Knowledge Spaces as Networks

David Stelter
Thomas Heavey
Knowledge Space Theory
Definitions

- **item** - a specific topic or skill
- **domain** - a related set of items, typically for a given field or class
- **knowledge state** - a set of items that constitutes a possible set of things a person could know
- **knowledge space** - the set of all possible knowledge states over a domain
- **fringe** - the set of knowledge states one item away from a certain state

The field was pioneered by Jean-Paul Doignon and Jean-Claude Falmagne at Université Libre de Bruxelles and UC Irvine, respectively.

*Knowledge Spaces: Applications in Education*; 2013.
Small Sample Knowledge Space

A. Quotients of expressions involving exponents
B. Multiplying two binomials
C. Plotting a point on the coordinate plane
D. Writing the equation of a line given a point and the slope
E. Solving a word problem using a set of linear equations
F. Graphing a line given its equation
G. Multiplication of a decimal by a whole number
H. Integer addition
I. Equivalent fractions
J. Graphing integer functions
Small Sample Knowledge Space

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Our Studied System

- **domain** - 571 items for a general chemistry class
  - maximum possible set of knowledge states is $2^{571} = 8 \times 10^{171}$

- knowledge states from Assessment and Learning in Knowledge Space (ALEKS), an adaptive online instruction tool
  - largely NSF funded
  - they care about their intellectual property

- we got a sample of knowledge states of BU students over the past four years
Network Structure
Single Class Visualization

Total Edges = 36162
Total Unique Learning States = 64444
\langle k \rangle = 1.1223
Degree Distribution
Largest Component

5819 nodes, diameter = 9

Second Largest

Third Largest

Fourth Largest

Fifth Largest
Components (Item Counts)
Components (Item Counts)
Components (Item Counts)

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Centralities
Future Work

Understand structure of large components and meaning of the central node.

Identify states crucial to student learning

Add assessments to get a finer progression of student learning progress to increase overall network connectivity

Propose analysis to possibly be run on the full network

Obtain full subset of the knowledge space to analyze

Add weights by assessing time spent between knowledge states
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