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Analysis of damage clusters in fracture processes

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Abstract

We present numerical simulations of two-dimensional models of electric breakdown and fracture in disordered systems subject to an increasing external stress. We provide a geometrical characterization of the damage by studying the scaling behavior of connected bonds clusters. The average cluster size and the lattice conductivity show features characteristic of a first order phase transition. The obtained results are discussed within the spinodal nucleation scenario recently proposed for fractures.

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The understanding of fracture in disordered materials [1,2] has recently progressed, due to new experiments focusing on the statistical properties of precursor events. The main tool used in these experimental studies is the measure of the acoustic emission (AE) signal produced by microfractures occurring before breakdown. The experimental observations of scaling laws suggest a description of fracture as a critical phenomenon. The response of the material to an increasing external stress takes place in bursts (avalanches) distributed over a wide range of scale; i.e. with power-law behavior. Examples are found in the fracturing of wooden composite [3,4], cellular glass [5] and concrete [6], in hydrogen precipitation [7], in dislocation motion in ice crystals [8] and in volcanic activity [9]. The avalanche-like response and the scaling properties

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of these phenomena, pose many fundamental theoretical questions. What kind of critical dynamics is underlying the fracture process? Is it possible to understand fracture phenomena within the wide context of phase transitions? What is the role of disorder and temperature in these phenomena? All these questions can be summarized in the ambitious goal of formulating a statistical thermodynamics of fracture.

The presence of disorder inside real materials is a fundamental point that can strongly affect the microcracks nucleation process [10–12]. For example, cracks may start from different defects and coalesce [13], in contrast with the assumptions of Griffith-like theories [14]. There are situations, encountered for example in material testing, in which the system is driven by an increasing external stress and the time scale of thermal fluctuations is larger than the time scale induced by the driving. In those cases, the system can be effectively be considered as being at zero temperature and only quenched disorder is relevant. This is the situation we have investigated in a series of papers [15–17]. In these works, we numerically studied the random fuse model [18–21] and a spring network [22–24]. For these models, we have shown that the precursor behavior near the breakdown in disordered systems is analogous to the formation of droplets observed close to a spinodal instability in first-order phase transitions [25], (for a review see [26]).

Here we provide new evidences in favor of the spinodal nucleation scenario by analyzing the geometrical and connectivity properties of damage clusters. We find that the average damage cluster size does not show any singular behavior and that the lattice conductivity has a discrete jump, indicative of a first-order transition. These results can be reconciled with the scaling behavior of the precursor avalanche distribution by invoking the occurrence of a first-order transition close to a spinodal point [21,26].

We will consider for simplicity the random fuse model, but the following results are valid also for the spring network. We simulate the random fuse model [18] on a tilted square lattice, with periodic boundary conditions in the transverse direction. To every bond ij of the lattice we associate a fuse of unit conductivity $\sigma_{ij}=1$. An external current I or voltage V is then applied to the system by imposing suitable boundary conditions on two opposite edges of the lattice. When the current in the bond exceeds a randomly distributed threshold D_{ij} the bond becomes an insulator ($\sigma_{ij}=0$). The voltages V_i for each node are computed solving the Kirchhoff equations for the network

$$\sum_j \sigma_{ij}(V_i - V_j) = 0\,, \tag{1}$$

where the sum is restricted to the nearest neighbors of site i. The distribution of thresholds is chosen to be uniform in the interval $[1-\Delta, 1+\Delta]$. We impose an external current I through the lattice and we increase it at an infinitesimal rate. When a bond fails, we recompute the currents to see if other failures occur. The process is continued until a path of broken bonds spans the lattice and no current flows anymore.

We have presented in Ref. [15,16] the distribution of avalanche sizes, i.e. number of bonds m that break for a given value of the current, in a particular realization of the process. We found that $P(m,I) \sim m^{-3/2} f(m(I_c - I))$, where I_c is the average

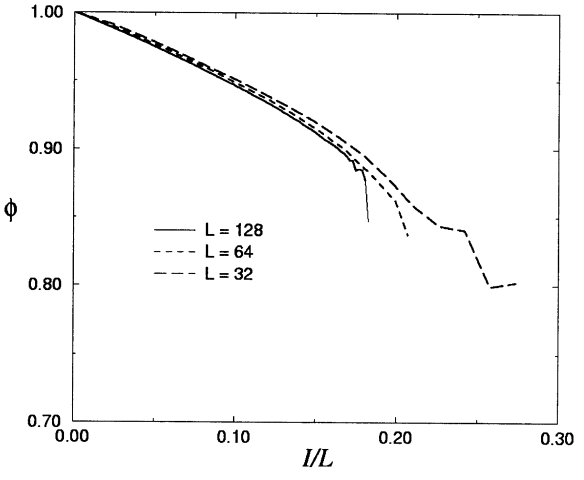

Fig. 1. The fraction of intact bonds as a function of the current.

breakdown current. By integrating over all the values of the current we get a *global* avalanche distribution $P(m) \sim m^{-5/2}$. We have also checked that the cut-off of the distribution increases with the system size. A similar result ($\tau \simeq 2.7$) for smaller lattice sizes ($L = 40$) was previously reported by Hansen and Hemmer [27], who also pointed out the similarity with the predictions of the FBM.

A more detailed analysis can be performed by inspecting the cluster size distribution $n(s,I)$, which is defined as the number of clusters formed by s neighboring broken bonds when the applied current is I. The moments ($M_k(I) \equiv \int s^k n(s,I)\,\mathrm{d}s$ is the kth moment) of $n(s,I)$ describe much of the physics associated with the breakdown process. We determine $n(s,I)$ by averaging over the various threshold distribution configurations. The first moment $M_1(I)$ is the total number of broken bonds due to the current I. We plot $\phi = 1 - M_1(I)$ in Fig. 1 and we note that the curve reminds the variation of the order paramter with the external field in a first-order transition. The average cluster size $S \equiv M_2/M_1$ increases with I. However, by plotting S for different system sizes, we observe that the cluster size is not diverging (Fig. 2). To clarify this point, we confirm that $S(I_c)$ does not show scaling with the lattice size L. We find similar results for the spring network, where the cluster size distribution has an exponential cut-off that does not change with the lattice size. We also study the number of clusters $n_c \equiv M_0$ as a function of the current and for different system sizes (see Fig. 3). We observe that n_c scales as

$$n_c = L^2 g(I/L)\,, \tag{2}$$

which is expected for a first-order transition.

Next, we study the behavior of the lattice conductivity in the fuse model. We plot the conductivity averaged over different realizations of the disorder and we observe a smooth curve with a slope at the breakdown that becomes sharper as the system size increases (Fig. 4).

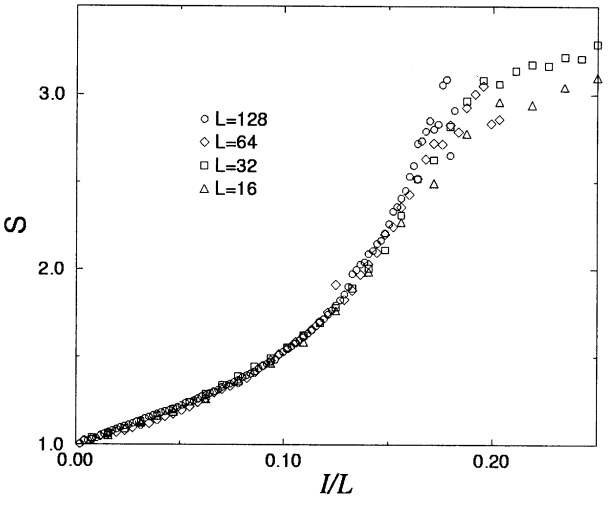

Fig. 2. The average cluster size as a function of the current for different system sizes. Note that the cluster size does not diverge.

Two principal scenarios have been proposed to explain the scaling behavior of avalanches prior to rupture. The first scenario invokes a continuous phase transition with a diverging characteristic length. The various cracks inside the lattice should grow until one of them finally rules over the others, becoming the incipient spanning cluster. This is precisely what happens in percolation when the occupation probability p is increased toward the percolation threshold p_c. If this scenario is true for fracture, we would expect the cluster characteristic size to diverge approaching the critical damage density, contrary to our results. In the random fuse network a percolation transition is expected only in the limit of infinitely wide disorder distributions [28], when the strength of the disorder clearly dominates over the interactions.

The second scenario, in favor of which we presented numerical and theoretical evidences [15,16], describes fracture as a first-order phase transition close to a spinodal-like instability. The elastic state is considered to be metastable, as soon as a non-zero stress is applied. Due to the presence of disorder, the system evolves through a series of metastable states towards the final instability. This occurs with the nucleation of cracks growing up to a critical size s_c at which they *coalesce* forming the macroscopic crack. Contrary to percolation, in this case there is no incipient spanning cluster prior to rupture. When nucleation occurs close to a spinodal the characteristic length divergence is not naively related to the fluctuations of a geometrical quantity such as the crack size, which is not diverging at the spinodal. In order to describe geometrically the critical properties, it is necessary to define the clusters in a peculiar way, considering each site connected with all the others within the range of interactions [29,30]. These fluctuations are therefore different from those encountered in a second-order phase transition. It is worth remarking that the spinodal point is a quite peculiar critical point which, rigorously speaking, exists only in mean-field theory, but can be detected when long-range

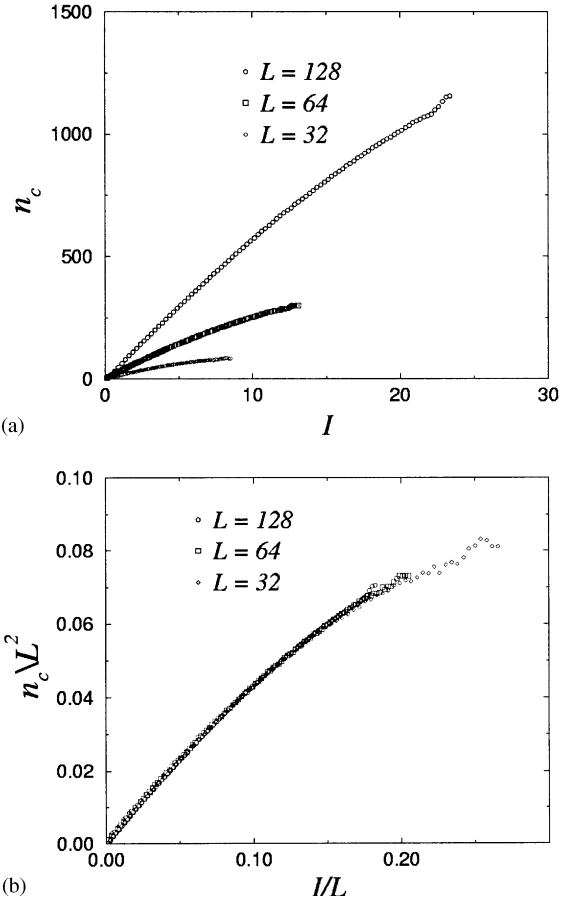

Fig. 3. (a) The number of clusters as a function of the current in the fuse model for different system sizes. (b) The corresponding scaled plot.

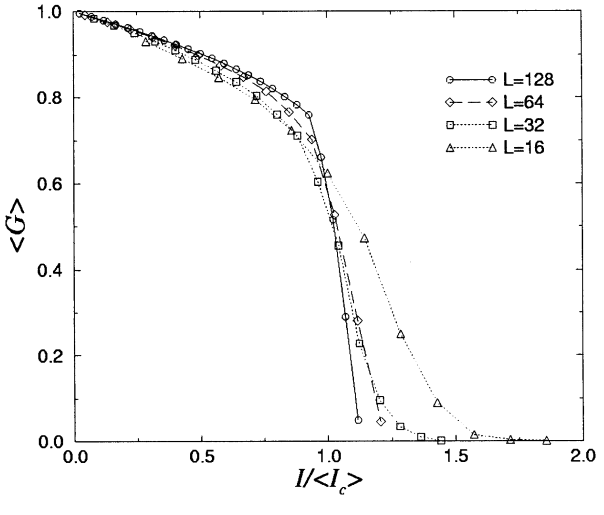

Fig. 4. The conductivity as function of I/I_c averaged over different realizations of the disorder. Note that the discrete jump, indicative of a first-order transition, is smoothed for small system sizes.

interactions are present. In this respect, no scaling is observed in the avalanche distributions when the stress transfer after breaking is local, such as in the *local* load-sharing fiber bundle model studied in Refs. [31,32,27].

In summary, we have shown that in disordered materials breakdown processes, only *globally* defined quantity such as $\langle m(I)\rangle$ and $P(m)$ display scaling. On the contrary, locally defined quantities, such as S, do not show any singular behavior. For a critical phase transition, we would expect also local quantities to show scaling. An exception to this rule is represented by first-order transitions close to a spinodal point, for which some global quantities display scaling [29,30]; we argue that this case is relevant to the behavior observed before breakdown.

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