Zipf's Law in the Dynamical Importance of Network Nodes and Links

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Outline

- Question about the Importance of Network Nodes and Links
- Dynamical Importance
- Simulations of Different Networks
- Zipf’s Law in the Dynamical Importance
- Take Home Message

Question about the Importance of Network Nodes and Links

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Question about the Importance of Network Nodes and Links

- How to quantify the importance of a node or a link in a network?
  Degree? or Clustering Coefficient?
  Degrees of the linked nodes?
- A higher degree doesn't always mean a higher importance.
  They are all local!
- We need a universal quantity to measure the importance of network nodes and links.
  It should reflect the structure of the whole network!

Dynamical Importance

Background

- The largest eigenvalue of the network adjacency matrix λ turns out to be very important in the properties of different dynamical networks. [4]
  Examples are discussed in [3, 5, 1, 2].

Dynamical Importance

Background

- The largest eigenvalue of the network adjacency matrix λ turns out to be very important in the properties of different dynamical networks. [4]
  Examples are discussed in [3, 5, 1, 2].
- λ is proven to be always real and positive. [2]
  It is about the whole network.

Dynamical Importance

Definition

- The dynamical importance of a node is defined as

$$I_k \triangleq = -\frac{\Delta\lambda_k}{\lambda},$$

  where $\Delta\lambda_k$ is the change of λ upon removal of node k. [4]
- The dynamical importance of a link is defined as

$$I_k \triangleq = -\frac{\Delta\lambda_{ij}}{\lambda},$$

  where $\Delta\lambda_{ij}$ is the change of λ upon removal of the link between i and j. [4]

Simulations of Different Networks

Methods

- What are the dynamical importance distributions of nodes or links?

Simulations of Different Networks

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- There are four types of network models in the Python library 'networkx':
  Regular Graph
  Erdos Renyi Random Graph
  Watts Strogatz Small World Graph
  Barabasi Albert Scale Free Graph

Simulations of Different Networks

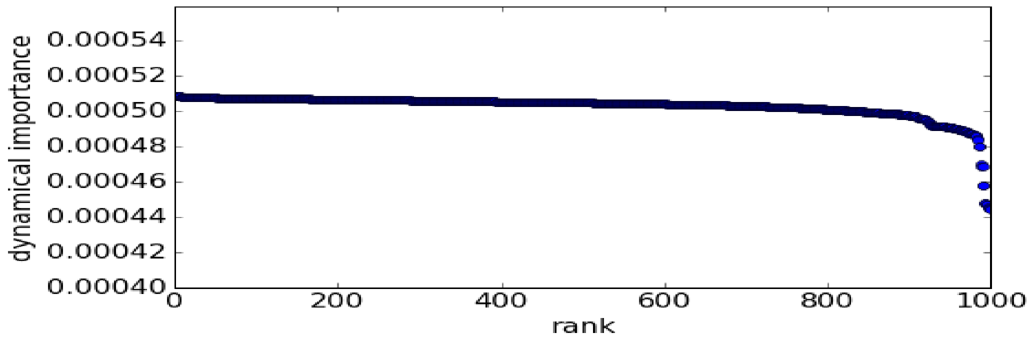
Methods

- What are the dynamical importance distributions of nodes or links?
- There are four types of network models in the Python library 'networkx':
  Regular Graph
  Erdos Renyi Random Graph
  Watts Strogatz Small World Graph
  Barabasi Albert Scale Free Graph
- Simulations are done to calculate I_k for each node or I_{ij} for each link, and then sorted in descending order.

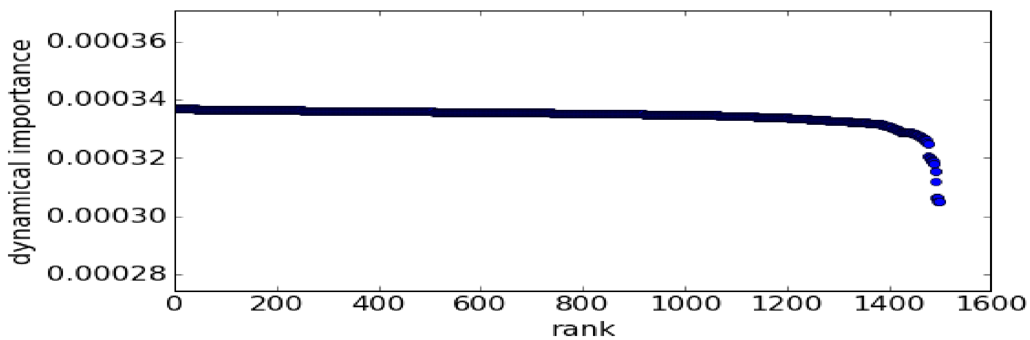
Simulations of Different Network

Results – Regular Graph

- For nodes:



- For links:

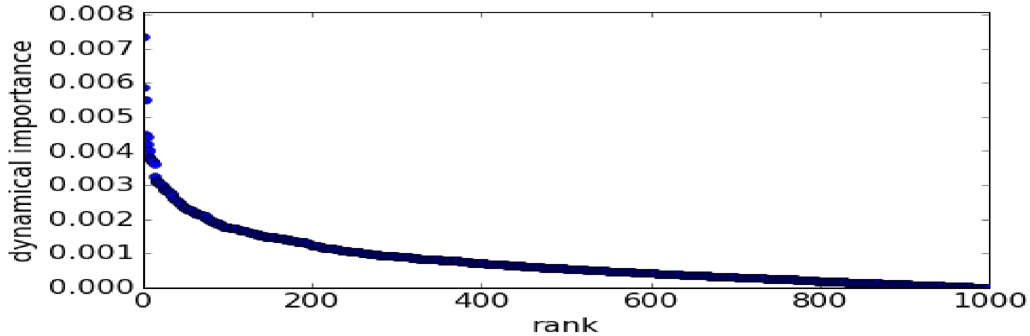


- All the nodes and the links are almost equally important.

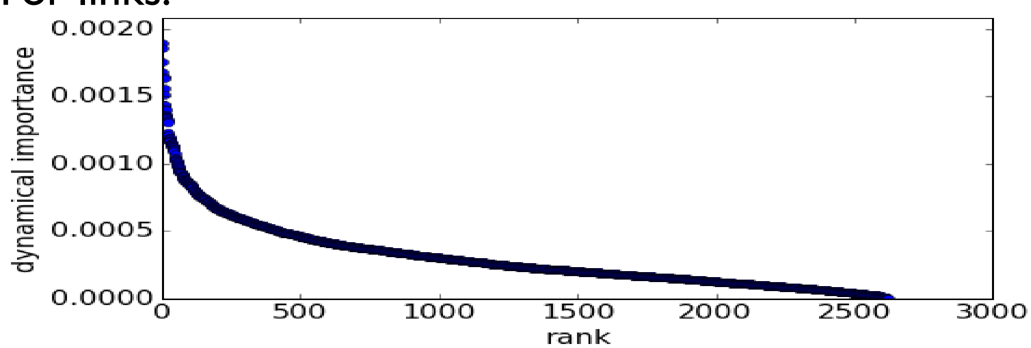
Simulations of Different Network

Results – Erdos Renyi Random Graph

- For nodes:



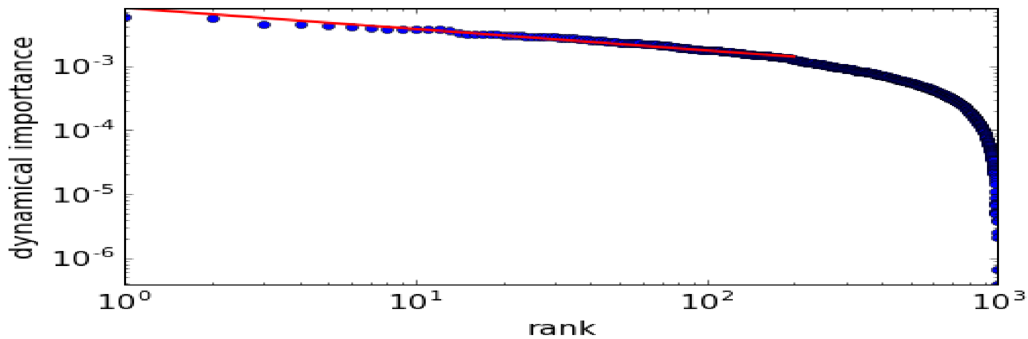
- For links:



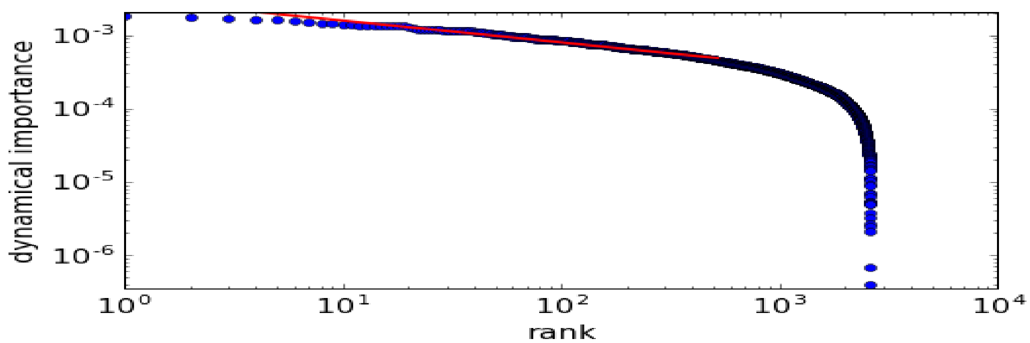
Simulations of Different Network

Results – Erdos Renyi Random Graph

- For nodes (log-log, $s = -0.33$):



- For links: (log-log, $s = -0.30$)



Simulations of Different Network

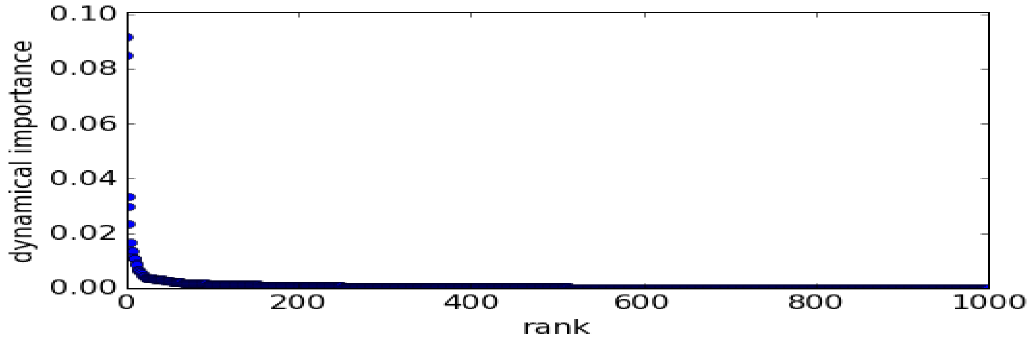
Results – Watts Strogatz Small World Graph

- Doesn't converge when the largest eigenvalue is calculated.

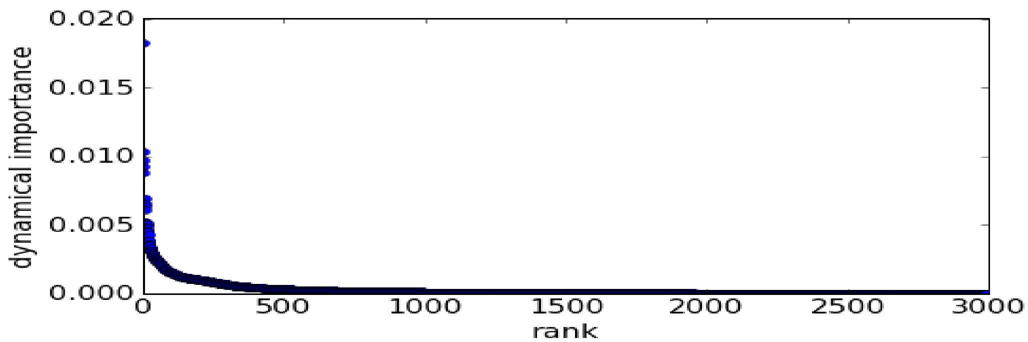
Simulations of Different Network

Results – Barabasi Albert Scale Free Graph

- For nodes:



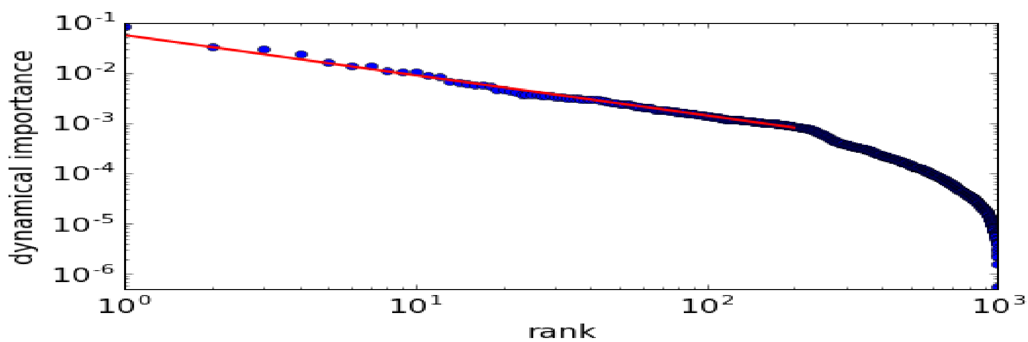
- For links:



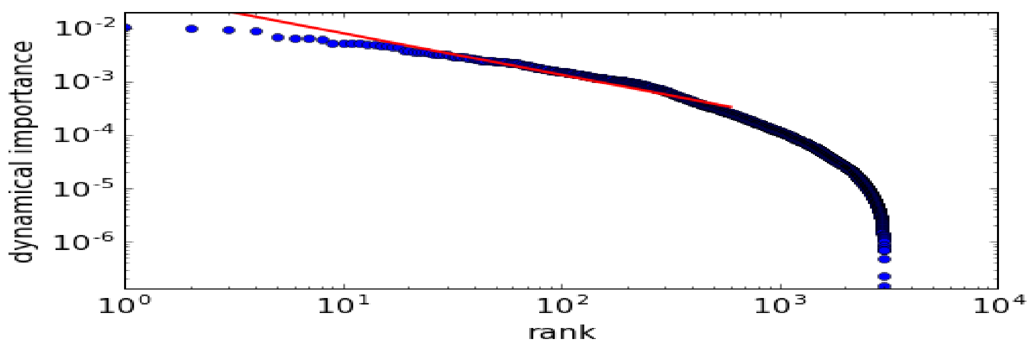
Simulations of Different Network

Results – Barabasi Albert Scale Free Graph

- For nodes: (log-log, $s = -0.80$)



- For links: (log-log, $s = -0.78$)



Zipf's Law in the Dynamical Importance

Zipf's Law

- ''Many types of data studied in the physical and social sciences can be approximated with a Zipfian distribution, one of a family of related discrete power law probability distributions.'' [6]

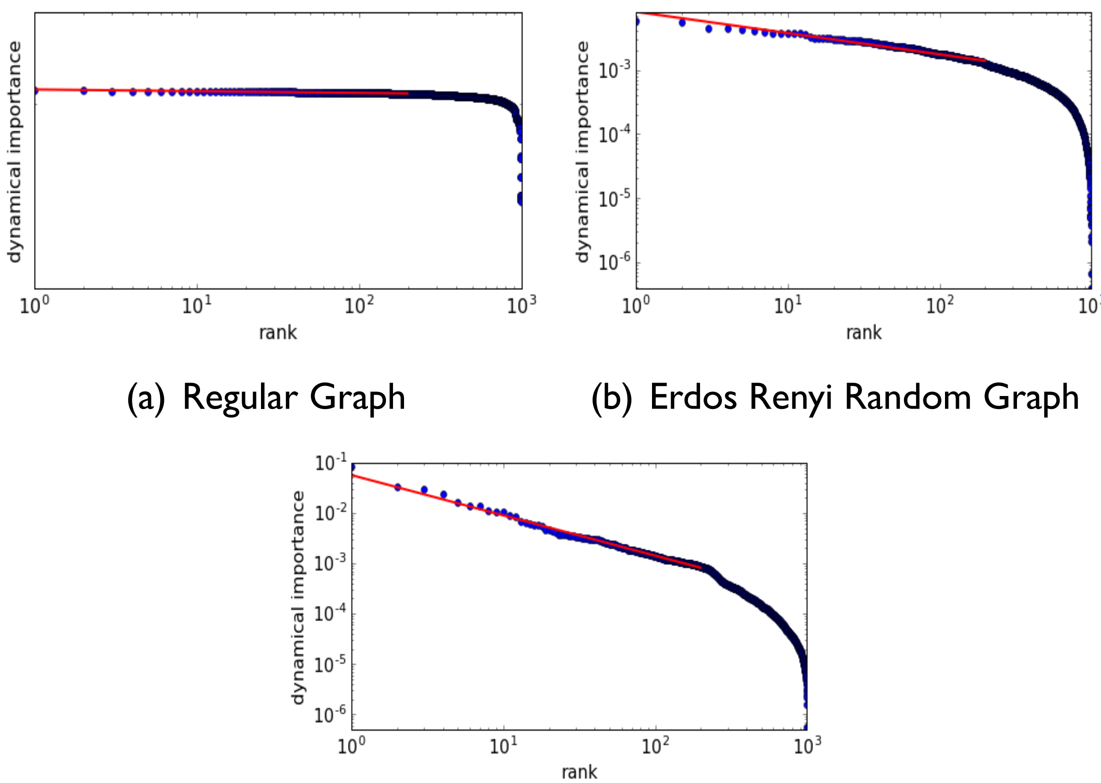
Zipf's Law in the Dynamical Importance

Zipf's Law

- "Many types of data studied in the physical and social sciences can be approximated with a Zipfian distribution, one of a family of related discrete power law probability distributions." [6]
- A quantity is in a power law in terms of its rank.

Zipf's Law in the Dynamical Importance

Nodes (log-log)

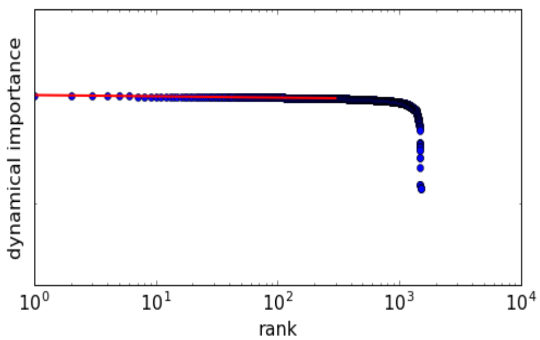


(a) Regular Graph (b) Erdos Renyi Random Graph

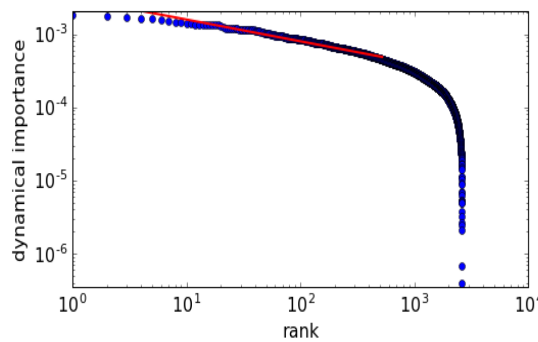
(c) Barabasi Albert Scale Free Graph

Zipf's Law in the Dynamical Importance

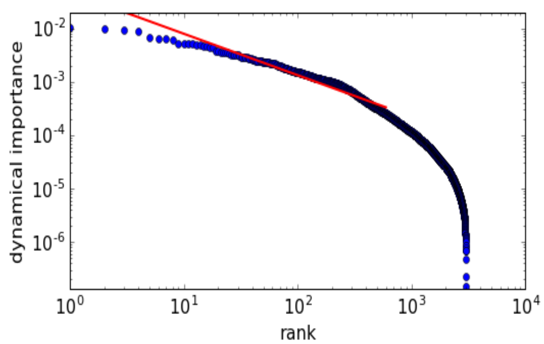
Links (log-log)



(d) Regular Graph



(e) Erdos Renyi Random Graph



(f) Barabasi Albert Scale Free Graph

Take Home Message

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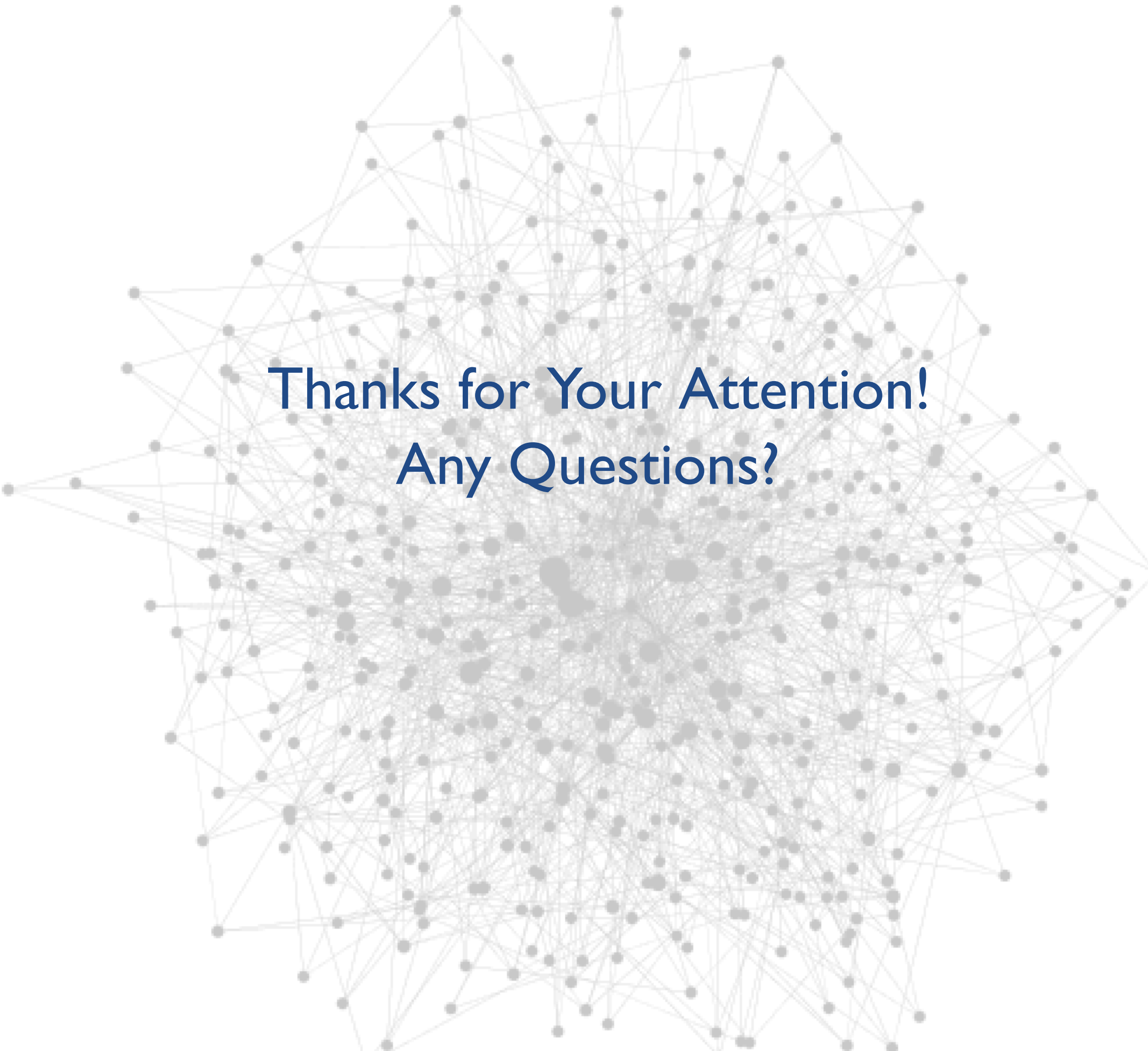
- The dynamical importance of a node or a link can be defined as the percentage decrease in λ upon its removal.
- The dynamical importance of nodes obeys perfect Zipf's Law.
  The dynamical importance of links obeys less-than-perfect Zipf's Law.



Thanks for Your Attention!
Any Questions?

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