Dynamic of the Stock Index: the Predictive Model with Relation to Linear Spring Equation

Nutthakorn Intharacha

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Abstract

In this paper, we will introduce a new stochastic indicator that can be modeled to predict the momentum of the stock index movement. Employing Ragnar's idea, we then formulate a new stochastic indicator function structurally similar to an underdamped solution of a simple harmonic oscillation with external force. We test the model by fitting it to the daily closing index in a certain short term period of S&P500 since 2000 in three different scenarios depending on the index movement—uptrend, downtrend, and sideway. However, to an extent of Ragnar's framework, we also take an account into how well the model can predict the movement of the stock index in some certain period in the future. The result shows, in short term period (about 25 to 35 trading days), the model capture the movement pretty well based on the current information. Moreover, it performs well enough to predict the index movement in next five days.

1. Introduction

1.1 Economic theory on price determination

In economic theory, the law of demand and supply is considered one of the fundamental principles that plays a large role governing an economy. This law appears everywhere there is a situation of trading and stock market is no exception. When demand of the stock increase, the price will then tend to increase, and vice versa. In the stock market, we can think of demand represented by volume. Therefore, normally in the stock market, when volume goes up, we expect to see the upward movement of the stock price.

Graph 1: Supply & Demand

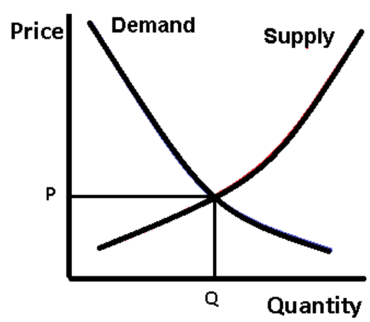


Figure 1 presents the graph of the law of demand-supply. If demand increases, demand curve will result in being shifted to the right. Hence the equilibrium, which is the point where the two curve intersect, changes resulted in higher price. And the result follows the same logic in the case of decreasing in demand.

We can mathematically derive equations for quantity demand and quantity supply in the function of price based on this dynamic.

- $Q_d = \alpha_0 - \alpha_1 P + \alpha_2 P' - \alpha_3 P''$ ---(1)
- $Q_s = \beta_0 + \beta_1 P - \beta_2 P' + \beta_3 P''$ ---(2)

Note that all parameters are non-negative. The (1) equation intuitively means when the price increase, the quantity demand tends to decrease. And when the tendency of price (P') tends to increase, the quantity demand tends to increase because people expect the price could rebound in the future. Lastly when there is a decreasing sign of tendency of price (P'') that could go up in the future, the quantity demand tends to decrease in response to that. The equation (2) can be interpreted by the same logic except that it is for supply side. We can think of the supply side as a trader who view the stock undervalue. So if the price goes up, the number of trader, who is willing to sell, increase. When the tendency of price in the future signals a down sign (p'), the supply side will shrink. When there is an increasing sign of the tendency of price (P'') that could go up in the future, the quantity supply will increase. However we have to consider how these quantity demand and supply can affect the market price. Therefore one additional condition will be placed in this analysis;

$$P' = \boldsymbol{\lambda}(Q_d - Q_s) \qquad \text{---(3)}$$

$\boldsymbol{\lambda}$ is non-negative. This equation simply means if quantity demand exceeds quantity supply. In other words, there are more people want to buy than people want to sell. The price tends to go up.

2. Mathematical analysis

We now have three different differential equations and two of them are second-order. So we wish to combine these three equations into one single second-order differential equation. Simply we combine (1) and (2) together and substitute $Q_d - Q_s$ by $P'/\boldsymbol{\lambda}$. Then we obtain;

$$\boldsymbol{\lambda}(\alpha3 + \beta3)P'' + \big(1 - \boldsymbol{\lambda}(\alpha2 + \beta2)\big)P' + \boldsymbol{\lambda}(\alpha1 + \beta1)P = \boldsymbol{\lambda}(\alpha0\text{- }\beta0) \; (3)$$

This yield us a very nice form of second-order differential equation. Note that quantity variable become instrumental. It affects the model through price determination. There are three types of solutions depending on the parameters of the model. Let A be a combination of parameters that determine type of solution.

$$A = 4\lambda^2(\alpha3 + \beta3)(\alpha1 + \beta1) - [1 - \boldsymbol{\lambda}(\alpha2 + \beta2)]2$$

For A > 0, we have overdamped that is,

$$P(t) = k_1e^{c1^*t} + k_2e^{c2^*t} + k_3 \qquad \text{---(4)}$$

k_1 , k_2 are constants,

$$c_1 = (-[1- \boldsymbol{\lambda}(\alpha 2 + \beta 2)] - \sqrt{A}) / 2\boldsymbol{\lambda}(\alpha 3 + \beta 3) \ ,$$
$$c_2 = (-[1- \boldsymbol{\lambda}(\alpha 2 + \beta 2)] + \sqrt{A}) / 2\boldsymbol{\lambda}(\alpha 3 + \beta 3)$$

and

$$k_3 = (\alpha 0- \beta 0) / (\alpha 1+ \beta 1)$$

For A =0, we have critical-damped,

$$P(t) = k_1e^{c^*t} + k_2\, te^{c^*t} + k_3 \qquad \text{---(5)}$$

k_1 , k_2 are constants,

$$c_1 = (-[1- \boldsymbol{\lambda}(\alpha 2 + \beta 2)]) / 2\boldsymbol{\lambda}(\alpha 3 + \beta 3)$$

For A<0, we have underdamped,

$$P(t)= k_1e^{-bt} \cos(wt- n) + k_3 \qquad \text{---(6)}$$

K_1, n are constants,

$$b = (1- \boldsymbol{\lambda}(\alpha 2 + \beta 2)) / 2\boldsymbol{\lambda}(\alpha 3 + \beta 3) \ , \ w = (\sqrt{-A}) / (2\boldsymbol{\lambda}(\alpha 3 + \beta 3))$$

We are particularly interested in the (5) because it describes oscillating movement of the P(t). However, we know that this solution would diminish over time, so it might work in a certain time period.

When looking at (3), it raises the question if the model is responsible for either positive or negative shock due to external force. Because really on the right hand side of the equation (3), that only constant term as a non-homogeneous part is the only one parameter that explains the whole exogenous shocks outside the market. It does not seem right because we know shock would happen and disappear really fast, so it is reasonable to add another term like $\mathcal{E}e^{-r^*t}$ that is responsible for a shock. We then obtain

$$\boldsymbol{\lambda}(\alpha 3 + \beta 3)P'' + \big(1 - \boldsymbol{\lambda}(\alpha 2 + \beta 2)\big)P' + \boldsymbol{\lambda}(\alpha 1 + \beta 1)P = \boldsymbol{\lambda}(\alpha 0- \beta 0) + \mathcal{E}e^{-r^*t}$$

We solve for this and the solution for underdamped case become

$$P(t)= k_1e^{-bt} \mathit{cos}(wt- n) + k_3 + \mathcal{E}e^{-r^*t}$$

Rewrite this, we have

$$P(t)= A+ Ce^{-Bt} \mathit{cos}(\Theta t- \phi) + De^{-G^*t}$$

Intuitively each parameter represents

A : initial value / staring point
B : how fast oscillations diminish over the time
C : magnitude of endogenous shock (amplitudes of the oscillations)
Θ : how fast it oscillates (frequency/period—related to Volatility)
Φ: the phase (determined by initial condition)
D : strength of the exogenous shock
G : how fast exogenous shock decreases

3. Data

We then fit the solution we have into the real data of the stock index. Using log of the daily closing
index of S&P500 since 2000, we then can see what percentage change of index in each day. Note that since we are dealing small time interval, so normalization could be ignored. Then the model will be fitted into three different situation depending on the log price including uptrend, downtrend, and sideways.

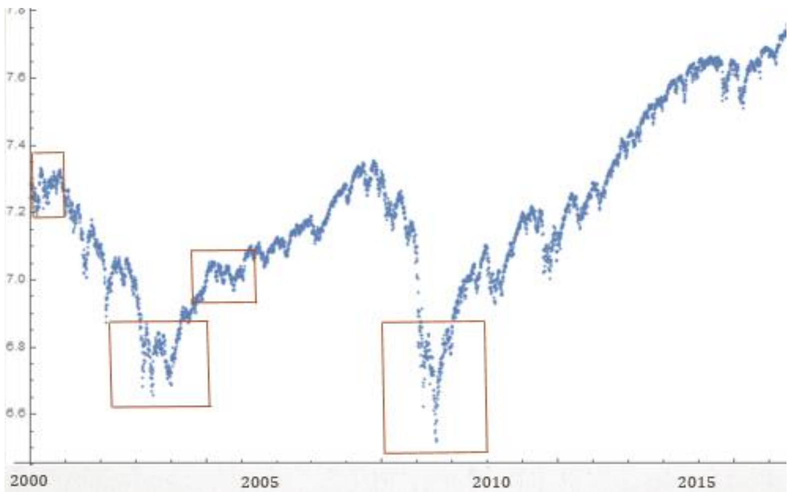


Figure 2 shows the period that we choose to test the model.

There are two of sideways period, two of uptrend, and two of downtrend. For the two sideways, we choose one period is data collected during Jan 2004 and the other one is data collected during March 2001. For the two uptrends, the first one is during November to December 2003. This is about two years after 9/11 attack and one year after 37.8 % decline from the peak. At that point no one can be sure that the market had recovered and resumed again. The second uptrend is during October to November 2009. This is about one year after the first round of quantitative easing (QE1) that aim to help stimulate market to recover from 2008 financial crisis. Lastly there are two downtrends. First, it is during September to October 2002, one year after 9/11 attack. We know that S&P500 hit bottom on October 2002, so I want to see if the model can explain pre-shock movement of the index. Then we have another downtrend which is right before 2008 financial crisis. It is during January 2007.

4. Results and findings

4.1 Sideways analysis

4.1.1 30 data points: January to February 2004

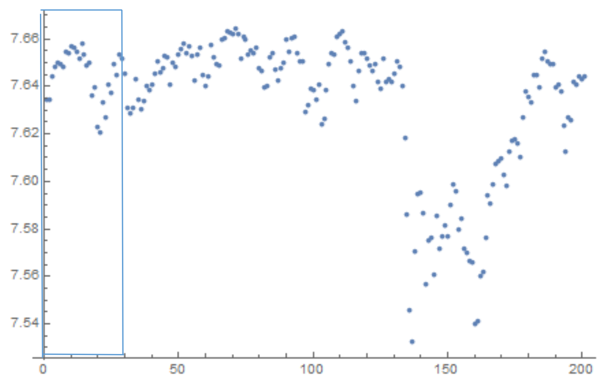


Figure 3 shows the data in January that we fit the model

We can see that model well fits the data for this given period of 30 data points (4.1.1a). We then want to see if it still fits data well for next five days. So we plot 4.1.1b. And 4.1.1c confirms that the actual data follows the plot. But what if this is just a coincident. More precisely is it still performing well if we shift the data. Figure 4.1.1d shows the plot after 5-day shifted and 4.1.1e tries to predict what would happen to the index in another next five days. Figure 4.1.1f ensures this finding as actual data follows.

4.1.2 30 data points: March to April 2001

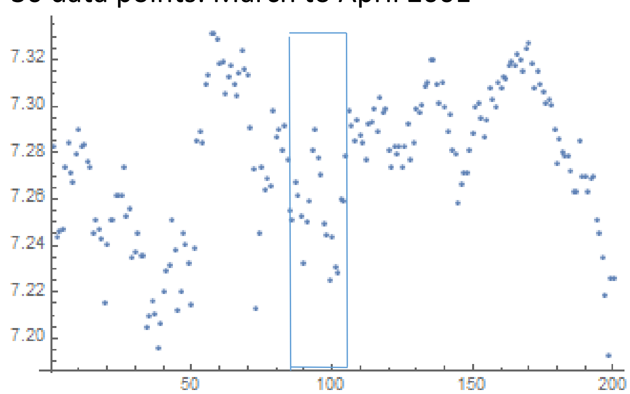


Figure 4, data in the box represents log index in March to April 2001

We performing the similar analysis to 4.1.1. It can be seen that the plot in 4.1.2a shows that actual data follow fitted plot very well. Note that the value of B is -0.0161 suggesting that the oscillating could blow up. We then make adjustment by doing 5-day shifted though 4.1.2b and 4.1.2c shows that the model can well predict data in next five days. The result of shifted plot can be seen in 4.1.2d. The value of B even shows a larger negative value which suggests large variations of movement in the future. The actual data in 4.1.2e and 4.1.2f confirm this prediction.

4.2 Uptrend analysis

4.2.1 30 data points: November to December 2003

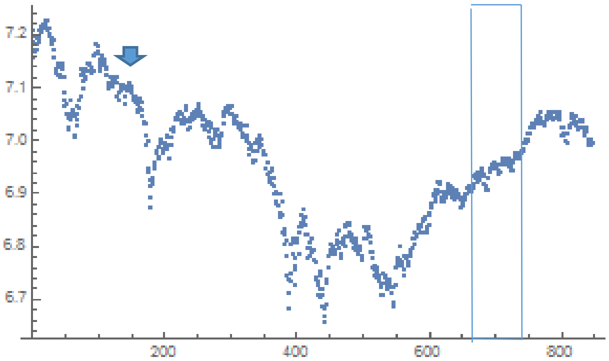


Figure 5, data in the box represent the log index of s&p500 two years after 9/11 incident (where the arrow points down) and it is about one year after S&P500 hit the bottom in October 2002.

The path of the fitted plot shows strongly uptrend, but the value B suggest it would be diminishing over time to some equilibrium log index as showed in 4.2.1a. But for this uptrend, we then question if it is able to predict the movement of the log index in next five days. Diagram 4.2.1b suggests upward movement, but the actual data presented in 4.2.1c dos not really follow the trend. This comes to common sense that we would have to change strategy here. By doing 5-day shifted plot, this would allow us take 5 additional data points into account as showed in 4.2.1d. But how well this adjusted plot can predict in another next five days. Diagram 4.2.1e shows the evolution of next five-day log index which is matching to the actual data showed in 4.2.1f.

4.2.2 30 data points: October to November 2009

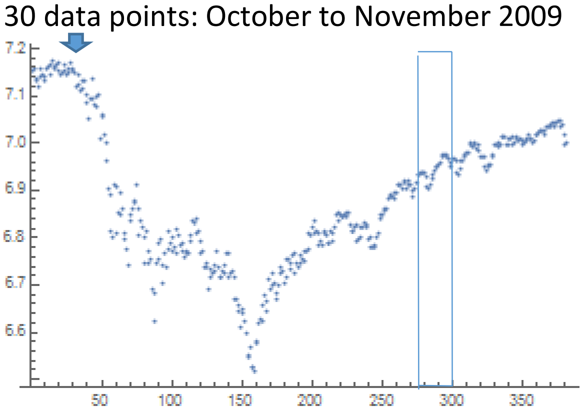


Figure 6, in the box, data in October to November 2009, is used to analyze the index movement. We decide to choose this period because it is about the end of the first round of quantitative easing (QE1) as the Fed would purchase$600 billion in mortgage-backed securities and debt. The point, where arrow points down, is the when the 2008 crisis happened to be recognized.

When looking at fitted plot in diagram 4.2.2a, it can well explain the data in the first 30 data points. However, now the large negative value of G is dominating the oscillating behavior, so as we see in 4.2.2b, the plot shows no convergence of the pattern and it blows up, while the actual data presented in 4.2.2c do not really follow the trend. We then adjust the model by doing 5-day shifted plot. The new plot is showed in 4.2.2d. Now G becomes very small, so we can ignore its value, but B significantly shows convergence sign because its value is positive. Diagram 4.2.2e shows the predictive direction of another next five-day movement of the 5-day shifted plot. Diagram 4.2.2f confirms that actual data follows this new shifted plot.

4.3 Downtrend analysis

4.3.1 30 data points: Post 9/11 incident—September to October 2002

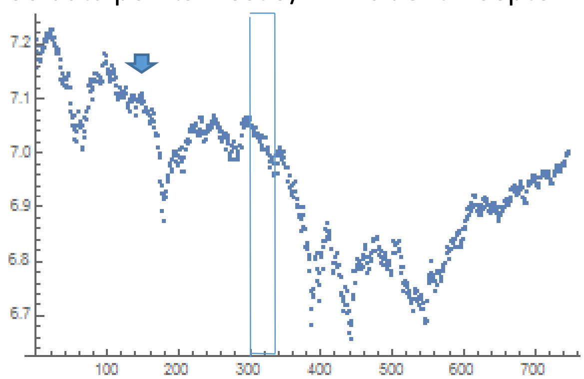


Figure 7, the data in the box is chosen to be analyzed. They are exactly one year after 9/11 attack. At this point, nobody knows that S&P500 would hit the lowest point in decade in three months later.

The fitted plot in 4.3.1a shows nice and smooth downtrend, but notice that value of G as an exogenous shock is not significant here. Then 4.3.1b suggest that the log index would go down. 4.3.1c confirms that prediction as actual data follow the trend. However we do not want to take an account of the first five days, so we do shifted plot by 5 days that can be seen in 4.3.1d. Now the value of B become negative which suggests that the plot would diverge in long run. Moreover, the value of G is now significant as e can see a big drop in log index. 4.3.1e shows the prediction of new shifted plot in next five days. And 4.3.1f confirms that actual data follows the predictive plot.

4.3.2 25 data points: Post 2008 crisis ---January to February 2009

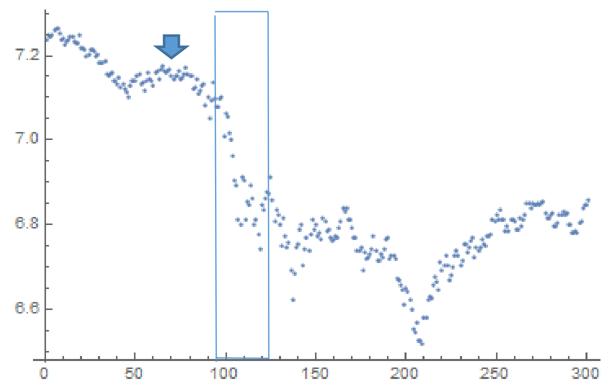


Figure 8, the data in the box presents early post crisis. It is only about 3 months after (the crisis starts where the arrow points down). We can see the huge drop in this time interval.

The fitted plot in 4.3.2a shows a big drop of log index as the value of B and G are positive 0.0607 and 0.1185 respectively. Then we want to see if this current information can predict the short-run movement in next five days. Diagram 4.3.2b shows both convergence and downtrend movement. However in 4.3.2c presents the actual data that quite follows the downtrend, but does not seem to be converged. We then do 5-day shifted plot as usual. The result can be seen in 4.3.2d in which this new plot yields negative value of B while G stays positive. This suggests variation of oscillating behavior and the predictive plot will blow up over time. However 4.3.2e and 4.3.2f is sufficient to validate the predictive model as the actual data follows the trend.

5. Limitation

5.1 From several experiments, we found that only short term time interval (about 25 to 35 data points) the predictive model can well explain and predict the data. When exceeding 35 data points, the model breaks down and become uninterpretable.

5.2 Our assumptions are somehow too weak. (1), (2), and (3) are assumption based on the justification that traders are rational. In other words, they would immediately sell if price of the stock goes up and buy if it goes down. In fact, we know that people do not behave rationally in the stock market. There are speculators who may risk their profit. And surely no individual would respond to the market the same way.

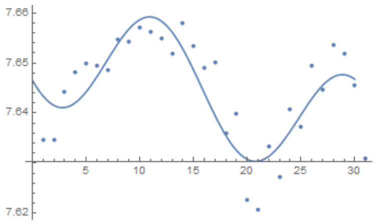
5.3 Our model cannot explain the movement in every situation. Data need to be bounded. Specifically, straight-up/down would cause of model's failure. For example in 4.2.1 and 4.2.2, the uptrends, the model fail to predict next five days.

6. Conclusion

This work shows how the stock market may be modeled as damped harmonic oscillators with arbitrary shock terms. In fact, it does a great job predicting the market movement during the short term period, but model is somehow too simple to be realistically used in some case—an uptrend for example. Sometimes the model fails to fit data due to lack of market movement. But afterall, we can use this along with other stochastic indicators in real trading. I would consider this model successive in a certain level because it takes an account of how volume could affect the stock market index.

7. Reference

Frisch, Ragnar (1933). "Propagation problems and impulse problems in dynamic economics". Economic Essays in Honour of Gustav Cassel: 171–205.



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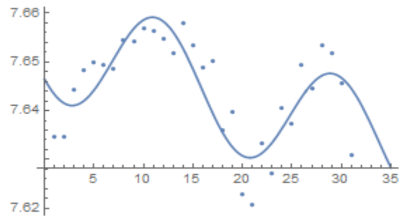

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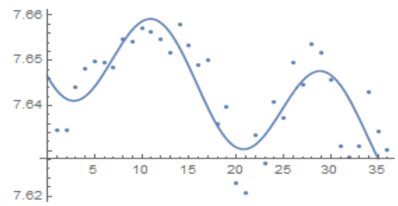

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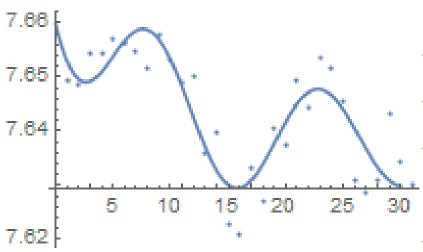

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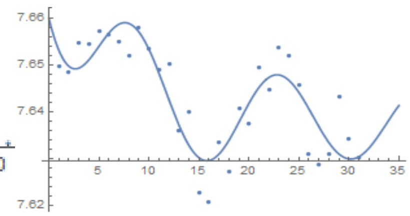

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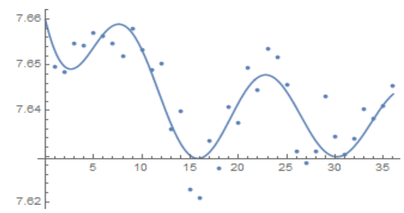

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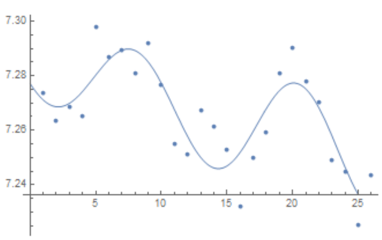

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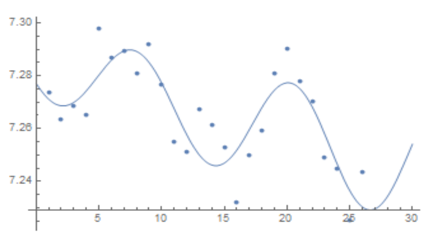

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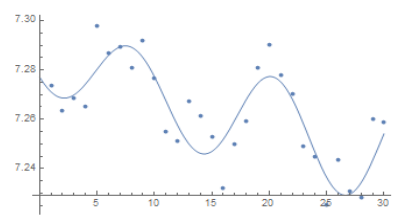

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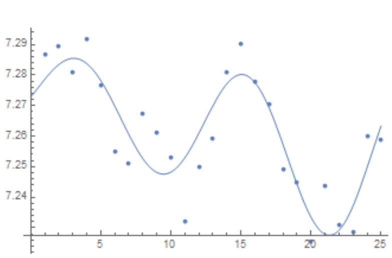

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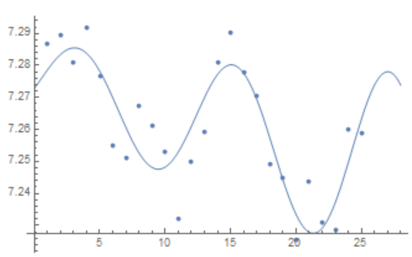

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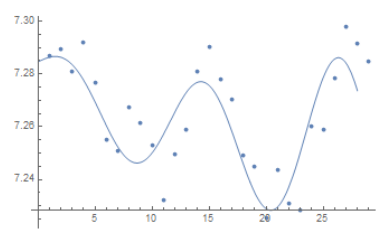

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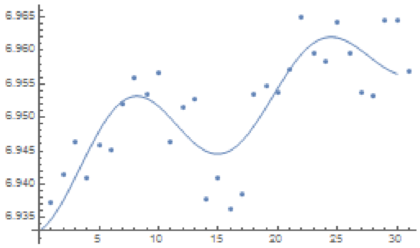

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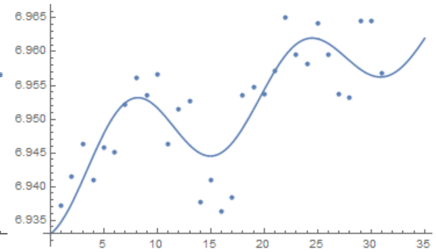

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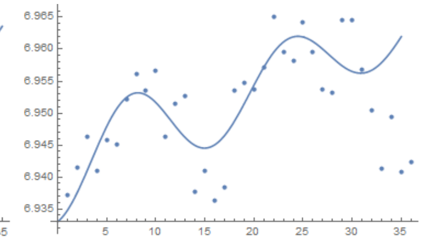

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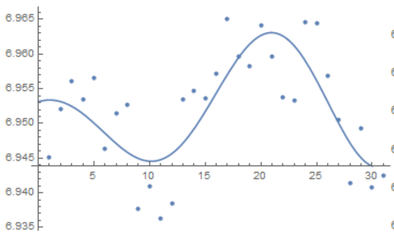

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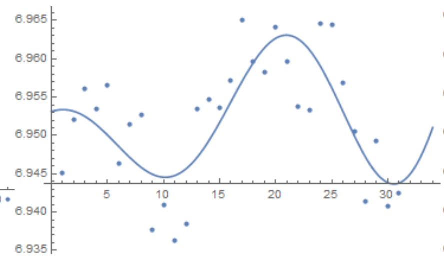

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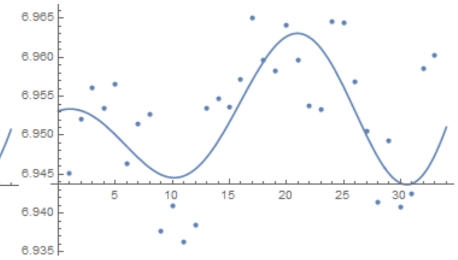

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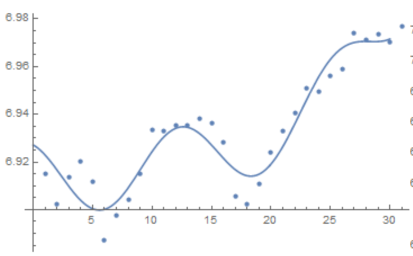

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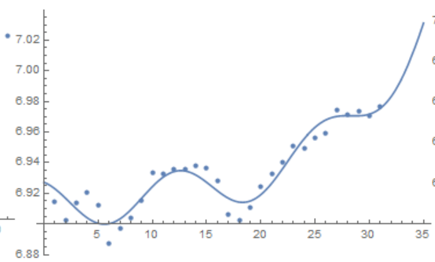

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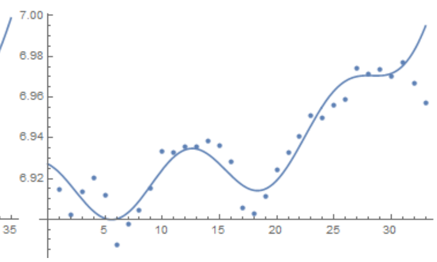

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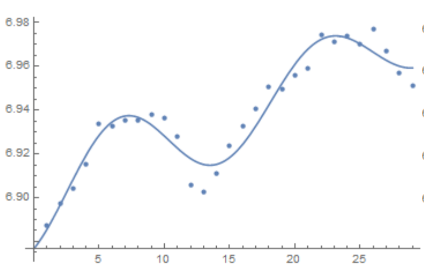

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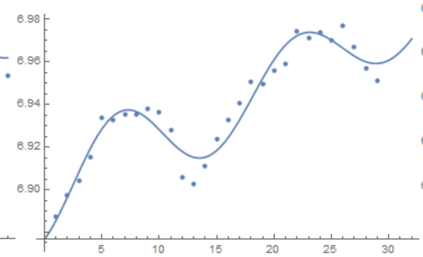

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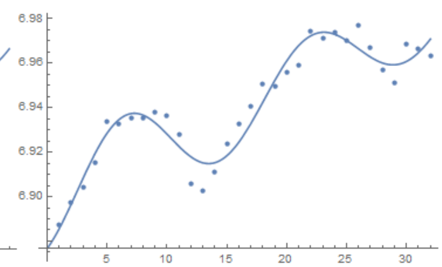

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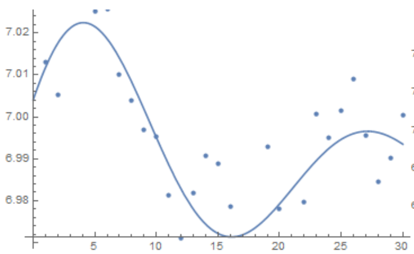

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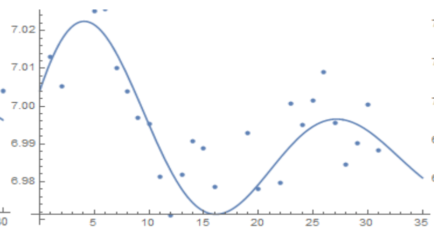

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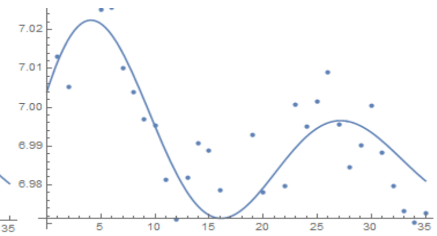

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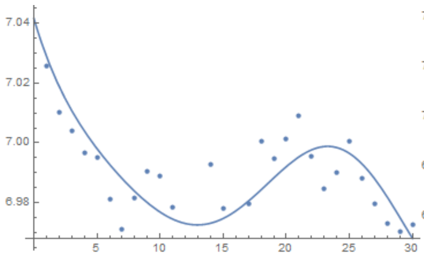

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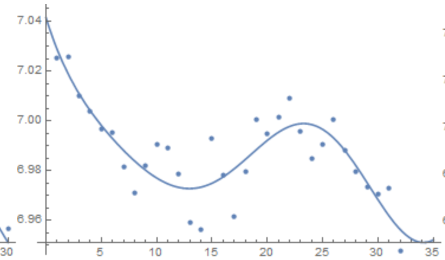

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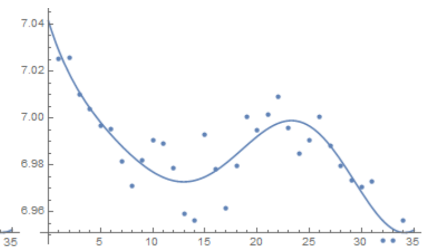

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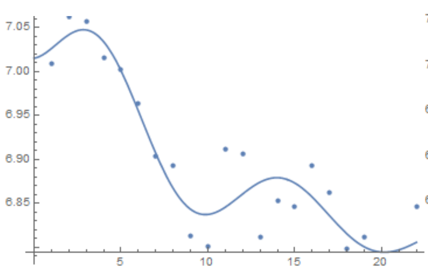

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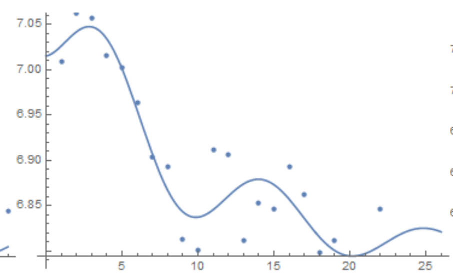

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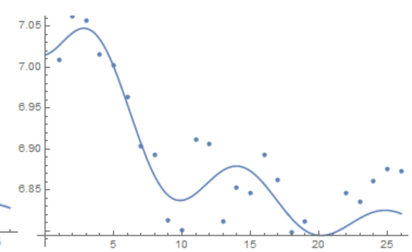

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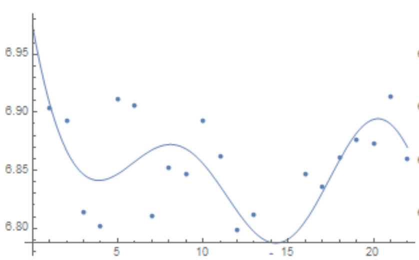

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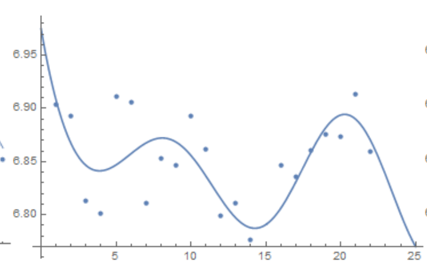

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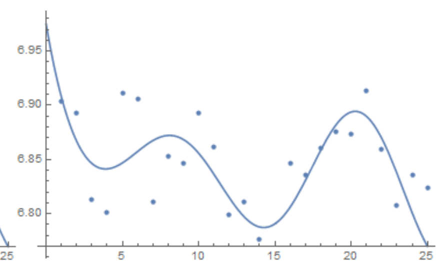

(4.3.2f)