Network Science, Fall 2016

Dr. Jianxi Gao, Professor H. E. Stanley

Homework 2, due by October 7th, 9:30AM (before class)

Write your name at the top of your homework before handing it in. Staple all pages together.

1. Erdős-Rényi graph

In an Erdős-Rényi graph with $N = 3000$ nodes, the linking probability is $p = 10^3$.

(a) What is the expected number of links $<L>$?

(b) In which regime is the network (Subcritical Regime, Critical Point Supercritical Regime, or Connected Regime) ?

(c) Provide the linking probability p_c to have a network with the same number of nodes $N = 3000$ at the critical point.

(d) Assuming the same linking probability of (a) and (b) ($p = 10^3$), provide the number of nodes N^{sc} of a network that has only one component (one example).

(e) For the same example network of point (d), calculate the average degree $<k^{sc}>$ and the average distance among two randomly chosen nodes $<d>$.

(f) Provide the degree distribution $P(k)$ of this network (approximate with a Poisson degree distribution).

2. Cayley tree

A Cayley tree is a symmetric regular tree, constructed starting from a central node of degree k. Every node in the network at distance d from the central node has degree k, until we reach the nodes at distance P that have degree one and are called the "leaves"of the network. Assume here $k = 4$.

(a) Calculate the number of nodes reachable in d steps from the central node for $d \in [1,P]$.

(b) Calculate the degree distribution of the network.

(c) What the diameter D?

(d) Find an expression for the diameter D of the network in terms of the total number of nodes N.

(e) Does the network display the small-world distance property?