financial mathematicians, econophysicists provide a unique solution for the use of stable Lévy processes.

[56] Madan, Carr, and Chang (1998) introduced the variance gamma process defined by an arithmetic Brownian motion with drift q and volatility s, time-changed by an increasing gamma process with unit mean and variance n, resulting in the three-parameter process.

[57] Ole Barndorff-Nielsen and Cul Halgreen (1977) show that hyperbolic distribution can be represented as a mixture of normals, where the mixing distribution is a generalized inverse Gaussian.

[58] All these Lévy processes share the property of being jump stable processes and having infinite activity, but, unlike α-stable processes such as those used by econophysicists, they do not present continuous properties to be applied in complete-market situations. In this perspective, the statistical properties of stable Lévy processes appear to be more interesting since they are continuous processes and can describe the leptokurticity of financial markets.

[59] Rosser's research focuses, partly, on complexity in economics, meaning that he shows considerable open-mindedness to the approach proposed by econophysicists.

allowing econophysicists to publish their work. Two further economic journals regularly publish econophysics articles: *Quantitative Finance*, launched in 2001, and the *Journal of Economic Interaction & Coordination*, created in 2006. As implied in section 1, the latter journal was created to promote research combining economics, physics, and computer science. It is mainly directed by physicists, and its editorial team features a substantial number of physicists and artificial intelligence specialists. *Quantitative Finance* is a finance journal directed by an econophysicist and a mathematician,[60] with a majority of econophysicists on the editorial team.[61] A further sign of the growing influence of econophysics is the International Conference on Econophysics, a platform for the presentation of interdisciplinary ideas coming from different communities, especially economics, finance, and physics.

This progressive incursion of econophysicists into economics journals would appear to herald certain future developments in modern financial theory and consequently in financial economics.

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[60] Jean-Philippe Bouchaud and Michael Dempster.

[61] A fact that doubtless explains why the journal most cited in *Quantitative Finance* is none other than *Physica A* (Gingras and Schinckus 2012).

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