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Statistical characteristics of breakthrough discoveries in science using the metaphor of black and white swans

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HIGHLIGHTS

- We define "black swan" and "white swans" against the background of the history of scientific research.
- We describe how the "black swan" has become a metaphor for scientific breakthroughs and how its appearance changes the citation patterns of "white swans."
- We combine scientific discovery with scientometric data to verify the accuracy of the qualitative and quantitative indications of a breakthrough discovery.

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ABSTRACT

A publication that reports a breakthrough discovery in a particular scientific field is referred to as a "black swan", and the most highly-cited papers previously published in the same field "white swans". Important scientific progress occurs when "white swans" meet a "black swan", and the citation patterns of the "white swans" change. This metaphor combines scientific discoveries and scientometric data and suggests that breakthrough scientific discoveries are either "black swans" or "grey-black swans".

1. Introduction

Before black swans were discovered in Australia, Europeans believed that all swans were white. The discovery of black swans changed this belief. Extending this to science, breakthrough discoveries that change scientific views and promote scientific progress are the "black swans" [1] of research.

Unlike black swans in art or economics, black swans in science have no benefit of hindsight. When white swans meet a black swan, only scientific views are changed.

The philosophy of science [2] indicates that scientific discoveries may cause scientific revolutions and promote scientific progress [3]. With the advent of citation analysis [4] we know more about the inheritance and transformation of scientific knowledge [5], and the discipline of scientometrics has become a useful tool.

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Scientometrics uses citation curves [6], and its citation statistics [7] reveal the "sleeping beauty" phenomenon [8] and related behaviour [9–12]. These "sleeping beauties" are pure statistical patterns, however, and do not reveal scientific content. The same is true of "smart girls" [13].

If we combine scientific discovery with scientometrics we can use the history of scientific research and citation data to locate "black swans". We define a black swan to be the publication of an unanticipated scientific breakthrough that has a major impact. To locate black swans we use scientometric data.

We can determine whether a discovery is a black swan through the use of qualitative scientific factors, e.g., whether it results in a Nobel Prize. It also can be determined through a scientometric analysis of its impact as quantitatively measured by citations. Here we develop a methodology for identifying black swans that uses the identification of scientific discoveries and the qualitative and quantitative analysis of scientometric data.

2. Methods

A 'black swan' and a 'white swan' are defined as follows.

Black swan: Qualitatively, a black swan is defined as a publication that presents a breakthrough discovery in a particular field of science. Quantitatively, a black swan is a publication that receives more than Cb (black swan citations) citations within five years of its publication, during which time its citations increase and the corresponding citations of the white swans decrease.

White swan: Qualitatively, the white swans comprise group of publications that were the highly-cited papers in the scientific field prior to the publication of the black swan. Quantitatively, each white swan is a publication that receives more than Cw (white swan citations) citations in the five years prior to the publication of the black swan. Following the publication of the black swan its citation rate decreases.

Black and white swans exist in relation to each other because the publishing event of the black swan links them. Because this is a black-and-white swan pair, we examine the five years prior to this publishing event and the five years following it. Because there can be many white swans, for simplicity we compare and analyse only the top two.

We begin with one black swan published at year T_S and two white swans, white swan 1 published at T_1 and white swan 2 at T_2. Here $T_W = \min\{T_1, T_2\}$ and $T_S - T_W \geq 5$. Examining the five years prior we discover the two white swans with the highest number of citations and compare them with the black swan using the swan-index $Sw = Cb/(Cw_1 + Cw_2)$, where Cb represents black swan citations and Cw_i $(i = 1, 2)$ the citations of white swan i. $Sw = 1$ is the reference standard for the black swan. When $Sw > 1$, the black swan is strong or "plump". When $Sw < 1$, the black swan is weak or "thin". If $T_S - T_W < 5$ (during the five-year period the number of citations to the black swan is not higher than those of the white swans) or $Sw < 0.5$, the black swan is atypical and becomes a grey-black swan (or briefly a grey swan, but ultimately remains a black swan).

Qualitatively these definitions rely on the records of scientific discovery (scientific history), and quantitatively on scientometric data. The citation counts are from such data sources as the Web of Science (WoS), Scopus, or Google Scholar. Here white swans appear as groups and black swan as individuals.

We set $Cb > 100$ and $Cw > 50$ because a "sleeping beauty" can also receive 10 citations during a five-year sleeping period and then receive >20 citations in the four years after waking [10]. These settings mean that we are enlarging by five times the difference between the typical white swans and the black swan during the five-year period. If during the five-year period the number of citations to the black swan is not higher than those of the white swans, the black swan merges into the swan group and becomes a grey swan.

When white swans meet a true black swan, important scientific progress happens, the citation pattern of the white swans changes, and the discovery is a breakthrough that alters traditional approaches within the field. Even when white swan meet grey swan, also important scientific progress happens.

Combining the qualitative and quantitative definitions, the black-swan/white-swan metaphor is supported by both scientific discovery and scientometric data. In the following case study we focus on scientific discoveries that received Nobel Prizes, and we collect the scientometric data from the WoS.

Finding black and white swans involves three steps, (i) using the list of Nobel Prizes in Physics to identify black swans (see Appendix), (ii) examining the reference list, the citations to the items in the reference list, and the citations to the black swan (using WoS in this study), and (iii) determining the top two most highly cited references and designating them white swans. We then apply the following indicators to check and identify the black and white swans.

(1) *Indicators*

To quantify black and white swans we use a swan-index indicator,

$$Sw = \frac{Cb}{Cw_1 + Cw_2}, \tag{1}$$

where Sw is the swan-index, Cb is the total expected citations of the black swan in the first five years following its publication (the publishing year is Ts), and Cw_i $(i = 1, 2 \ldots)$ is the citations of white swan i five years prior to Ts. The Ts is also designated the scientific finding breakthrough year. In every case we search for the top two white swans. As Cb increases and Cw_i decreases, we expect that $Cb > Cw_i$.

When $Sw = 1$ there is a standard black swan. When $Sw > 1$, Cb is larger than $(Cw_1 + Cw_2)$, i.e., the integral area of the citation curve of the black swan is larger than those of the top two white swans, then the black swan is strong or "plump".

Table 1
The indicators for identifying black and white swans.

Type	Cb	Cw	$T_S - T_W$	Sw
Black swan	>100		≥ 5	>0.5
White swan		>50		(Cw follows Cb)
Grey-black swan (Grey swan)			<5	<0.5

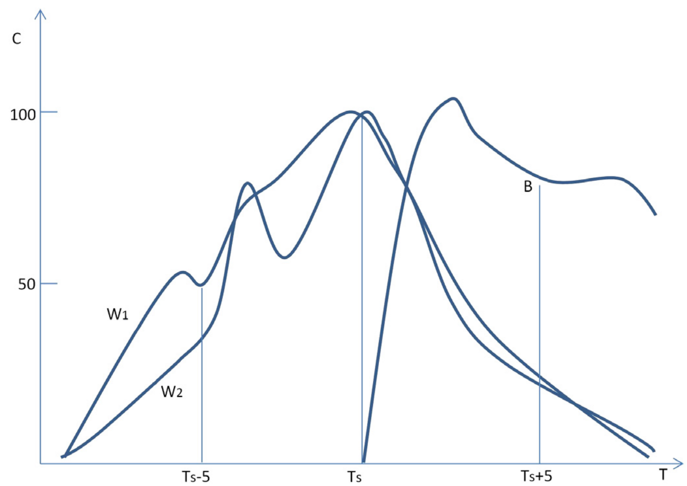


Fig. 1. Modelling black and white swans.

When $0.5 < Sw < 1$, the Cb is smaller than $(Cw_1 + Cw_2)$, i.e., the integral area of the citation curve of the black swan is smaller than those of the top two white swans, then the black swan is weak or "thin". When $Cb \ll (Cw_1 + Cw_2)$, $Sw \ll 1$. When $Sw < 0.5$, or $T_S - T_W < 5$, the black swan becomes a grey-black swan (or briefly a grey swan, but ultimately remains a black swan). Table 1 shows the indicators.

(2) *Modelling*

Fig. 1 shows an ideal figure of the black-white swan model in terms of citations (C) and publishing years (T).

Fig. 1 shows that scientific discovery occurs when black swan (B) is published at Ts. Before B is published, two white swans (W_1 and W_2) are highly cited papers. After B is published its citations increase and those of W_1 and W_2 decrease.

If we assume that all yearly citation curves can be described using the Avramescu formula, the time distributed equations of citation curves for W_1, W_2 and B become

$$c_{W1}(t) = A_{W1}[\exp(-\alpha_1 t) - \exp(-m_1\alpha_1 t)] \tag{2}$$

$$c_{W2}(t) = A_{W2}[\exp(-\alpha_2 t) - \exp(-m_2\alpha_2 t)] \tag{3}$$

$$c_{\mathrm{B}}(t) = A_B[\exp(-\alpha_B t) - \exp(-m_B\alpha_B t)] \tag{4}$$

where t is the age of publications, A_x denotes the amplitude, α_x is the age decrement and m_x is the initial increment. Here we set $\alpha_x > 0$ and $m_x > 1$, $x = W1, W2, B$.

If white swan 1 is published at T_1 and white swan 2 at T_2, the integrals as theoretical values of Cb and Cw_i are

$$C_{W1} = \int_{T_s-T_1-5}^{T_s-T_1} c_{W1}(t)dt = \frac{A_{W1}}{\alpha_{W1}}(1-e^{-5\alpha_{W1}})e^{-(T_S-T_i-5)\alpha_{W1}} - \frac{A_{W1}}{m_{W1}\alpha_{W1}}(1-e^{-5m_{W1}\alpha_{W1}})e^{-(T_S-T_1-5)m_{W1}\alpha_{W1}} \tag{5}$$

$$C_{W2} = \int_{T_S-T_2-5}^{T_S-T_2} c_{W1}(t)dt = \frac{A_{W2}}{\alpha_{W2}}(1-e^{-5\alpha_{W2}})e^{-(T_S-T_2-5)\alpha_{W2}} - \frac{A_{W2}}{m_{W2}\alpha_{W2}}(1-e^{-5m_{W2}\alpha_{W2}})e^{-(T_S-T_2-5)m_{W2}\alpha_{W2}} \tag{6}$$

$$C_B = \int_0^5 c_B(t)dt = \frac{A_B}{\alpha_B}(1-e^{-5\alpha_B}) - \frac{A_B}{m_B\alpha_B}(1-e^{-5m_B\alpha_B}). \tag{7}$$

Geometrically, Cb is the integral area of curve B along Ts to Ts + 5, and C_{w1} and C_{w2} are the integral areas of curves W_1 and W_2 between Ts − 5 to Ts, respectively. Sw can now be written

$$Sw = \frac{\frac{A_B}{\alpha_B}(1-e^{-5\alpha_B}) - \frac{A_B}{m_B\alpha_B}(1-e^{-5m_B\alpha_B})}{\sum_{i=1}^{2}\left[\frac{A_{Wi}}{\alpha_{Wi}}(1-e^{-5\alpha_{Wi}})e^{-(T_S-T_i-5)\alpha_{Wi}} - \frac{A_{Wi}}{m_{Wi}\alpha_{Wi}}(1-e^{-5m_{Wi}\alpha_{Wi}})e^{-(T_S-T_i-5)m_{Wi}\alpha_{Wi}}\right]}. \tag{8}$$

Sw relates to T_1 and T_2 and, assuming $T_1 < T_2$, $T = T_s - T_1$ to be the period from the old finding to the new breakthrough, then

$$f(T_1, T_2) = \sum_{i=1}^{2}\left[\frac{A_{Wi}}{\alpha_{Wi}}(1-e^{-5\alpha_{Wi}})e^{-(Ts-Ti-5)\alpha_{Wi}} - \frac{A_{Wi}}{m_{Wi}\alpha_{Wi}}(1-e^{-5m_{Wi}\alpha_{Wi}})e^{-(Ts-Ti-5)m_{Wi}\alpha_{Wi}}\right]. \tag{9}$$

When $f(T_1, T_2)$ decreases, *Sw* increases. The partial derivative is

$$\begin{aligned}\frac{\partial f(T_1,T_2)}{\partial T_1} &= \frac{\partial\left[\frac{A_{W1}}{\alpha_{W1}}(1-e^{-5\alpha_{W1}})e^{-(T_S-T_1-5)\alpha_{W1}} - \frac{A_{W1}}{m_{W1}\alpha_{W1}}(1-e^{-5m_{W1}\alpha_{W1}})e^{-(T_S-T_1-5)m_{W1}\alpha_{W1}}\right]}{\partial T_1}\\ &= A_{W1}[e^{-(T-5)m_{W1}\alpha_{W1}} - e^{-Tm_{W1}\alpha_{W1}}] - A_{W1}[e^{-(T-5)\alpha_{W1}} - e^{-T\alpha_{W1}}].\end{aligned} \tag{10}$$

Introducing two decreasing functions

$$g_1(T) = A_{W1}e^{-m_{W1}\alpha_{W1}T} \tag{11}$$

$$g_2(T) = A_{W1}e^{-\alpha_{W1}T}. \tag{12}$$

Eq. (10) can be written in a finite interval $[T-5, T]$ as

$$\frac{\partial f(T_1,T_2)}{\partial T_1} = [g_1(T-5) - g_1(T)] - [g_2(T-5) - g_2(T)]. \tag{13}$$

Considering the change ratio of two functions

$$r = \frac{g_1'(T)}{g_2'(T)} = \frac{-m_{W1}\alpha_{W1}A_{W1}e^{-m_{W1}\alpha_{W1}T}}{-\alpha_{W1}A_{W1}e^{-\alpha_{W1}T}} = m_{W1}e^{-(m_{W1}-1)\alpha_{W1}T}, \tag{14}$$

we find that $r < 1$ when T is longer. Hence we have

$$g_1(T-5) - g_1(T) < g_2(T-5) - g_2(T) \quad \text{i.e.}\ \frac{\partial f(T_1,T_2)}{\partial T_1} < 0. \tag{15}$$

This means that *Sw* will increase as T_1 decreases. A theoretical analysis reveals that *Sw* is larger when the white swans are published earlier.

3. Results

We explore some typical black swan examples in physics. We identify black swans by (i) examining the list of Nobel Prizes in Physics awarded during the period 1950–2010 (http://www.nobelprize.org/nobel_prizes/physics/laureates/), (ii) analysing the references of the key paper in the Nobel lecture of each Nobel laureate, and (iii) searching the citations of the key paper and all references in the key paper using WoS. This key paper then becomes a black swan and the two most highly cited references in the same field in the five years prior to the publication of the key paper are the white swans.

In 118 key papers we found 16 black swans and 8 grey swans, i.e., 20.34% of the discoveries that were awarded Nobel Prize (NP) in Physics were black swans (13.56% were breakthrough discoveries) or grey swans (6.78% were also important discoveries). Although we also found some sleeping beauties and all-elements-sleeping-beauties [9], we did not include them in our study because a sleeping beauty is a scientometric concept with no direct link to scientific discovery.

Fig. 2 shows two typical black swans in which the left vertical coordinate is the number of black swan citations and the right vertical coordinate is the number of white swan citations. Fig. 2 also shows two atypical black swans, i.e., grey-black swans.

In Fig. 2, case (a) is the discovery of Bose–Einstein condensation, where the black swan is

B: Davis, K.B., Mewes, M.-O., Andrews, M.R., van Druten, N.J., Durfee, D.S., Kurn, D.M. and Ketterle, W. Bose–Einstein Condensation in a Gas of Sodium Atoms. *Physical Review Letters* **75**, 3969 (1995).

The top two white swans are

W1: Doyle, J.M., Sandberg, J.C., Ite, A.Y., Cesar, C.L., Kleppner, D., & Greytak, T.J. Hydrogen in the submillikelvin regime: Sticking probability on superfluid He 4. *Physical Review Letters* **67**(5), 603 (1991).

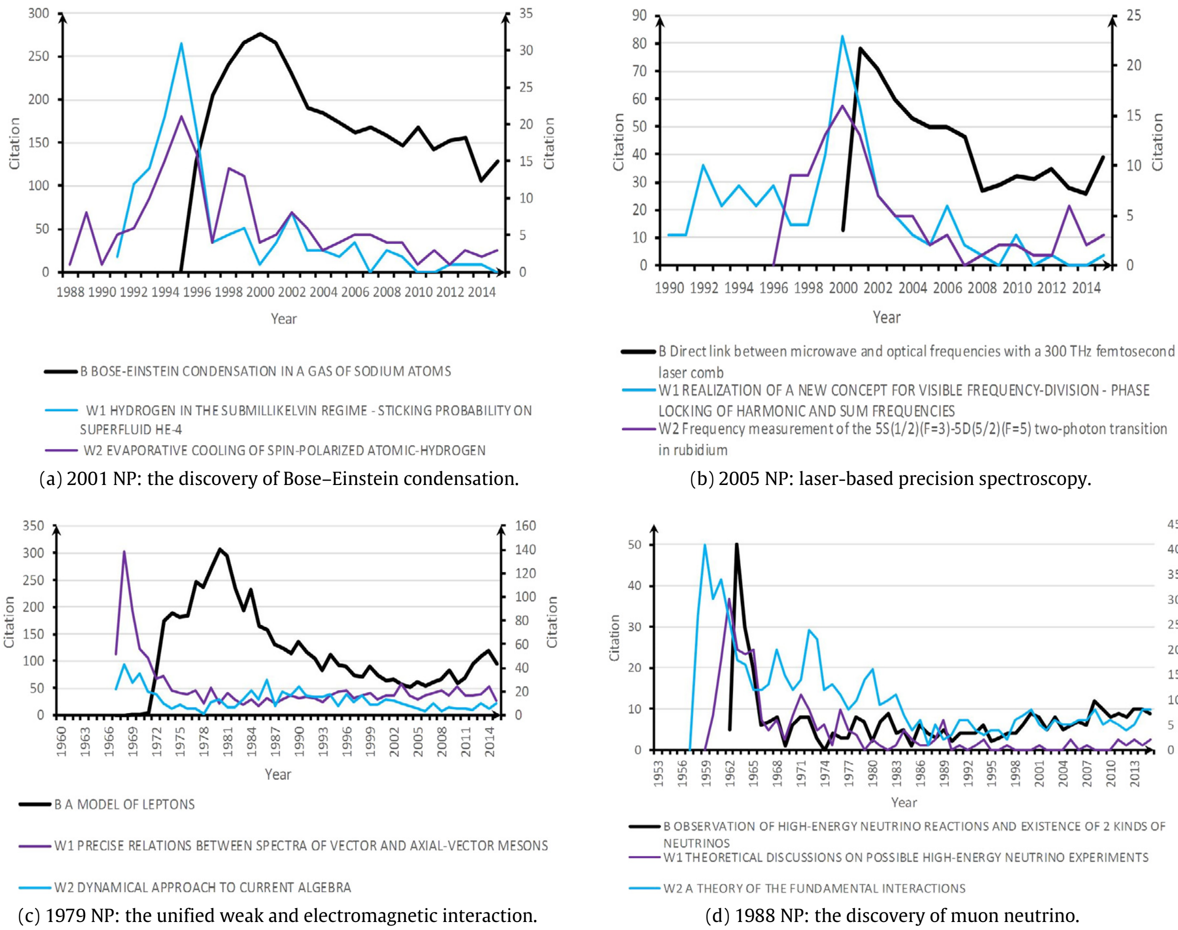
W2: Masuhara, N., Doyle, J.M., Sandberg, J.C., Kleppner, D., Greytak, T.J., Hess, H.F., & Kochanski, G.P. Evaporative cooling of spin-polarized atomic hydrogen. *Physical review letters* **61**(8), 935 (1988).

Its swan index is $Sw = 840/(80+57) = 6.13 > 1$ and its $T_S - T_W = 7$. This is a "plump" black swan, a typical black swan.

When the Bose–Einstein condensation was introduced it changed the physics of condensed matter. A similar case was another 'black swan': M.H. Anderson, J.R. Ensher, M.R. Matthews, C.E. Wieman, and E.A. Cornell, Observation of Bose–Einstein Condensation in a Dilute Atomic Vapor, *Science* **269**, 198 (1995).

Case (b) is a contribution to laser-based precision spectroscopy, where the black swan is

B: Diddams, S.A., Jones, D.J., Ye, J., Cundiff, T., Hall, J.L., Ranka, J.K., Windeler, R.S., Holzwarth, R., Udem, T. and Hänsch, T.W. Direct Link between Microwave and Optical Frequencies with a 300 THz Femtosecond Laser Comb. *Physical Review Letters* **84**, 5102–5105 (2000).



(a) 2001 NP: the discovery of Bose–Einstein condensation.
(b) 2005 NP: laser-based precision spectroscopy.
(c) 1979 NP: the unified weak and electromagnetic interaction.
(d) 1988 NP: the discovery of muon neutrino.

Fig. 2. Typical and atypical black swans.

The top two white swans are
W1: Telle, H.R., Meschede, D., & Hänsch, T.W. Realization of a new concept for visible frequency division: phase locking of harmonic and sum frequencies. *Optics letters* 15(10), 532–534 (1990).
W2: Touahri, D., Acef, O., Clairon, A., Zondy, J.J., Felder, R., Hilico, L., …& Nez, F.. Frequency measurement of the 5S12 ($F = 3) - 5D52$ ($F = 5$) two-photon transition in rubidium. *Optics communications* 133(1), 471–478 (1997).
Its swan index is $Sw = 275/(50 + 47) = 2.84 > 1$ and its $T_S - T_W = 10$. This is also a "plump" black swan.

Physically, laser-based precision spectroscopy changed precision spectroscopy, including the optical frequency comb technique.
Case (c) is the discovery of the unified weak and electromagnetic interaction, where the (grey) black swan is
B: Weinberg, S. A model of leptons. *Physical Review Letters* **19**(21), 1264 (1967).
The top two white swans are
W1: Weinberg, S. Precise Relations between the Spectra of Vector and Axial-Vector Mesons. *Physical Review Letters* **18**(13), 507–509 (1967).
W2: Weinberg, S. Dynamical Approach to Current Algebra. *Physical Review Letters* 1**8**(5), 188–191 (1967).
Its swan index is $Sw = 253/(279 + 93) = 0.68 < 1$. Since its $T_S - T_W = 0$ and $T_1 = T_2$, this is a "grey" swan in which the white swans and the black swan were both published in (1967).

The discovery of electroweak unification is a special grey swan, and its white swans are by the same author.
Case (d) is the discovery of muon neutrino, where the black swan is
B: Danby, G., Gaillard, J.M., Goulianos, K., Lederman, L.M., Mistry, N., Schwartz, M., & Steinberger, J. Observation of High-Energy Neutrino Reactions and the Existence of Two Kinds of Neutrinos. *Physical Review Letters* **9**(1), 36 (1962).
The top two white swans are
W1: Lee, T.D., & Yang, C.N. Theoretical discussions on possible high-energy neutrino experiments. *Physical Review Letters* **4**(6), 307 (1960).
W2: Schwinger, J. A theory of the fundamental interactions. *Annals of Physics* **2**(5), 407–434 (1957).

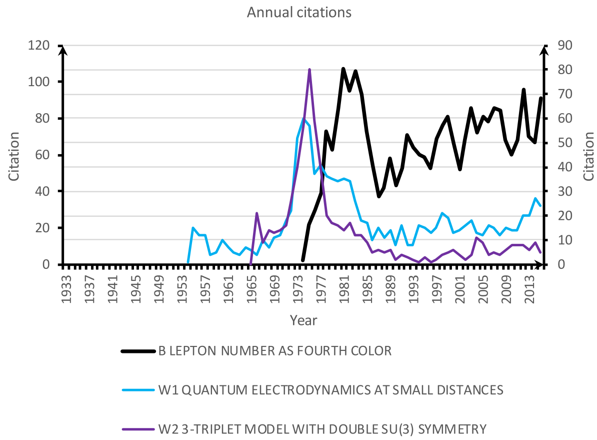


Fig. 3. A case of a new finding.

In Fig. 3, its swan index is $Sw = 111/(55 + 158) = 0.52–0.5$ and $T_S - T_W = 5$. This is a "thin" black swan. After its publication the citations to the black swan are lower than to one of white swans and its swan index is ~ 0.5 with $T_S - T_W = 5$. Thus this black swan is a grey-black swan.

The discovery of the muon neutrino revealed the doublet structure of the leptons, which benefited electroweak unification.

4. Discussion

The black-white swan metaphor provides an interesting interpretation of important scientific findings and can be used to identify and verify important scientific discoveries. Combining the identification of important scientific discoveries using Nobel Prizes with scientometric data allows us to doubly verify the status of a black swan.

Nobel discoveries can be improved by subsequent findings and these may become black swans that change a previous black swan into a white swan. An example is a new finding for improving the discovery of electroweak unification.
Here the black swan is
B: Pati, J.C., & Salam, A. "Lepton Number as the Fourth Color," *Physical Review D* **10**(1), 275 (1974).
The top two white swans are
W1: Gell-Mann, M., & Low, F.E. Quantum electrodynamics at small distances. *Physical Review*, 95(5), 1300 (1954).
W2: Han, M.Y., & Nambu, Y. Three-triplet model with double SU (3) symmetry. *Physical Review*, 139(4B), B1006 (1965).
Its swan index is $Sw = 166/(164 + 153) = 0.52 < 1$. This is also a thin black swan.

Electroweak unification was awarded the Nobel Prize in Physics in 1979. When the paper "Lepton Number as the Fourth Color" as a new black swan emerged and the electroweak interaction was improved. All the cases provided an interesting evidence chain for verifying the discovery of electroweak unification.

Note that every black swan and every white swan is linked an important publication and exist as pairs. The metaphors 'black swan' and 'white swan' provide a statistical pattern for understanding their impact distribution in the context of scientific history.

5. Conclusion

The black-white swan metaphor allows us to identify important scientific discoveries and measure the effects of scientific progress qualitatively and quantitatively. A black swan generates scientific progress and changes old paradigms, a plump black swan having a strong effect and a thin black swan a relatively weak effect. Grey-black swans also generate progress in mainstream science. The quality of scientific discovery is not determined by quantity of publications or number of citations, and the black-white swan metaphor balances quality and quantity and this equilibrium makes the method valuable. In the black and white swan metaphor, the record of scientific discovery (scientific history) provides the foundation, and the scientometric data (citation curves) give quantitative complementary supports.

When white swans meet a black swan, important progress in science occurs and the citation pattern of the white swans changes, and this provides a useful method of identifying and verifying the importance of scientific discoveries.

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Author contributions

J.Z. initiated the idea and collected data, E.P.Q. collected references and assisted analysis, S.S.L processed data and coded program, H.E.S checked the research and wrote the paper, and F.Y.Y designed the research and wrote the paper.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.physa.2017.05.041.

References

[1] N.N. Taleb, Black Swans: The Impact of the Highly Improbable, Penguin Books, New York, 2007.
[2] K.R. Popper, The Logic of Scientific Discovery, second ed., Routledge, London & New York, 2002.
[3] T.S. Kuhn, The Structure of Scientific Revolution, second ed., University of Chicago Press, Chicago, 1970. Enlarged.
[4] E. Garfield, Citation Indexing: Its Theory and Application in Science, Technology and Humanities, Wiley, New York, 1979.
[5] N. Kaplan, The norms of citation behavior, prolegomena to the footnote, Am. Doc. 16 (3) (1996) 179–184.
[6] A. Avramescu, Actuality and obsolescence of scientific literature, J. Am. Soc. Inf. Sci. 30 (5) (1979) 296–303.
[7] S. Redner, Citation statistics for more than a century of physical review, Phys. Today 58 (6) (2004) 49–54.
[8] A.F.J. van Raan, Sleeping beauties in science, Scientometrics 59 (3) (2004) 467–472.
[9] J. Li, F.Y. Ye, The phenomenon of all-elements-sleeping-beauties in scientific literature, Scientometrics 92 (3) (2012) 795–799.
[10] J. Li, D. Shi, Sleeping beauties in genius work, when were they awakened?, J. Assoc. Inf. Sci. Technol. 67 (2) (2016) 432–440.
[11] T. Braun, W. Glänzel, A. Schubert, On sleeping beauties, princes and other tales of citation distributions, Res. Eval. 19 (3) (2010) 195–202.
[12] Q. Ke, E. Ferrara, F. Radicchi, A. Flammini, Defining and identifying sleeping beauties in science, Proc. Natl. Acad. Sci. USA 112 (24) (2015) 7426–7431.
[13] F.Y. Ye, L. Bornmann, "Smart Girls" versus "Sleeping Beauties" in the sciences: the identification of instant and delayed recognition by using the citation angle, J. Assoc. Inf. Sci. Technol. (2017) (in press): https://arxiv.org/pdf/161108849.