DYNAMIC OF THE STOCK INDEX: THE RELATION TO LINEAR SPRING EQUATION

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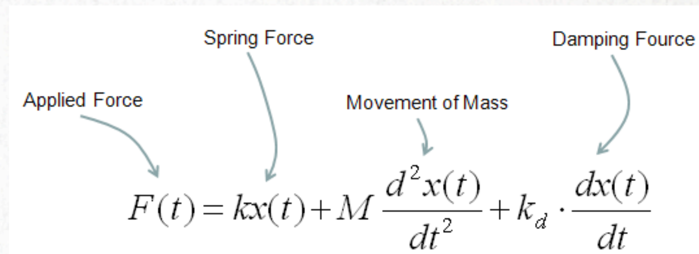
PY538

OUTLINE

- Introduction and motivation
- Goals
- Data collecting
- Mathematical method
- Findings and results
- Limitations
- Conclusion

INTRODUCTION AND MOTIVATION



$$F(t) = kx(t) + M\frac{d^2x(t)}{dt^2} + k_d \cdot \frac{dx(t)}{dt}$$

RELATED PAPER

- *Propagation problems and impulse problems in dynamic economics*, 1933, Ragnar Frisch
  - "We may perhaps start by throwing all kinds of **production** into one variable, all **consumption** into another, and so on, imagining that the notions 'production', 'consumption', and so on, can be measured by some sort of **total indices**. "

GOALS

- Borrow Frisch's idea and reconstruct a new simpler model
- That well predicts the movement (**momentum**) of the stock index
- Find the appropriate **time frame** for the oscillating equation
- Construct the trading strategies

DATA COLLECTING

Calculating **log return** of the daily closing index of S&P500 since 2000

Note that in this case we only consider small time interval, so I think **normalization** could be ignored.

Then categorize data into three different groups depending on the movement:

1. Sideways movement
2. Bullish (Up) trend
3. Bearish (Down) trend

BUT HOW WOULD WE IDENTIFY THE TREND ON THE GIVEN DATA PERIOD

- Momentum indicator called "ROC (Rate of Change)." In fact it is equivalent to drawing support/resistence of the trend.

- Rate of Change = Y/Yx
  Y represents the most recent closing value of index
  Yx represents the closing value of index a specific number of x days ago

- Notice: we have a choice here which is x. In general for short-term period, x could be 10 days up to 6 months

- After several experiments, I found that 25-30 data point is the most efficient for the predicting model

MATHEMATICAL METHOD

STARTING WITH DEMAND-SUPPLY EQUATION

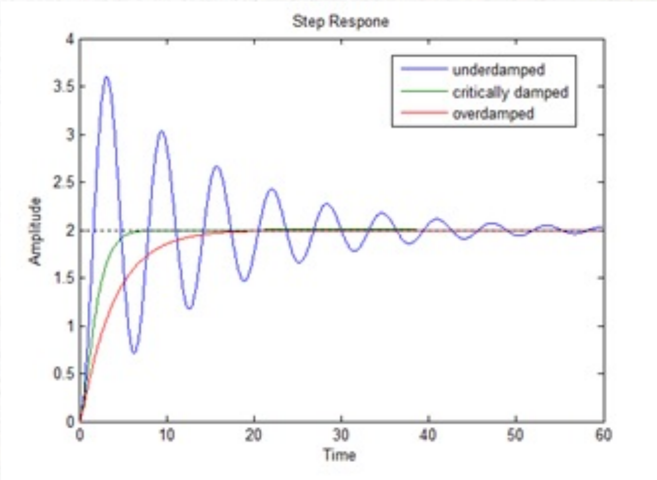
$$\begin{cases} Q_d = \alpha_0 - \alpha_1 P + \alpha_2 P' - \alpha_3 P'' & \text{where } \alpha_0, \alpha_1, \alpha_2, \alpha_3 \geq 0 \\ Q_s = \beta_0 + \beta_1 P - \beta_2 P' + \beta_3 P'' & \text{where } \beta_0, \beta_1, \beta_2, \beta_3 \geq 0 \\ P' = \boldsymbol{\lambda}(Qd - Qs) & \text{where } \boldsymbol{\lambda} \geq 0 \end{cases}$$

Then we derive a second-order differential equation,

$$\boldsymbol{\lambda}(\alpha 3 - \beta 3) P\uparrow'' + (1 - \boldsymbol{\lambda}(\alpha 2 - \beta 2)) P\uparrow' + \boldsymbol{\lambda}(\alpha 1 - \beta 1) P = \boldsymbol{\lambda}(\alpha 0 - \beta 0)$$

So there will be three solutions equation, depending on the parameters of the model

1. $P(t)=c_1 e^{r_1 t}+c_2 e^{r_2 t}+k$
2. $P(t)=c_1 e^{rt}+c_2 e^{rt}+k$
3. $P(t)= c_1 e^{-bt} \cos(\theta t- \phi) + k$



$$\boldsymbol{\lambda}(\alpha 3-\beta 3)P\upharpoonleft'' + (1-\boldsymbol{\lambda}(\alpha 2-\beta 2))p\upharpoonleft' + \boldsymbol{\lambda}(\alpha 1-\beta 1)p = \boldsymbol{\lambda}(\alpha 0 - \beta 0)$$

This homogeneous part describes only **endogenous effect** that is due to nature of the markets

This non-homogeneous part describes **exogenous effects** like shock outside the market

So I think it would make more sense if we add another term which describe exogenous shock (positive, negative) into the system:

$$\boldsymbol{\lambda}(\alpha 3-\beta 3)P''+(1-\boldsymbol{\lambda}(\alpha 2-\beta 2))p'+\boldsymbol{\lambda}(\alpha 1-\beta 1)p=\boldsymbol{\lambda}(\alpha 0-\beta_0)+de^{-at}$$

Then the solution becomes:

$$P(t)= c_1e^{-bt}\cos(\theta t-\phi)+k+de^{-at}$$

Rewrite,

$$P(t)= A+Ce^{-Bt}\cos(\theta t-\phi)+De^{-gt}$$

$$P(t) = A + Ce^{-Bt}\cos(\theta t - \phi) + De^{-Gt}$$

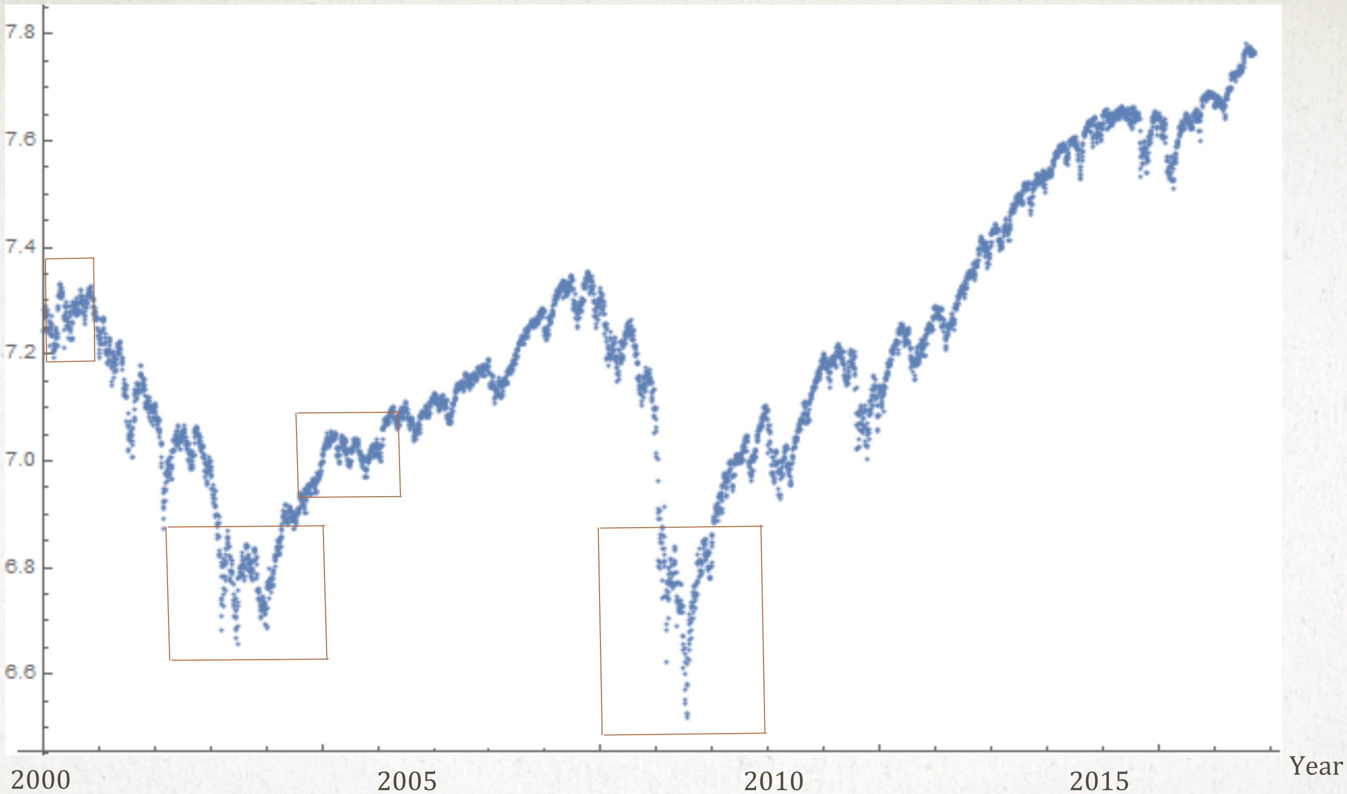
A : initial value of the S&P index at staring point
B : how fast oscillations diminish over the time
C : magnitude of endogenous shock (amplitudes of the oscillations)
θ : how fast it oscillates (frequency/period—related to Volatility)
Φ: the phase (determined by initial condition)
D : strength of the exogenous shock
G : how fast exogenous shock decreases

RESULTS AND FINDINGS

SOME REMARKS BEFORE WE BEGIN…

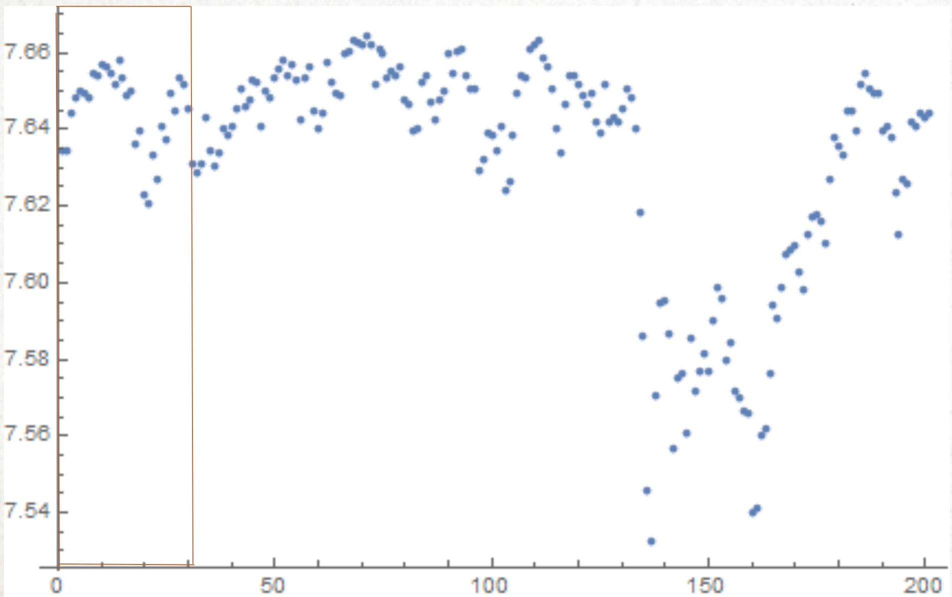
1. What does current information tell us?
   - Significance of the parameters (B, G, Θ, ..)

2. What will happen to the index in some certain (short-term) period in the future using the predictive model?
   - Does the actual data follow the dynamic predictive model?
   - Divergence sign? If so, how do we do?


7.8
7.6
7.4
7.2
7.0
6.8
6.6
2000
2005
2010
2015
Year

SIDEWAYS ANALYSIS #1

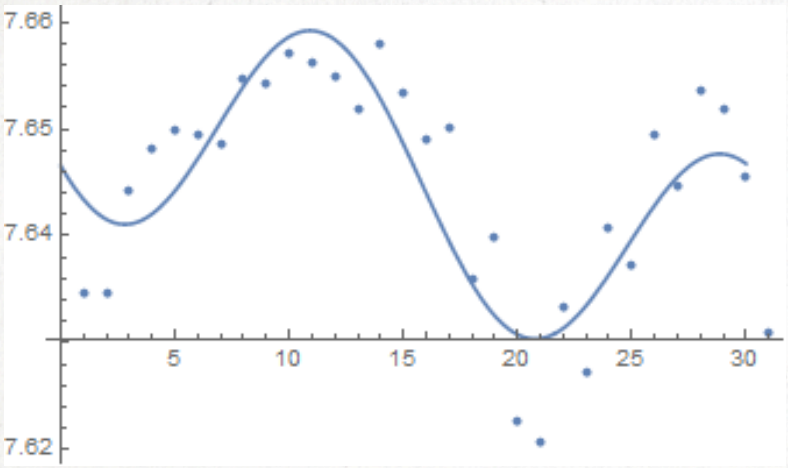
Return



Data : Jan 2004
Length : 30 data points

SIDEWAYS ANALYSIS #1

Return



A	-14224.1936
B	0.002007919
C	0.011864206
Θ	0.34997182
ϕ	3.97604607
D	14231.84815
G	4.35E-08

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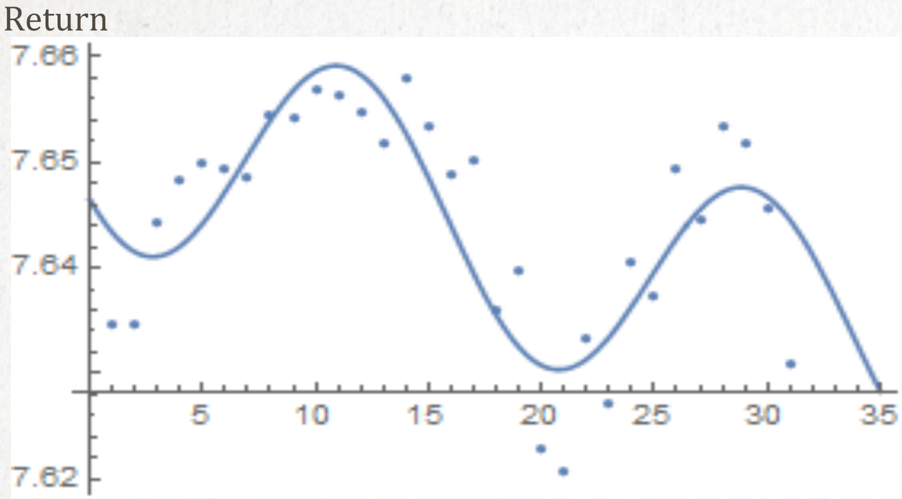

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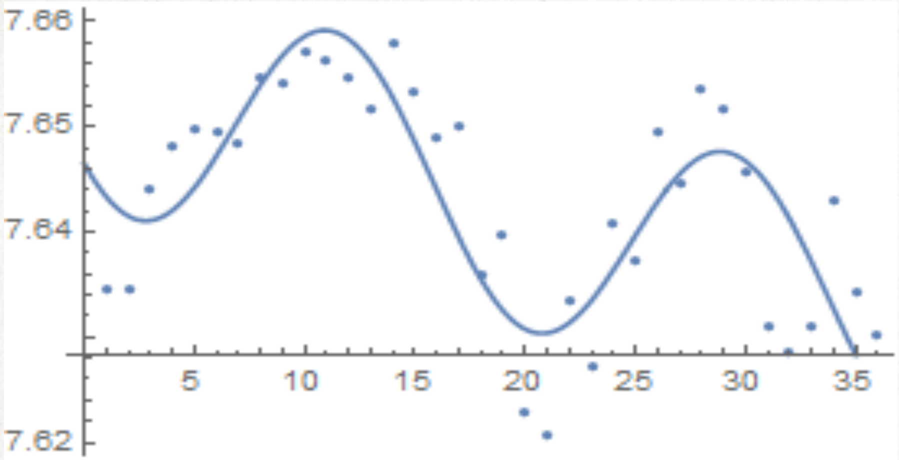

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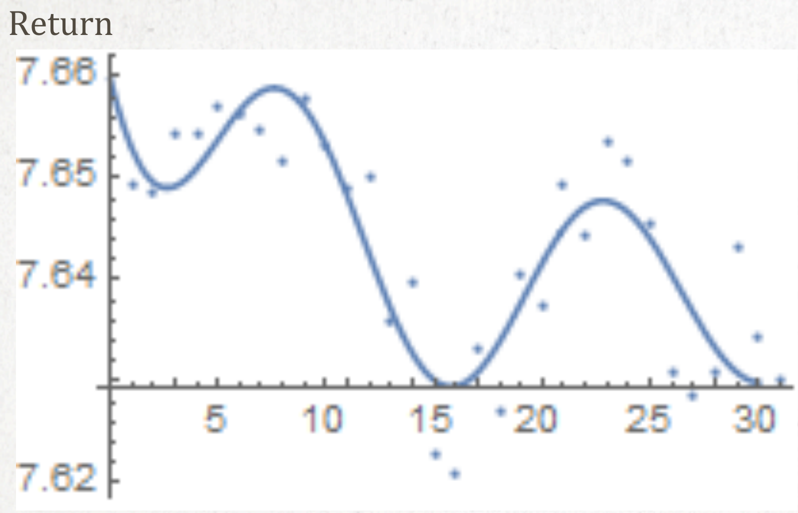

A	7.6378144
B	0.018685058
C	0.014291727
θ	0.431275046
ϕ	3.625591189
D	0.034664601
G	0.170811157

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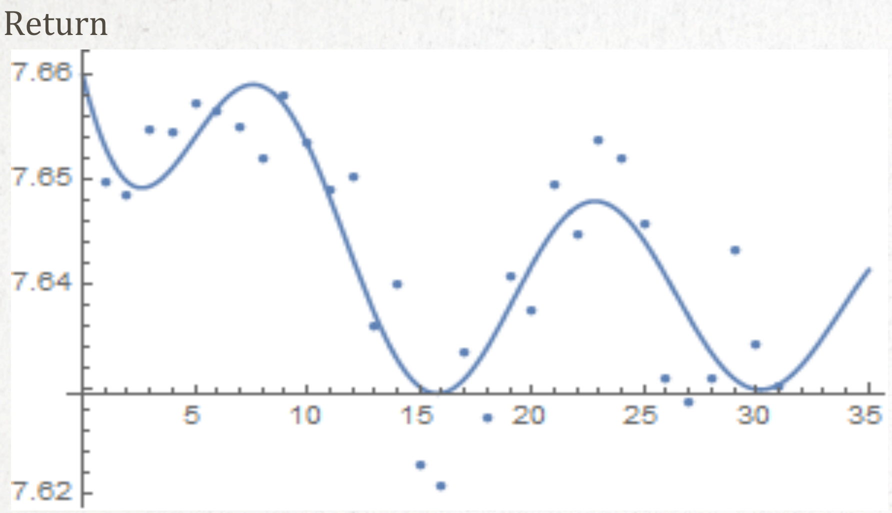

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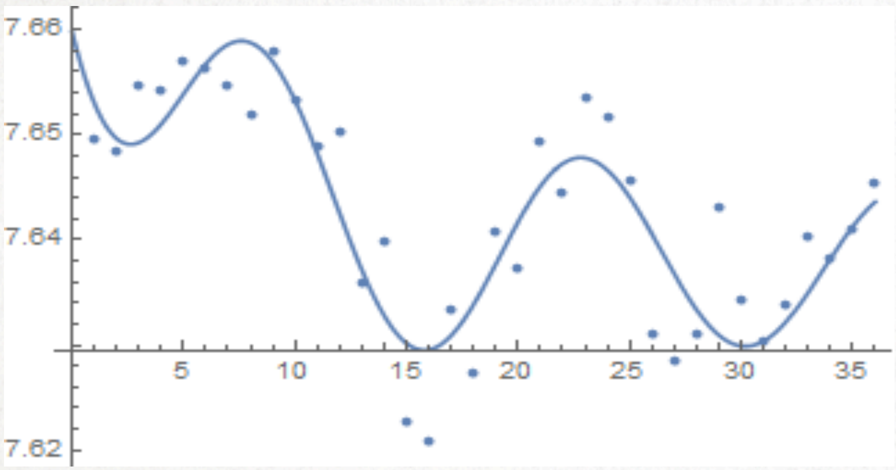

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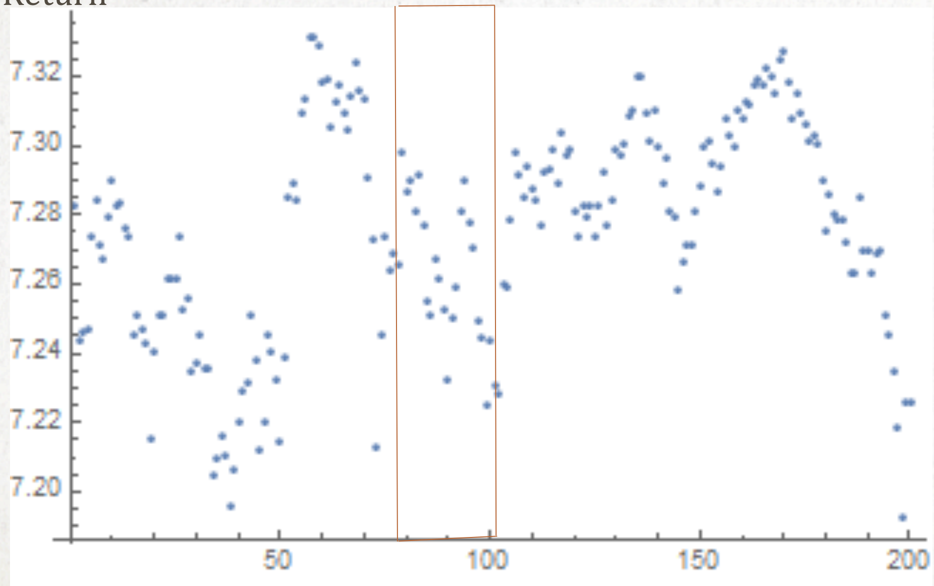
SIDEWAYS ANALYSIS #2

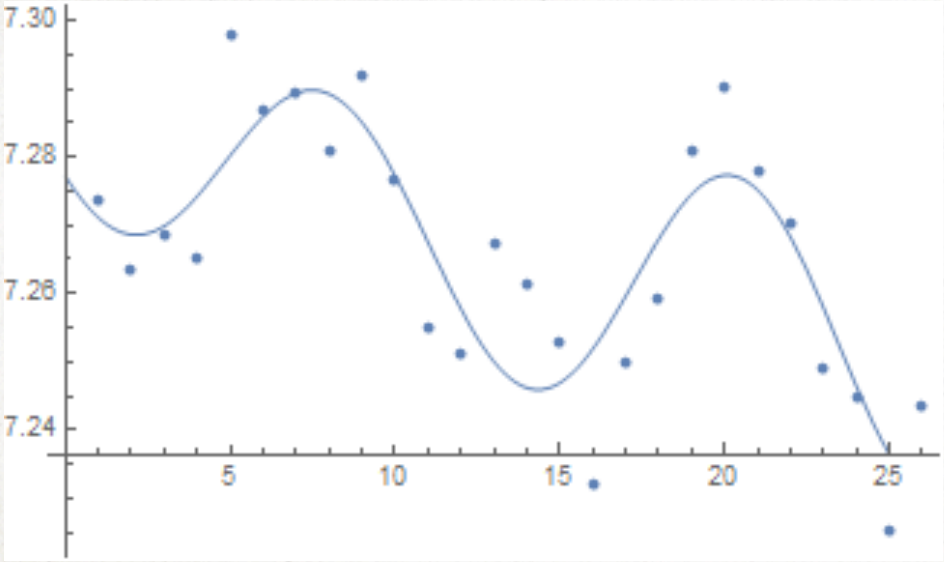
Return



Data : March 2001
Length : 25 data points

SIDEWAYS ANALYSIS #2

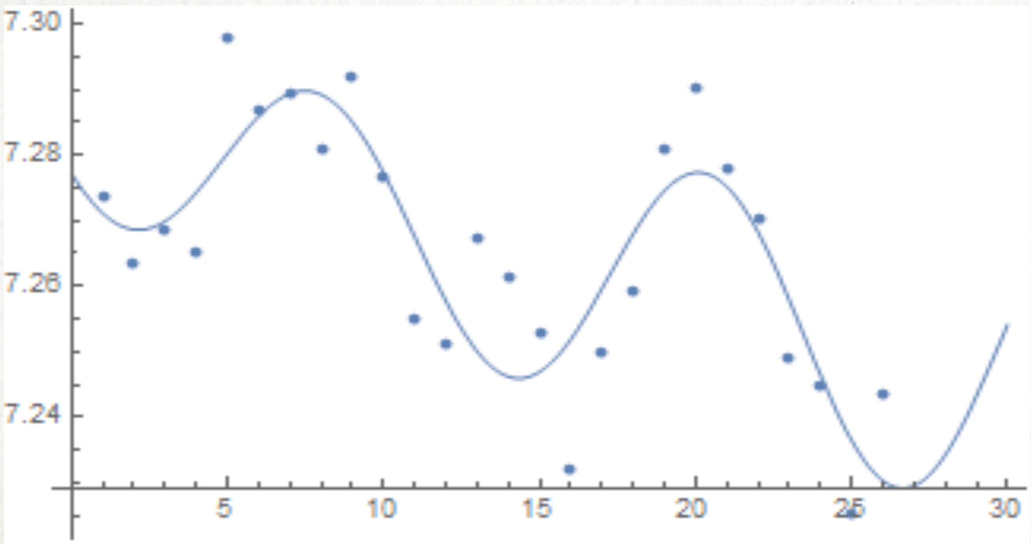
Return



A	7.2273
B	-0.0161
C	0.014476
θ	-0.5058
ϕ	-10.227
D	0.059911
G	0.034365

SIDEWAYS ANALYSIS #2

Return



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C	0.014476
Θ	-0.5058
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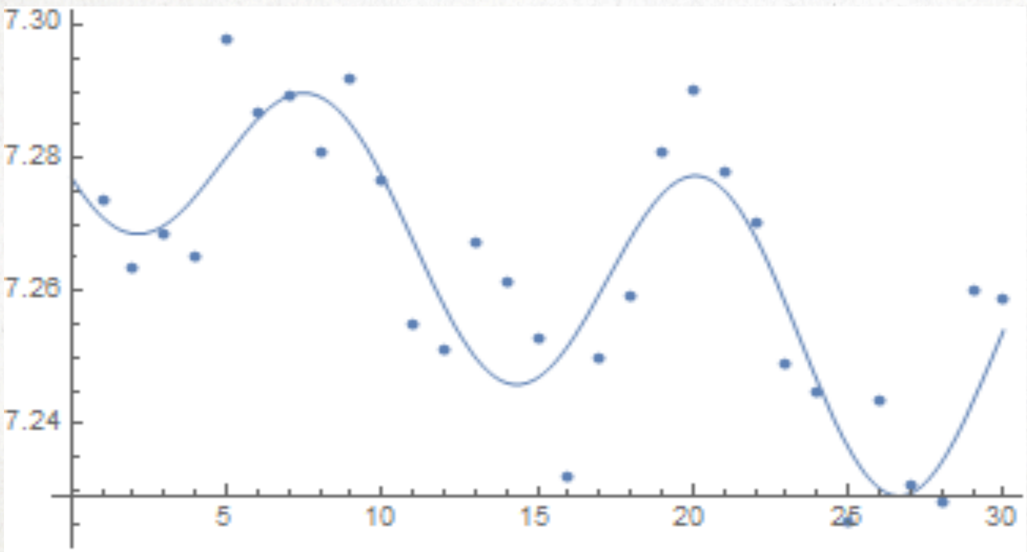

A	7.2273
B	-0.0161
C	0.014476
Θ	-0.5058
ϕ	-10.227
D	0.059911
G	0.034365

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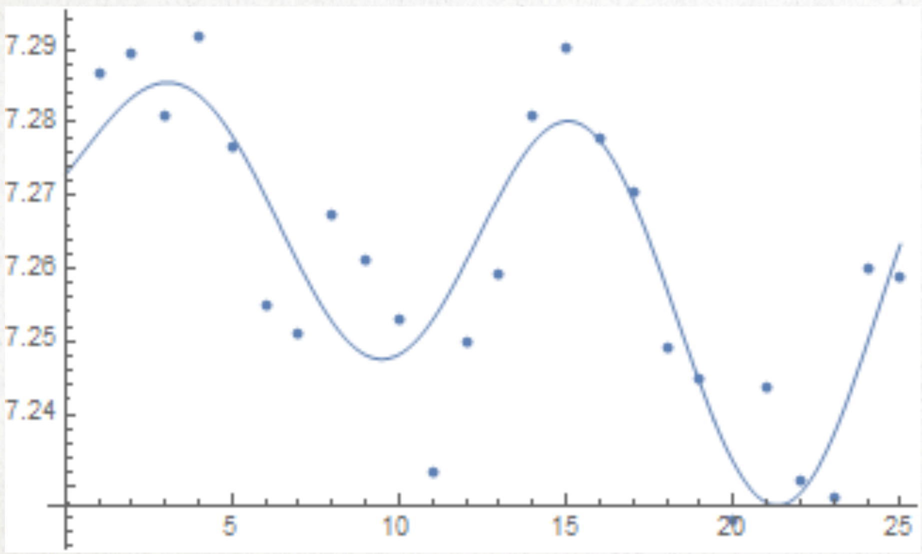
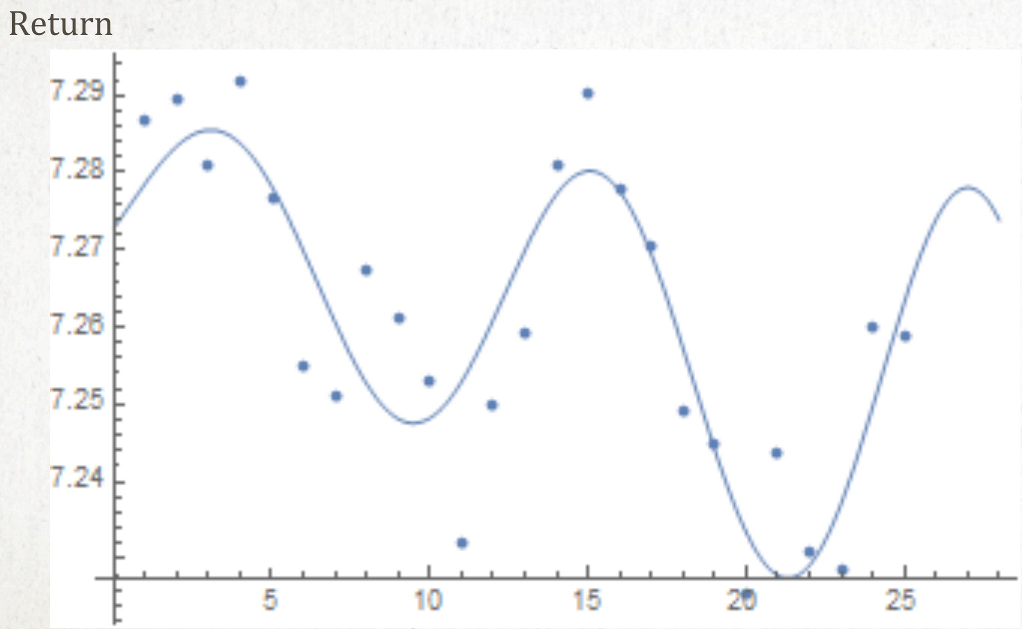


A	12701.5
B	-0.032208
C	0.0129133
θ	0.527938
ϕ	1.69233
D	-12694.2
G	-7.91811E-08

SIDE-WAY ANALYSIS #2



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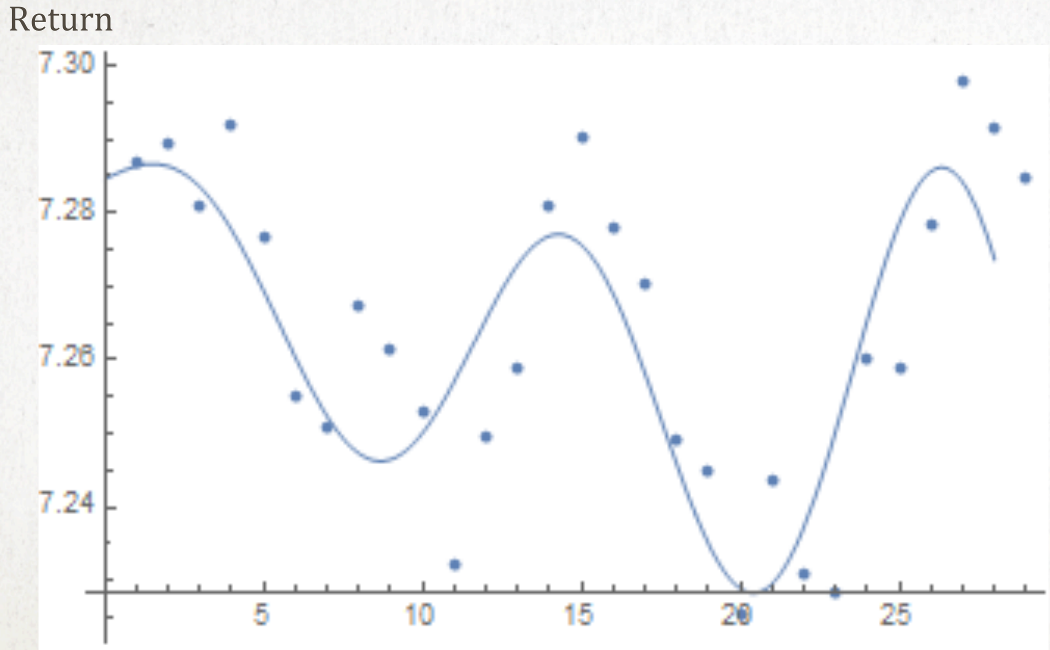

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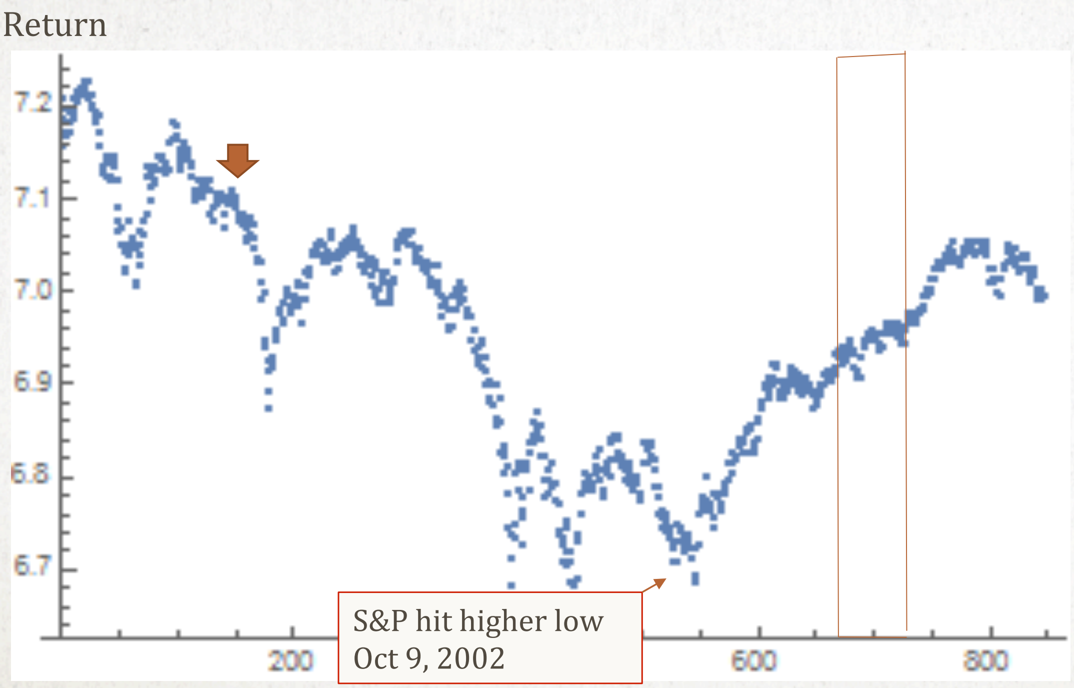
UPTREND #1 (2 YEAR AFTER 9/11 ATTACK)

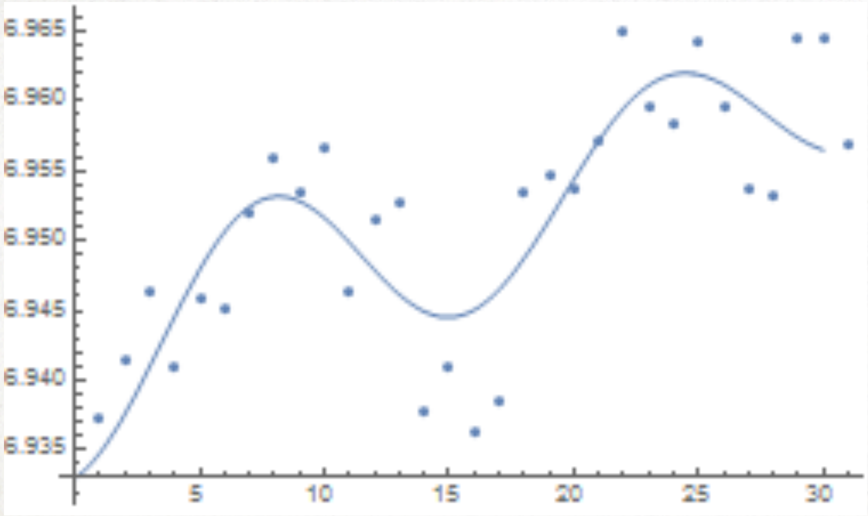


Data : Late 2003 –around November-December

Length : 30 data points

UPTREND #1 (2 YEAR AFTER 9/11 ATTACK)

Return



A	14044.2
B	0.0157616
C	0.00805801
ϴ	-0.39002
ϕ	-2.99482
D	-14037.3
G	4.59E-08

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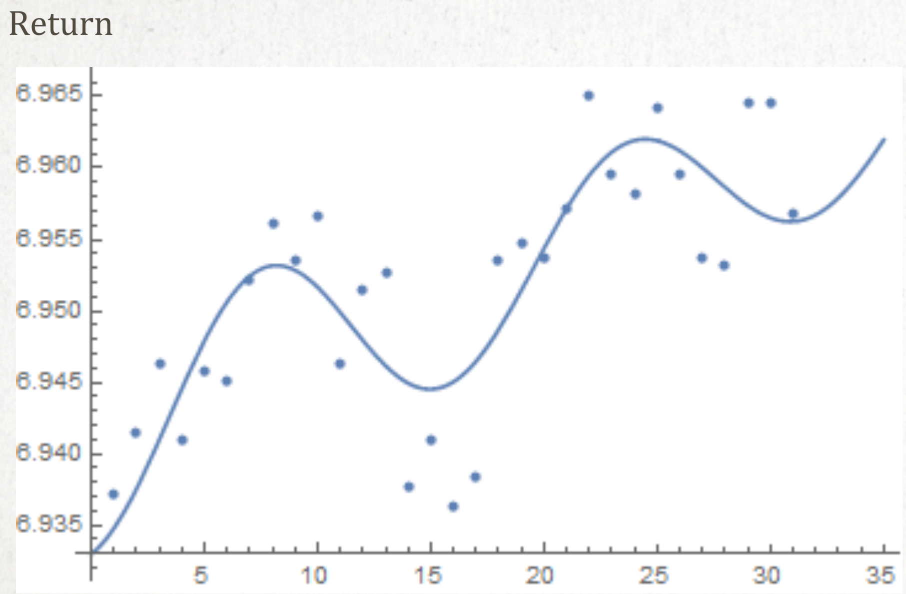

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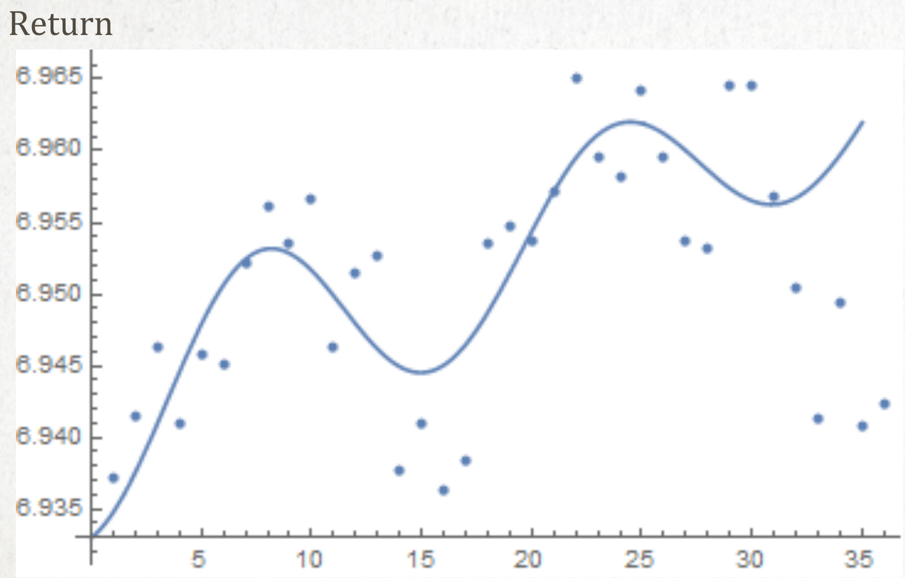

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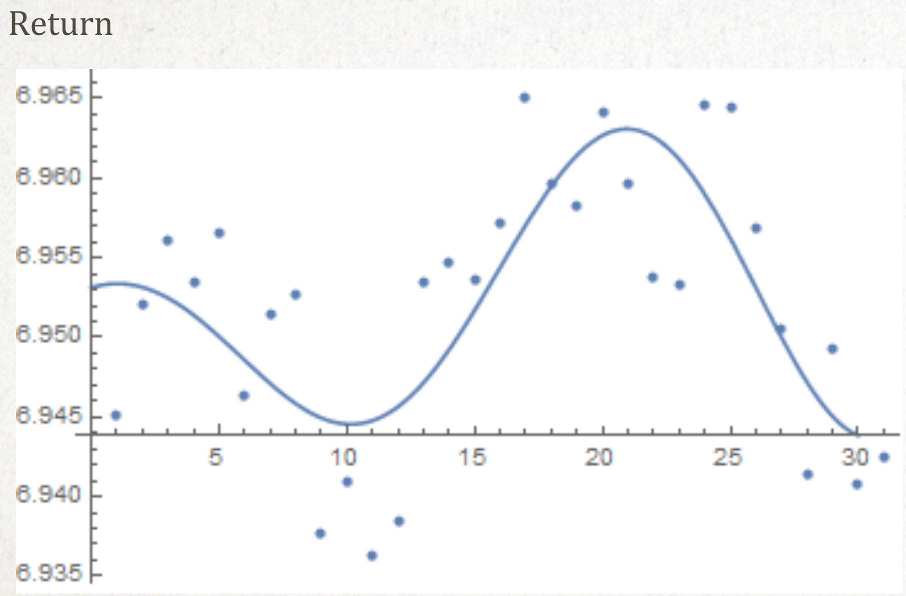

A	-2718.66
B	-0.0328128
C	0.00469519
θ	-0.311212
ϕ	-0.0419518
D	2725.61
G	-9.62505E-08

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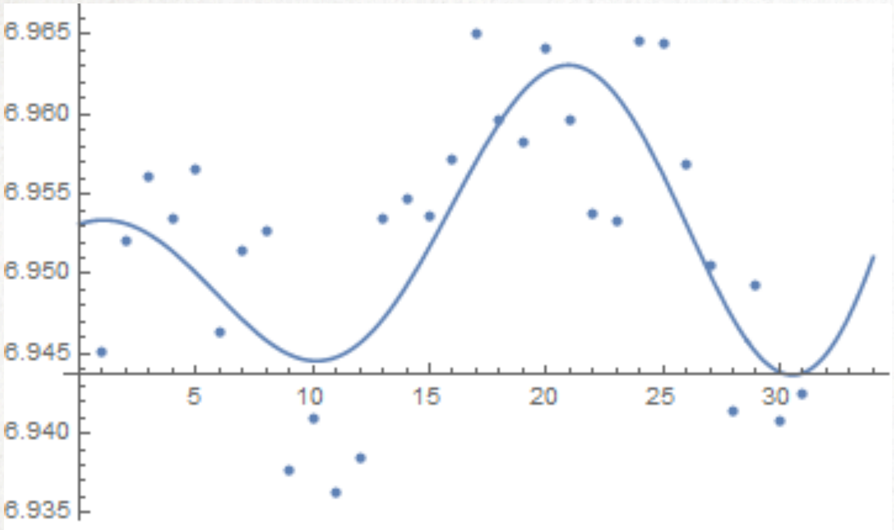

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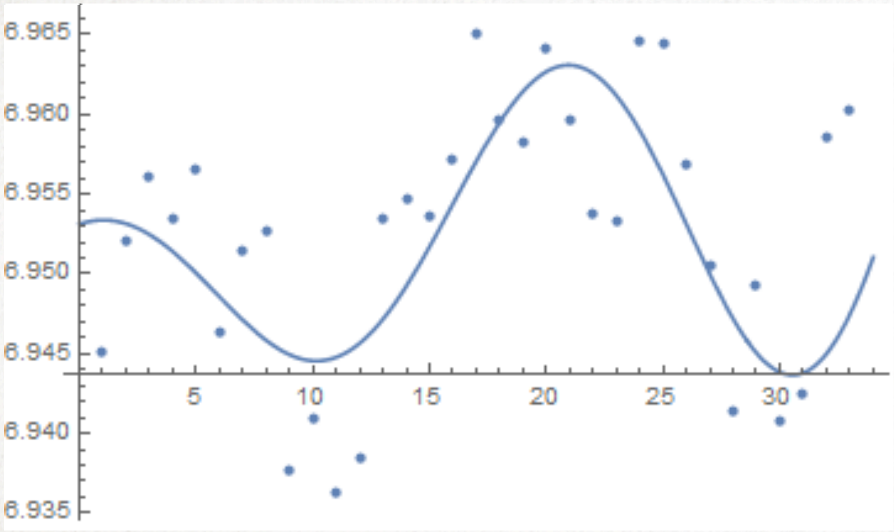

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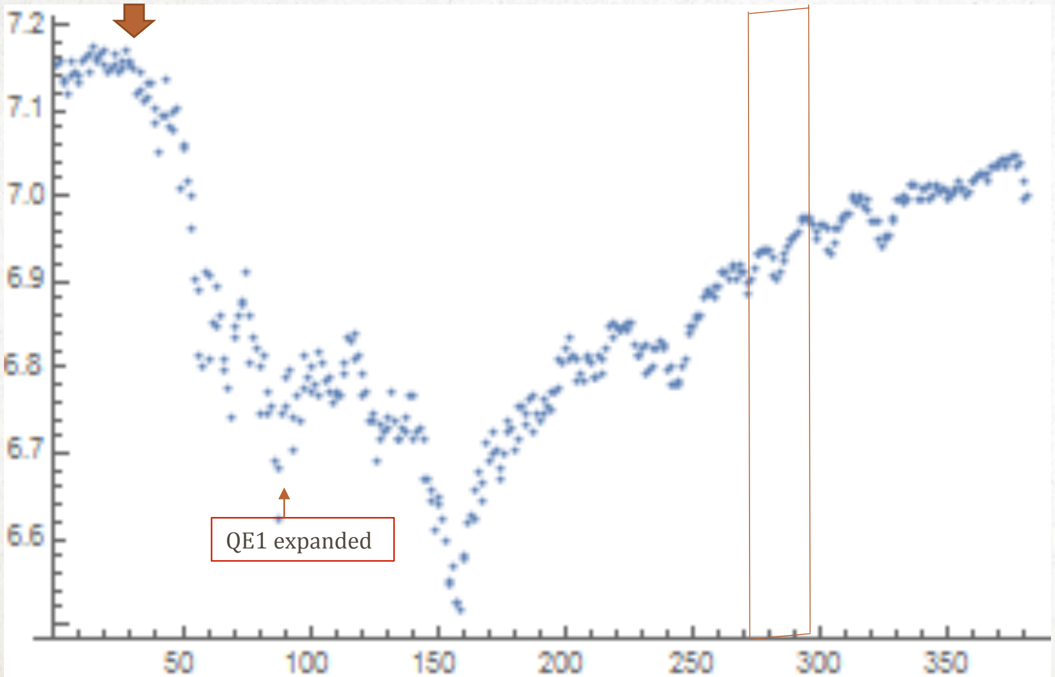
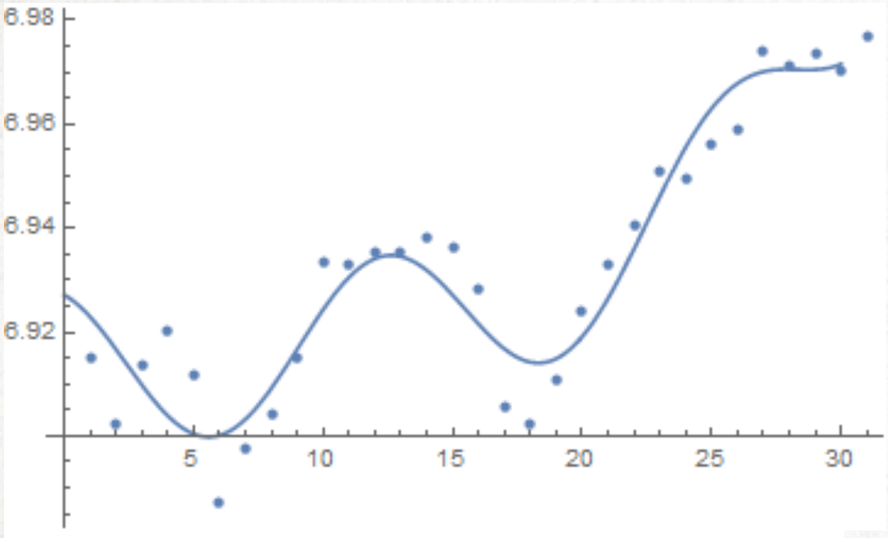
UPTREND#2 (A YEAR AFTER THE SEP-2008 CRISIS)



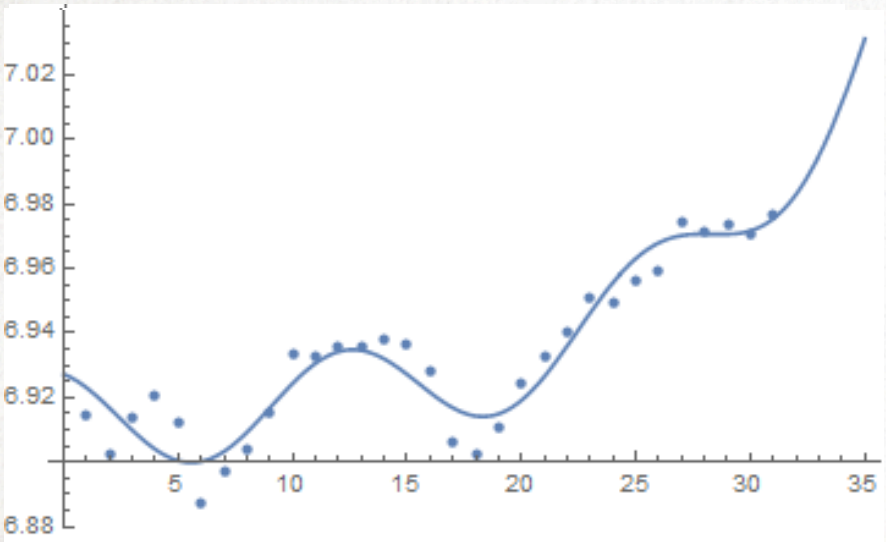
Data : Oct-Nov 2009
Length : 30 data points

UPTREND#2 (A YEAR AFTER THE SEP-2008 CRISIS)



A	6.9112
B	0.001805
C	-0.01535
θ	-0.47558
ϕ	-2.71952
D	0.001958
G	-0.11829

UPTREND#2 (A YEAR AFTER THE SEP-2008 CRISIS)



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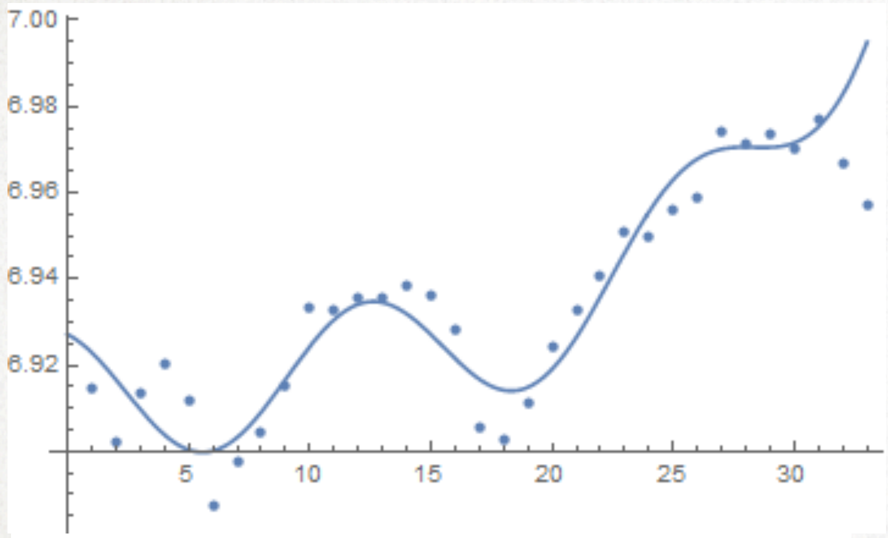

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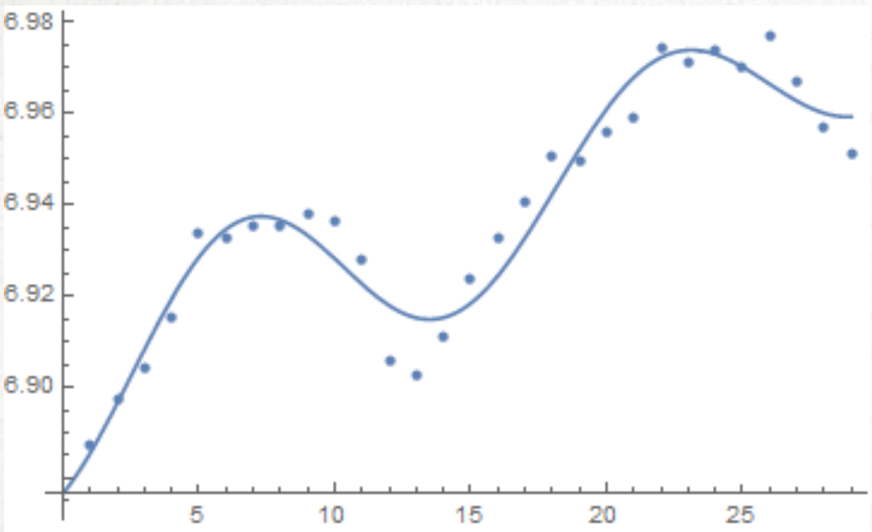
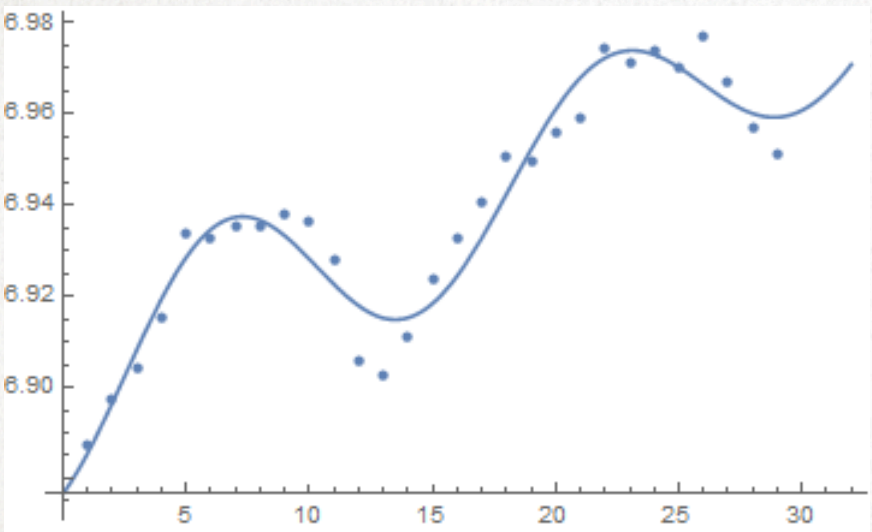


A	-19184.4
B	0.0149507
C	-0.0238661
θ	-0.402389
ϕ	0.481795
D	19191.3
G	-1.35835E-07

UPTREND#2 (A YEAR AFTER THE SEP-2008 CRISIS)



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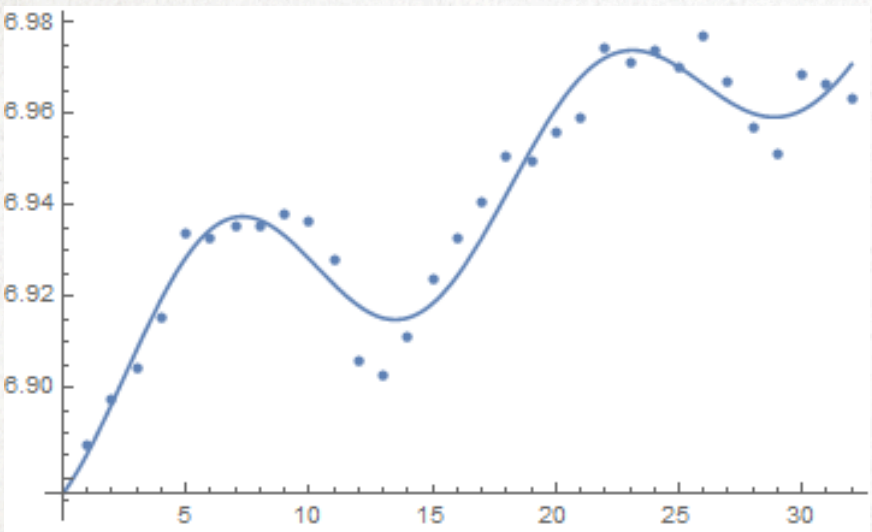

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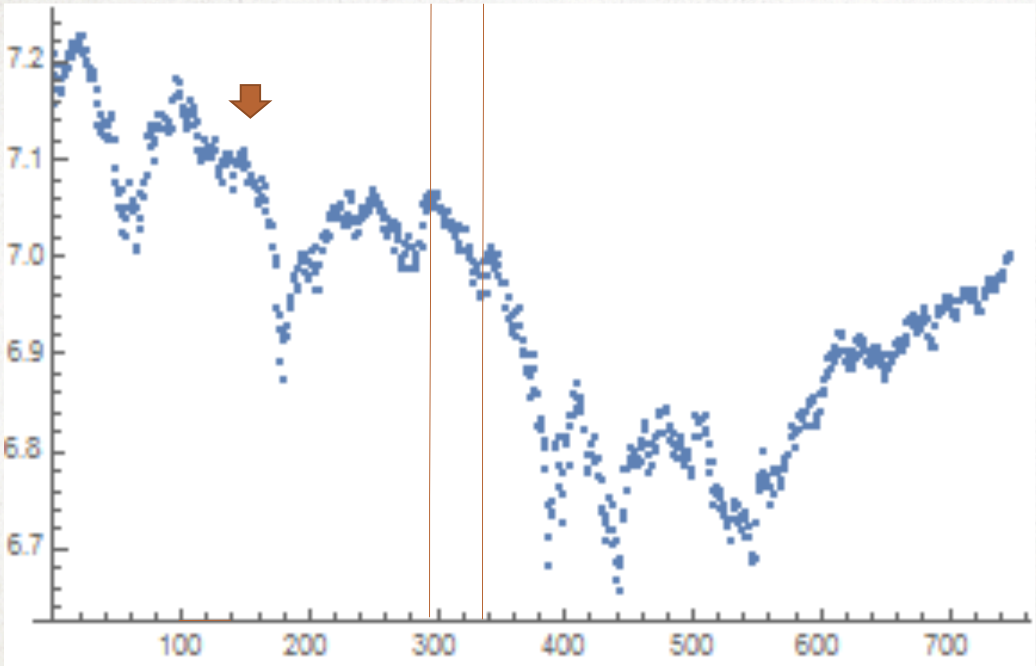
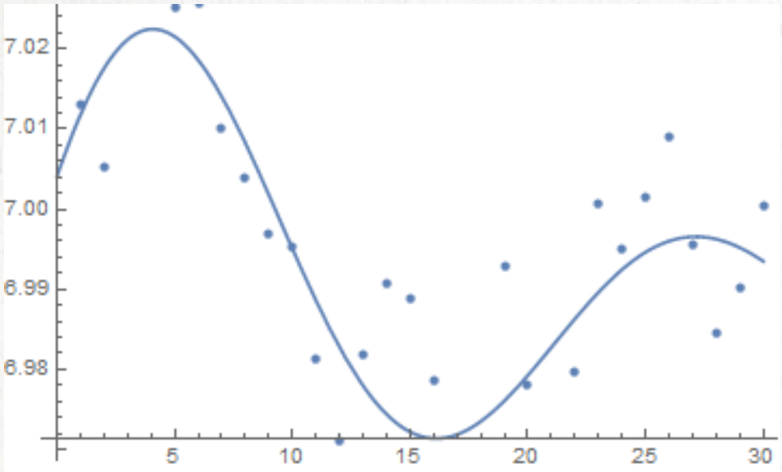
DOWNTREND #1 (THE POST 9/11 SHOCK)



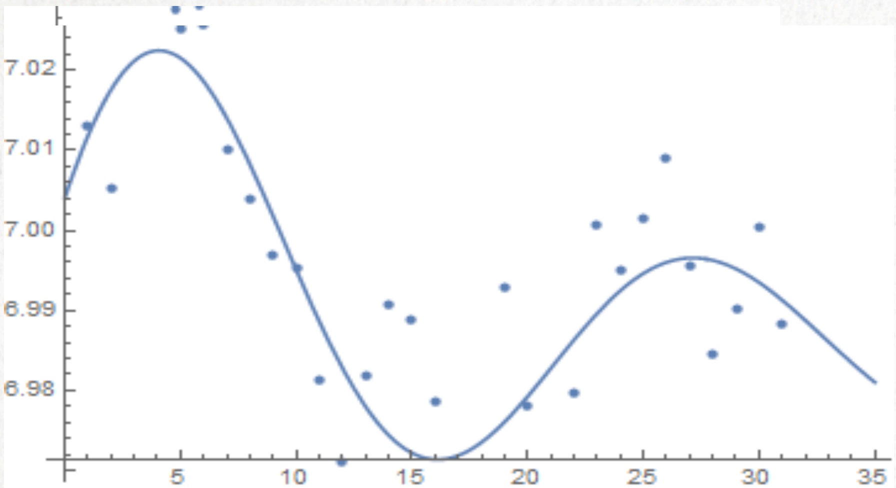
Data : Sep-Oct 2002
Length : 30 data points

DOWNTREND #1 (THE POST 9/11 SHOCK)



A	-1788.13
B	0.042827
C	0.035735
Θ	0.269533
ϕ	1.29321
D	1795.12
G	1.75E-07

DOWNTREND #1 (THE POST 9/11 SHOCK)



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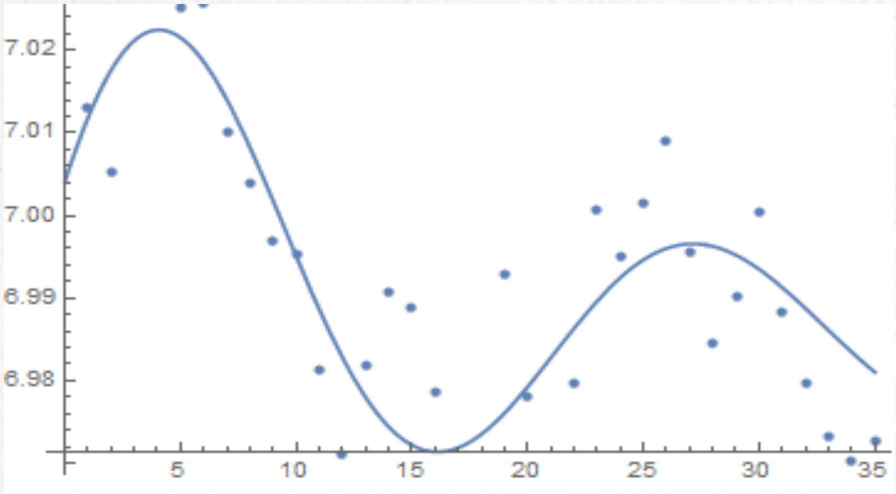

A	-1788.13
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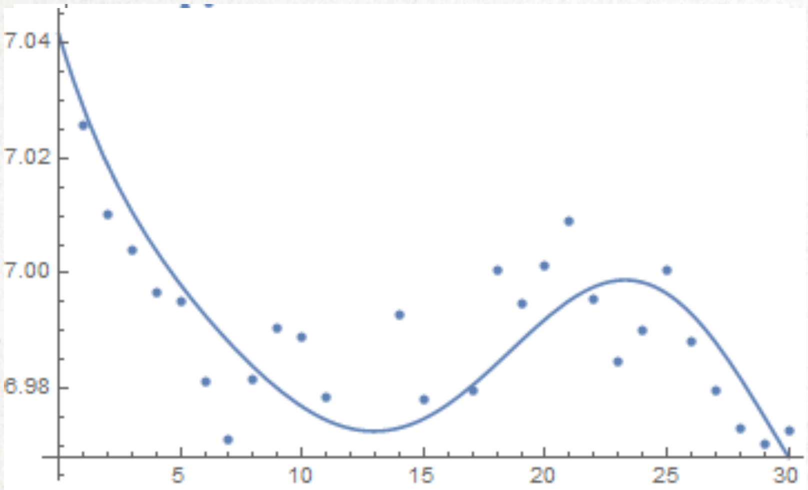

A	6.98126
B	-0.0503661
C	0.0055016
Θ	-0.2905
ϕ	-0.313406
D	0.0549065
G	0.274987

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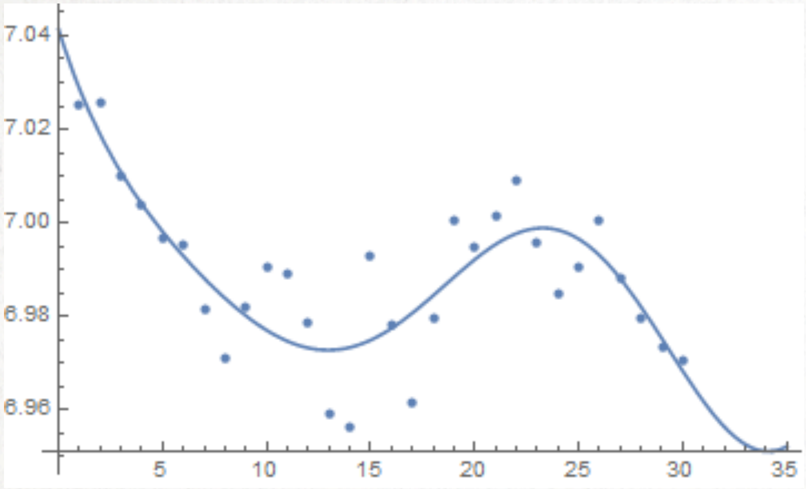

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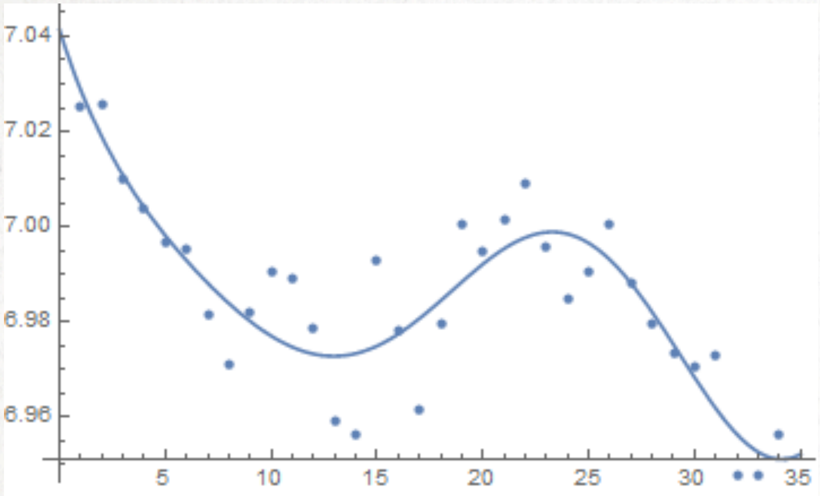

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B	-0.0503661
C	0.0055016
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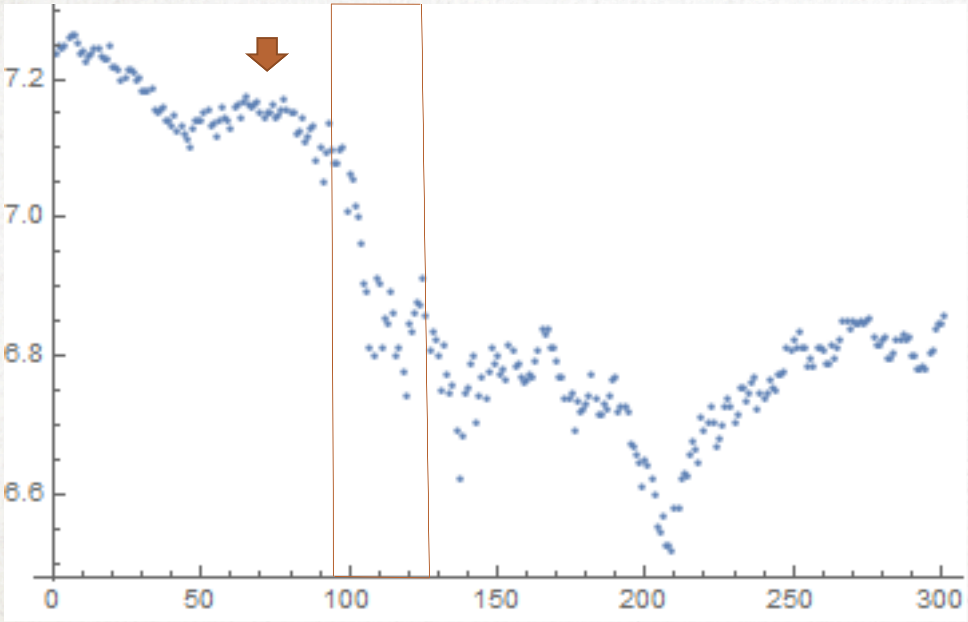
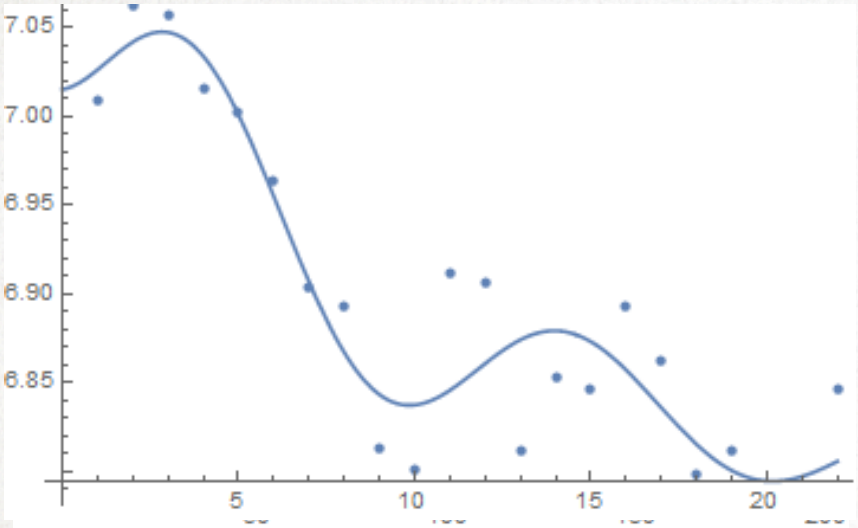
DOWNTREND #2 (THE 2008 CRISIS)



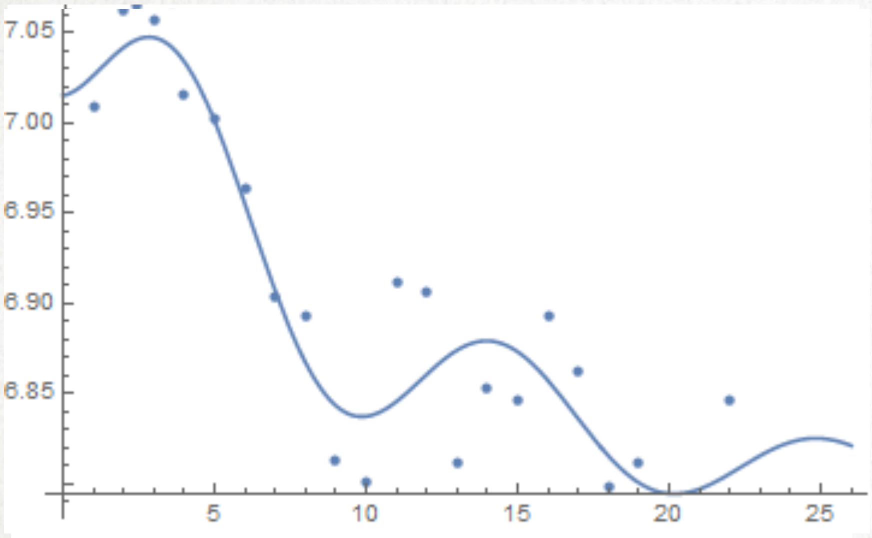
Data : Jan 2007
Length : 25 data points

DOWNTREND #2 (THE 2008 CRISIS)



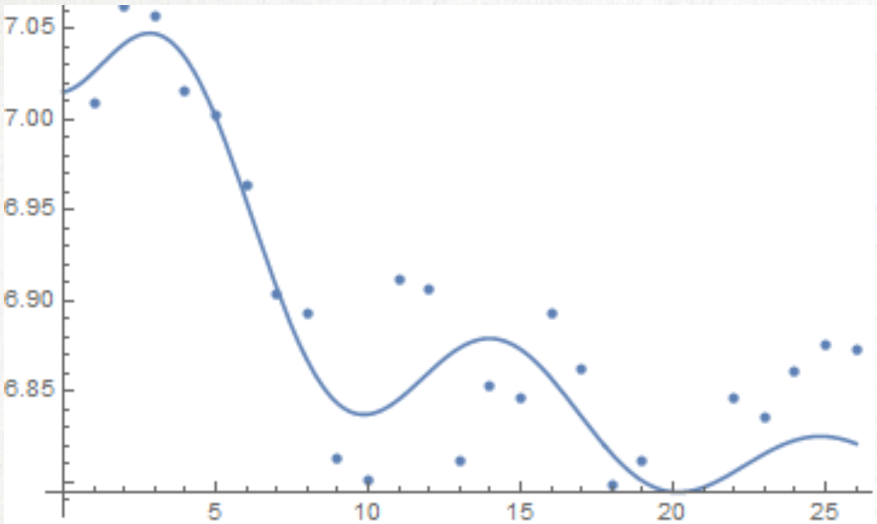
A	6.79265
B	0.0607648
C	0.0827193
ϴ	0.591906
ϕ	2.39608
D	0.283232
G	0.118508

DOWNTREND #2 (THE 2008 CRISIS)



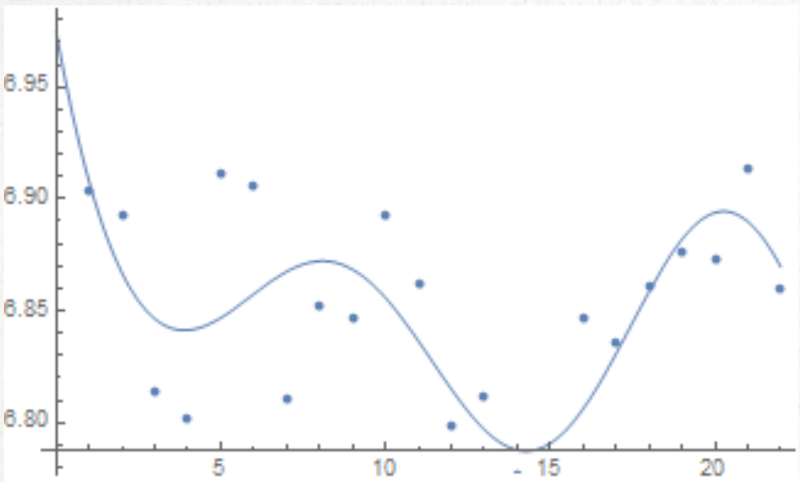
A	6.79265
B	0.0607648
C	0.0827193
ϴ	0.591906
ф	2.39608
D	0.283232
G	0.118508

DOWNTREND #2 (THE 2008 CRISIS)



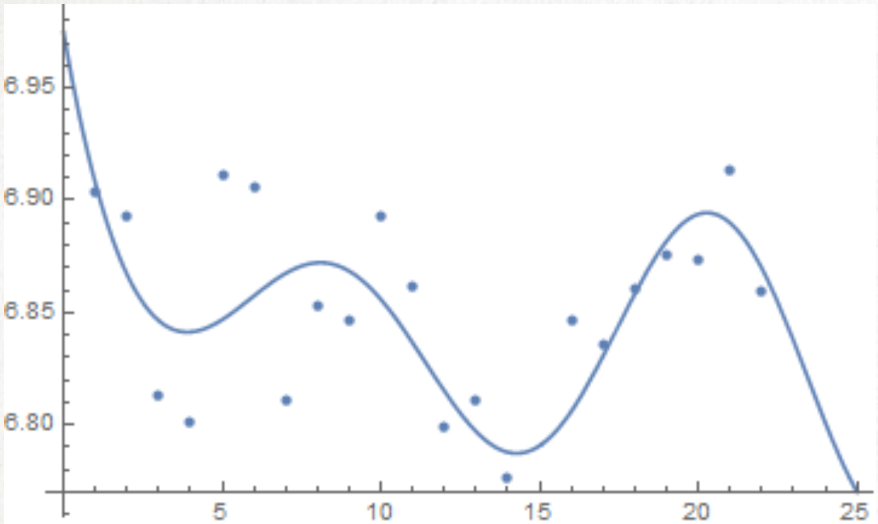
A	6.79265
B	0.0607648
C	0.0827193
Θ	0.591906
ϕ	2.39608
D	0.283232
G	0.118508

DOWNTREND #2 (THE PRE-2008 CRISIS)



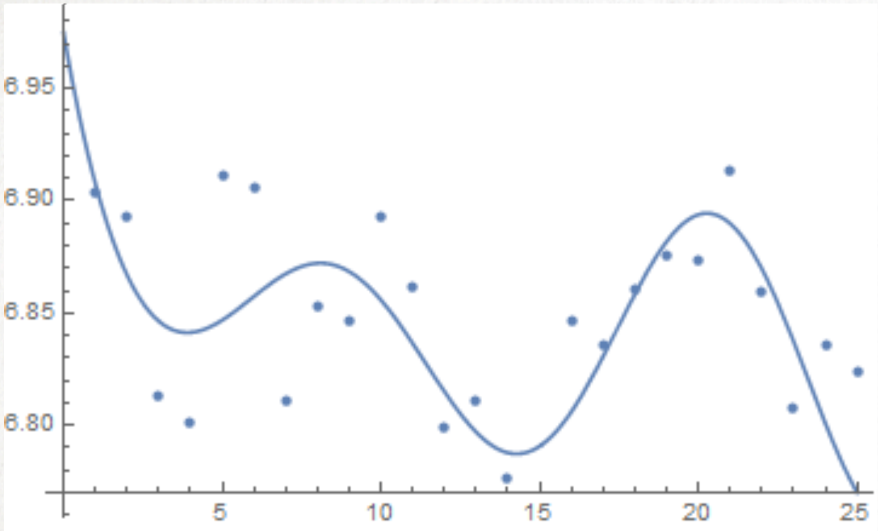
A	6.8335
B	-0.04536
C	0.024381
θ	0.523358
ϕ	4.23726
D	0.152507
G	0.46529

DOWNTREND #2 (THE 2008 CRISIS)



A	6.8335
B	-0.04536
C	0.024381
θ	0.523358
ϕ	4.23726
D	0.152507
G	0.46529

DOWNTREND #2 (THE 2008 CRISIS)



A	6.8335
B	-0.04536
C	0.024381
θ	0.523358
ϕ	4.23726
D	0.152507
G	0.46529

LIMITATIONS

1) Only works well in the short-time period (25-30 data points). The model collapses if number of data points approximately exceeds 35.
2) The third assumption, $P' = \lambda(Qd - Qs)$, is too ideal. In fact, no individual would respond to the market the same way
3) Data needs to be bounded –straight up/down will not work. In other words, when we deal with the situation in which stock index appear to go straight down/up, the models just become straight line and have no economic interpretation.

CONCLUSION

This work shows that how stock market may be modeled as damped harmonic oscillators with the arbitrary shock terms. It in fact does a great job in predicting the market movement during the short term period, but the model is somehow too simple to be realistically used. Sometimes the model fails to fit the data due to lack of market movement. However I would consider it a successful model in a certain level that answers my question in which how volume could affect stock market index.