Information Percolation and Wallet game

Gaoxiang Mei[1]

[1]*Department of Physics, Boston University*

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Information percolation is a simple model of information transmission through a large market. In this report we introduce the basic model of Information percolation and explain that how this model gives a relatively explicit solution for the distribution of posterior probability at each time t. We also explain that why the convergence of the distribution of probability to a common posterior is exponential and the rate of convergence is merely λ, the mean rate at which an individual agent is matched in two-agent meeting. In the end, we apply this model to a market example which is actually a special type of 'wallet game'.

INTRODUCTION

Information percolation refers to the model that a large number of asymmetrically informed agents are randomly matched into groups over time, exchanging their information with each other when matched. Over time, the conditional beliefs held across the population of agents regarding a variable of common concern converge to a common posterior.

Unlike the site percolation in physics, now the question we are interested in is not the connectivity or critical probability p_c any more. We focus on the evolution of distribution of probabilities to a common posterior. We are particularly interested in the type of convergence that the distribution has and the rate of convergence.

BASIC MODEL

The basic model of information percolation can be described as follow.

A random variable X of potential concern to all agents has two possible outcomes, H ("high") and L ("low"), with respective probabilities μ and $1-\mu$.

Each agent is initially endowed with a sequence of signals that may be informative about X. The signals $\{s_1, ..., s_n\}$ are, conditional on X, independent with outcomes zero and one.

The number of signals, as well as the probability distributions of the signals, may vary across agents.

Whenever agents meet they communicate to each other their posterior probabilities of the event that X is high. Meeting group size is m and rate of meeting is λ.

By Bayes rule, the logarithm of the likelihood ratio between states H and L conditional on signals $\{s_1, ..., s_n\}$ is[1]

$$\log\frac{P(X=H|s_1,...,s_n)}{P(X=L|s_1,...,s_n)} = \log\frac{\mu}{1-\mu} + \theta \qquad (1)$$

where the "type" θ of this set of signals is

$$\theta = \sum_{i=1}^{n} \log \frac{P(s_i|X=H)}{P(s_i|X=L))} \quad (2)$$

The higher the type θ of the set of signals, the higher is the likelihood ratio between states H and L and the higher the posterior probability that X is high.

INFORMATION TRANSMISSION DYNAMICS

Now that we have an idea of the basic model of information percolation, we can focus on the evolution of distribution of probabilities to a common posterior. Let's consider the simplest case : *Two-Agent Meetings*. This is the standard setting for search-based models of labour, money and asset markets.

We let $g(x,t)$ denote the distribution of type x in the population at time t. We can prove that if an agent of pre-posterior type θ meets an agent with pre-posterior type ϕ, and they communicate to each other their types, then both have posterior type $\theta+\phi$. Thus we have the distribution of types for this setting is determined by the evolution equation:[2]

$$g_t(x,t) = -\lambda g(x,t) + \int_{-\infty}^{+\infty} \lambda g(y,t) g(x-y,t)\, dy \quad (3)$$

Solving this equation by applying Fourier transformation on both sides, we have

$$\hat{g}_t(z,t) = -\lambda \hat{g}(z,t) + \lambda \hat{g}^2(z,t) \quad (4)$$

Where $\hat{g}(*,t)$ is the Fourier transform of $g(*,t)$.

Thus we have

$$\hat{g}(z,t) = \frac{\hat{g}(z,0)}{e^{\lambda t}[1-\hat{g}(z,0)] + \hat{g}(z,0)} \quad (5)$$

This solution for the distribution of types is converted to an explicit distribution for posterior probabilities that $X = H$, using the fact that

$$f(b,t) = g(\log \frac{b}{1-b} - \log \frac{\mu}{1-\mu}, t) \quad (6)$$

In our setting, it turns out that the beliefs of all agents converge to that of complete information, in that any agent's posterior probability of the event $\{X = H\}$ converges to one on this event and to zero otherwise. In general, we say that $f(b,t)$ converges to a common posterior distribution $f(b,\infty)$.

We can prove that[1]

$$\begin{aligned} f(b,0)e^{-\lambda t} &\leq |f(b,t) - f(b,\infty)| \\ &= f(b,t) \leq (\beta + e^c \frac{\gamma}{1-\gamma}) e^{-\lambda t} \end{aligned} \quad (7)$$

where β, c and γ are constants and $\gamma < 1$.

Thus, in the simplest case two-agent meeting, convergence of the distribution of probability to a common posterior is exponential and the rate of convergence is merely λ, the mean rate at which an individual agent is matched.

MARKET EXAMPLE AND THE WALLET GAME

Now let's apply the model of information percolation to a market example. In this market example, uninformed buyers hedge the uncertainty in X. Some risk-neutral sellers are initially given signals about X, so that there is an initial distribution of their types $g(*,0)$. The uninformed buyer conducts an auction with the two chosen informed sellers. The lower bidder sells the uninformed agent a forward financial contract that pays 1 at time T if X is high and 0 otherwise. After purchasing the contract, the uninformed buyer leaves the market.

This market example is actually a special type of wallet game. To have a better understanding of this example, we should figure out what is wallet game first. Select two students, Alice and Bob, and let them check how much money is in his or her wallet. Now Charlie auctions a prize equal to the sum of the money in their wallets to these two students. That is, Charlie will continuously raise the price until one of the students quits the bidding, and Charlie will then pay the other student an amount equal to the sum of the money in their wallets, in return for the student paying Charlie that final price.[3] This is called the wallet game.

In the market example, the buyer acts as Charlie and the sellers act as the students. The posterior probability of $X = H$ the sellers hold is the money in their wallet.

In the unique symmetric Nash equilibrium of each auction in wallet game, each student should remain in the bidding up to a price of double the money in his or her wallet. Thus, the information about money in his or her wallet is transmitted to the other student.

Similarly, in the unique symmetric Nash equilibrium of each auction in the market example, each seller should remain in the bidding until the price reaches the posterior probability of $X = H$ the seller hold. From the one-to-one correspondence between an seller's type and the seller's posterior probability of $X = H$, informed sellers learn each others' types from their bids. Thus, each auction in the market is just a two-agent meeting and the dynamics of information transmission in this market example are therefore as described in our 'Information Transmission Dynamics' part.

CONCLUSIONS

Information percolation is a simple model of information transmission through a large market. This model allows a relatively explicit solution for the distribution of posterior probability at each time t.

Convergence of the distribution of probability to a common posterior is exponential and that the rate of convergence does not depend on m, the size of the groups of agents that meet. The rate of convergence is merely λ, the mean rate at which an individual agent is matched.

[1] Duffie, Darrell, Gaston Giroux, and Gustavo Manso. "Information percolation." American Economic Journal: Microeconomics 2.1 (2010): 100-111.

[2] Duffie, Darrell, and Gustavo Manso. "Information percolation in large markets." The American economic review 97.2 (2007): 203-209.

[3] Klemperer, Paul. 1998. "Auctions with Almost Common Values: The 'Wallet Game' and Its Applications." European Economic Review, 42(3-5): 757-69.