

PAPER: Interdisciplinary statistical mechanics

Limit-order book resiliency after effective market orders: spread, depth and intensity

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Abstract. In order-driven markets, limit-order book (LOB) resiliency is an important microscopic indicator of market quality when the order book is hit by a liquidity shock and plays an essential role in the design of optimal submission strategies of large orders. However, the evolutionary behavior of LOB resilience around liquidity shocks is not well understood empirically. Using order flow data sets of Chinese stocks, we quantify and compare the LOB dynamics characterized by the bid-ask spread, the LOB depth and the order intensity surrounding effective market orders with different aggressiveness. We find that traders are more likely to submit effective market orders when the spreads are relatively low, the same-side depth is high, and the opposite-side depth is low. Such phenomenon is especially significant when the initial

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spread is 1 tick. Although the resiliency patterns show obvious diversity after different types of market orders, the spread and depth can return to the sample average within 20 best limit updates. The price resiliency behavior is dominant following aggressive market buy orders, while the price continuation behavior is dominant following less-aggressive market sell orders. Moreover, the resiliency stimulus of buy-sell shock is asymmetrical. The intensities of limit sell orders after market buy orders' shock are always higher than the intensities of limit buy orders after market sell orders' shock. The resiliency behavior of spread and depth is linked to limit order intensity.

Keywords: market microstructure, market impact, quantitative finance, scaling in socio-economic systems

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1. Introduction

Resiliency is an important measure of market liquidity. A market where prices recover quickly after a liquidity shock is defined as a resilient market [1]. Now with the popularity of electronic order-driven market, the definition of resiliency is extended. A limit order book is called resilient when it reverts to its normal shape promptly after trades [2].

Studies on LOB resiliency have been carried out on different time horizons. Many researchers focus on minutely and daily time scales. For the Swedish stock index futures, the shocks to depth are restored in less than 60 min [3]. For the NYSE and the NASDAQ stocks, the resiliency dynamics of volatility, volume and bid-ask spread are examined after experiencing a large liquidity shock [4]. Further, a power-law decay is observed for the resiliency of the bid-ask spread and the volatility [5–7]. In addition, the impact of institutional trading on stock resiliency during the financial crisis of 2007–2009 is also studied from a long horizon perspective [8].

Other researchers analyze market resiliency at order-event scales. Generally, there are two methods to conduct resiliency analysis on this shortest scale, i.e. Hawkes processes and global average measures. The first method views price changes, order submissions and cancelations as a single-variable or multi-variable Hawkes point process. The order-event intensities with a ten-variable point process model are estimated, showing that the order book does not replenish reliably after a large trade in over 60 percent of cases [2]. The trades-through model constructed by a bivariate Hawkes process suggest that the cross-exciting effect of buy and sell trades-through is weak [9]. Hawkes processes can also model the resiliency of high frequency financial price jump events [10]. More interestingly, the separation that how much of price reflexivity is due to endogenous feedback processes can even be quantified by the Hawkes model [11–13].

Although Hawkes process methods are more quantitative, they can only characterize one dimension of liquidity, namely the intensity of order events. In contrast, global average measures can include different dimensions of liquidity. The evolution of depth and spreads as well as the prices and durations at the best prices around aggressive orders is investigated by the average measures [14]. They show that the depth and spread return to their initial levels within 20 best limit updates after the shock. Around large intraday price changes in the Shenzhen stock exchange, the volatility, the volume of orders, the bid-ask spread, and the volume imbalance decay slowly as a power law [15]. Differently, liquidity is found quickly reverting to normal levels after large transactions, i.e. resiliency is high [16]. In addition, all dimensions of liquidity are found to revert to their steady-state values within 15 orders after a very aggressive market order shock, based on a VAR model [17].

We note that these empirical studies focus mainly on aggressive orders, usually defined as the set of market orders that move the best price, like ‘large transactions’, ‘trades-through’ and ‘extreme order events’, etc. This paper contributes to this literature by performing multi-dimensional analysis (bid-ask spread, depth and intensity) of limit order book resiliency around effective market orders. Our research differs in two ways from previous papers. First, we include all effective market orders, not only aggressive orders, but also less-aggressive orders, considering that less-aggressive orders (MB3 orders and MS3 orders below) account for the largest proportion of effective market orders. We will show that the LOB resiliency patterns caused by less-aggressive orders are quite different with patterns caused by aggressive orders. In addition, the price behavior after less-aggressive orders shows even opposite with that after aggressive orders. Second, there is an equilibrium strategy model of an order-driven market showing that the spread is negatively related to the proportion of patient traders and their order arrival rate [18], which indicates a predictable relation between trading intensity and spread and we try to examine empirically the relationship between spread/depth and order intensity.

2. Materials and methods

2.1. Dimensions of LOB resiliency

We describe the limit order book first. The order book right before the t -th event can be described as follows

$$\begin{aligned} \cdots, & b_2, b_1, a_1, a_2, \cdots \\ \cdots, & B_2, B_1, A_1, A_2, \cdots \end{aligned} \quad (1)$$

where b_i and a_i are respectively the i th bid and ask prices and B_i and A_i are the associated volumes at the corresponding quotes.

Kyle defines market liquidity along three dimensions: (i) tightness, ‘the cost of turning around a position over a short period of time’, measured by bid-ask spread $\tilde{s}(t) = a_1(t) - b_1(t)$, (ii) depth, ‘the size of an order flow innovation required to change prices a given amount’, measured by pending volume at the best quotes (B_1 and A_1) if the given amount is 1 tick, and (iii) resiliency, ‘the speed with which prices recover from a random, uninformative shock’ [1]. For order-driven markets, the definition of resiliency is extended as the speed with which the LOB reverts to its normal shape. Hence, LOB resiliency after shocks can be characterized by the evolution of bid-ask spread, depth and intensity, which is defined by the expected number of events in a unit time interval.

2.2. Data sets

Our data sets include the order flow data of 20 A-share stocks and 10 B-share stocks traded on the Shenzhen stock exchange in 2003. The key distinction is that A-shares are denominated in Renminbi and B-shares in Hong Kong dollar. The A-shares market was open only to domestic investors in 2003. The market consists of three time periods on each trading day, namely, the opening call auction, the cooling period, and the continuous double auction period. Here we only consider the order flow occurring in the continuous double auction period (9:30 AM to 11:30 AM and 1:00 PM to 3:00 PM, 240 min for each day).

2.3. Order types

We present the classification of orders. Assume that, right before the arrival of an effective market order, the sequences of prices and volumes on the bid side of the LOB are $\{b_i : i = 1, 2, \dots\}$ and $\{B_i : i = 1, 2, \dots\}$ and those on the ask side are $\{a_i : i = 1, 2, \dots\}$ and $\{A_i : i = 1, 2, \dots\}$, respectively. Without loss of generality, assume that $b_m < \dots < b_2 < b_1 < a_1 < a_2 < \dots < a_n$, where b_m and a_n are respectively the minimal bid price and maximal ask price. These four sequences determine the current status of the LOB right before the arrival of an effective market order.

Consider an effective market order of price π and size ω . This order can be decomposed into two parts, the executed part and the remaining part that is not executed, such that

$$\omega = \omega_e + \omega_r, \quad (2)$$

where ω_e is the size of the executed part and ω_r is the size of the remaining part. When $\omega_r = 0$, all shares of the order are filled. This type of orders is termed filled effective market orders, or *filled orders* for short. When $\omega_r \neq 0$, only part of the order is filled and we can call this type of orders as partially filled effective market orders, or *partially filled orders* for short.

Many empirical studies measure order aggressiveness by the position of the order price relative to that of the latest best quotes [19–22]. However, more precisely, the aggressiveness of an effective market order can also be partly captured by its penetrability. The penetrability p of an effective market order can be defined as the number of levels on the opposite LOB side it consumes:

$$p = \min_j \{ j : \omega_e \leq A_1 + \dots + A_j \} \quad (3)$$

for an effective buy market order and

$$p = \min_j \{ j : \omega_e \leq B_1 + \dots + B_j \} \quad (4)$$

for an effective sell market order. Therefore, effective market orders are orders with $p \geq 1$ and effective limit orders are characterized by $p = 0$. We find that, for an effective market order of penetrability p , $A_1 + \dots + A_{p-1} < \omega \leq A_1 + \dots + A_p$ and $\pi \geq a_p$ for filled buy orders, $\omega > A_1 + \dots + A_p$ and $\pi = a_p$ for partially filled buy orders, $B_1 + \dots + B_{p-1} < \omega \leq B_1 + \dots + B_p$ and $\pi \leq b_p$ for filled sell orders, and $\omega > B_1 + \dots + B_p$ and $\pi = b_p$ for partially filled sell orders.

We utilize the classification scheme consistent with that of [14]. Specifically, the effective buy (sell) market orders are classified into three types based on order price and penetrability. Orders of market buy 1 (MB1) correspond to bid orders with the order size greater than the best ask size and the order price higher than the best ask price. In our terminology, a MB1 order is an effective buy market order whose penetrability p is greater than 1. This means that these orders walk up the limit order book and result in multiple trades. An order of market buy 2 (MB2) is an order with the size greater than the best ask size, but it does not walk up the limit order book above the best ask. In other words, MB2 order is a partially filled order with $p = 1$. Orders of market buy 3 (MB3) are bid market orders whose order size is lower than the best ask size. Speaking differently, MB3 order is a filled order with $p = 1$. Similarly, Orders of market sell 1 (MS1) are effective sell orders with $p > 1$, orders of market sell 2 (MS2) are partially filled sell orders with $p = 1$, and orders of market sell 3 (MS3) are filled sell with $p = 1$. Obviously, MB1/MS1 are for the most aggressive market orders and MB3/MS3 are for the less aggressive ones. Degryse *et al.* focus on the orders of MB1/MS1 and MB2/MS2 [14].

The remaining order types are not executed immediately. The prices of limit buy 1 (LB1) orders are lower than the best ask, but higher than the best bid price, that is, LB1 orders are limit buy orders placed in the spread. The prices of limit buy 2 (LB2) orders are exactly at the best bid. The prices of limit buy 3 (LB3) orders are lower than the best bid. Symmetrically, limit sell 1 (LS1) orders are limit orders placed in the spread; limit sell 2 (LS2) orders are limit orders placed exactly at the best ask; The remaining orders are limit sell 3 (LS3) orders. Figure 1 illustrates all the 6 types of orders on the buy side.

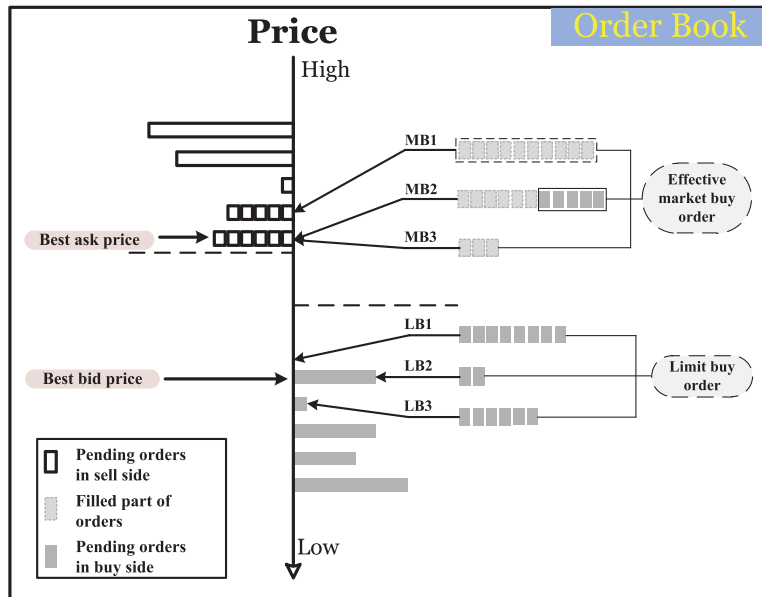


Figure 1. Classification of buy orders. The order prices are pointed by arrows. The number of small rectangles indicates order size.

Table 1 displays the distribution of effective market orders of 30 stocks. It is obvious that the least aggressive (MB3/MS3) orders are most popular for all stocks. To show more detailed order distribution under different liquidity condition, we take one A-share stock (SZ000858) and one B-share stock (SZ200002) for example. Table 2 shows the numbers of orders of all 12 types when placed with different values of bid-ask spreads. We can also confirm that less aggressive orders are most frequency for both effective market orders and limit orders. In addition, for all types of orders except LB1 and LS1, the vast majority are placed when the spread is 1 tick.

2.4. Methods

To quantify the limit order book resiliency, we average the spread, depth and intensity around effective market orders of the same type and compare the results among different types. Before doing the averaging measure, we should remove the intra-day seasonality of these 3 resiliency proxies. For spread \tilde{s} , depth at best bid/ask \tilde{d} and intensity λ , the intra-day seasonality $\{s(\tau), d(\tau), \lambda(\tau) : \tau = 1, 2, \dots, 240\}$ is simply determined by corresponding 1 min mean value of everyday in one year. Note that this paper only pay attention to the intensity of LB1/LS1 and LB2/LS2 orders, considering that the limit order book resiliency after an effective market order is mainly achieved by these 4 types orders.

Take SZ000858 and SZ200002 for example, figure 2 displays the estimated 1 min intra-day seasonality for bid-ask spread, depth, intensity of LB1/LS1 orders and intensity of LB2/LS2 orders. We can observe that the intra-day seasonality of bid-ask spread appears a reversed *J*-shaped pattern, which is consistent with many other limit-order markets. On the contrary, due to the accumulation of limit orders, the volumes at the best quotes show an increasing trend. For the intensities of limit orders in the spread, their seasonality generally show a *U*-shaped pattern, indicating that higher

Table 1. The distribution of effective market orders of 30 stocks.

Stock symbol	Market buy orders			Market sell orders		
	MB1	MB2	MB3	MS1	MS2	MS3
SZ000012	13 459	15 733	110 642	16 017	15 648	105 912
SZ000016	6663	5149	34 162	7152	4916	39 122
SZ000024	7405	8898	80 093	8918	8751	75 053
SZ000027	8886	15 153	120 136	10 646	15 225	101 343
SZ000063	3546	7411	47 484	4076	7313	47 595
SZ000066	11 852	8273	45 978	10 699	10 914	32 382
SZ000088	6990	6467	55 800	6166	7134	32 136
SZ000089	3269	5314	23 466	4041	5350	27 296
SZ000406	5313	6648	31 675	6164	6544	37 622
SZ000429	4036	9577	97 907	4661	9429	82 179
SZ000488	8892	10 213	70 886	4952	9088	28 205
SZ000539	7057	9850	59 980	7715	10 484	62 233
SZ000541	12 383	15 043	118 162	14 023	14 701	109 083
SZ000581	12 586	12 158	64 680	14 968	12 241	62 523
SZ000709	8522	12 095	74 626	9283	11 643	86 369
SZ000720	14 307	16 446	119 671	17 513	16 991	105 435
SZ000778	8008	12 690	73 272	9643	12 467	72 489
SZ000858	6735	9310	47 279	7550	9198	53 509
SZ000917	10 211	11 521	119 354	11 381	10 935	150 466
SZ000983	15 849	10 643	100 559	17 140	10 542	110 732
SZ200002	2520	4539	20 285	2652	4818	19 524
SZ200012	1757	5000	20 015	2088	5083	19 295
SZ200016	1298	3529	11 833	1628	3605	11 497
SZ200024	1901	4381	14 175	2294	4295	14 826
SZ200429	1078	3714	17 610	1457	3694	16 668
SZ200488	4711	9642	45 835	5472	9251	43 923
SZ200539	2953	7056	41 987	3254	6985	41 207
SZ200550	3455	6694	29 156	3898	6703	26 937
SZ200581	1766	2893	7876	1640	2845	10 399
SZ200625	8048	11 738	63 584	8873	11 147	61 980

Table 2. The distribution of 12 different types of orders for stock SZ000858 and SZ200002 with different values of bid-ask spreads.

Type	000 858				200 002			
	$s = 0.01$	$s = 0.02$	$s = 0.03$	$s \geq 0.04$	$s = 0.01$	$s = 0.02$	$s = 0.03$	$s \geq 0.04$
MB1	9346	1981	635	421	1688	467	162	203
MB2	12 568	1801	442	232	3673	598	152	116
MB3	97 038	14 916	3735	2473	16 026	2795	782	682
LB1	0	8302	3370	2957	0	1934	993	1265
LB2	69 485	14 331	4536	3189	12 844	3858	1384	1458
LB3	135 210	32 123	10 145	7281	23 297	7282	2740	3441
MS1	10 593	2311	672	447	1764	507	192	189
MS2	12 063	1935	451	252	3751	716	211	140
MS3	90 510	13 412	3346	1815	15 378	2718	771	657
LS1	0	7520	3065	2487	0	1600	807	1066
LS2	61 318	12 451	3474	2343	11 722	3117	982	1215
LS3	192 156	44 715	13 863	9711	30 682	8913	3352	4088

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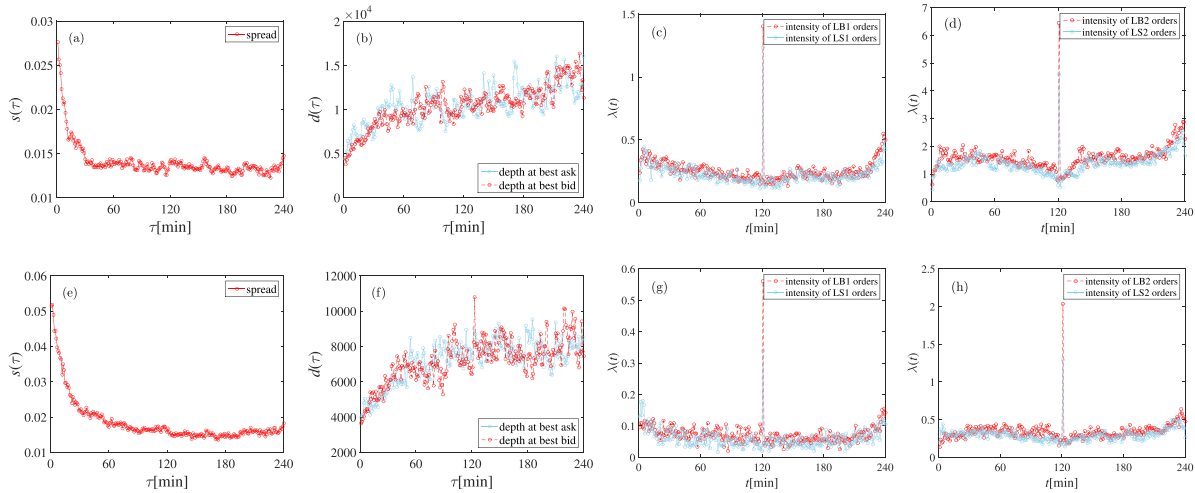


Figure 2. Estimated 1min intra-day seasonality factors for SZ000858 (top panel) and SZ200002 (bottom panel) traded from January to December, 2003. The columns from left to right correspond to the intra-day seasonality patterns of the bid-ask spread $s(\tau)$, the depth $d(\tau)$, the intensity $\lambda(\tau)$ of LB1/LS1 orders, and the intensity $\lambda(\tau)$ of LB2/LS2 orders.

order-in-spread intensity emerge near the opening time and the closing time. Note that the time at 120th min is also a opening/closing time. We also observe that the intensity of buy orders and sell orders fluctuates synchronously.

In order to account for the intra-day seasonality effects, we adjust the spread and depth correspondingly as follows,

$$S(t) = \frac{100\langle\tilde{s}(t)/s(\tau_t)\rangle}{\langle\tilde{s}(0)/s(\tau_0)\rangle}, t = -20, -19, \dots, 19, 20 \tag{5}$$

$$D(t) = \frac{100\langle\tilde{d}(t)/d(\tau_t)\rangle}{\langle\tilde{d}(0)/d(\tau_0)\rangle}, t = -20, -19, \dots, 19, 20 \tag{6}$$

where t means the best limit updates around the effective market order. The best limit updates are defined as the updates of either the best quotes or the depth at these quotes (or a combination of both). Time $t = 0$ corresponds to the spread/depth just before the effective market order. τ_t means the 1min interval in which the t th best limit update occurs. Note that all events will have equal weight in the averaging procedure. This adjustment also includes a normalized process compared with the value at time $t = 0$, and we set the average value as 100 at $t = 0$.

The adjustment for intensity is slightly different:

$$\Lambda([t]) = \langle 2\tilde{\lambda}([t]) / (\lambda(\tau_{[t]}^-) + \lambda(\tau_{[t]}^+)) \rangle, t = -30, -29, \dots, -1, 1, \dots, 29, 30, \tag{7}$$

where $[t]$ presents the t th 1min interval away from the event time $t = 0$. Because $[t]$ may intersect with two neighboring τ -intervals, we average two λ 's of the covered intervals ($\tau_{[t]}^-$ and $\tau_{[t]}^+$) as the intra-day seasonality. Note that, for intensity, neither $\tilde{\lambda}$ nor Λ has definition on $t = 0$, because intensity must be defined over an interval.

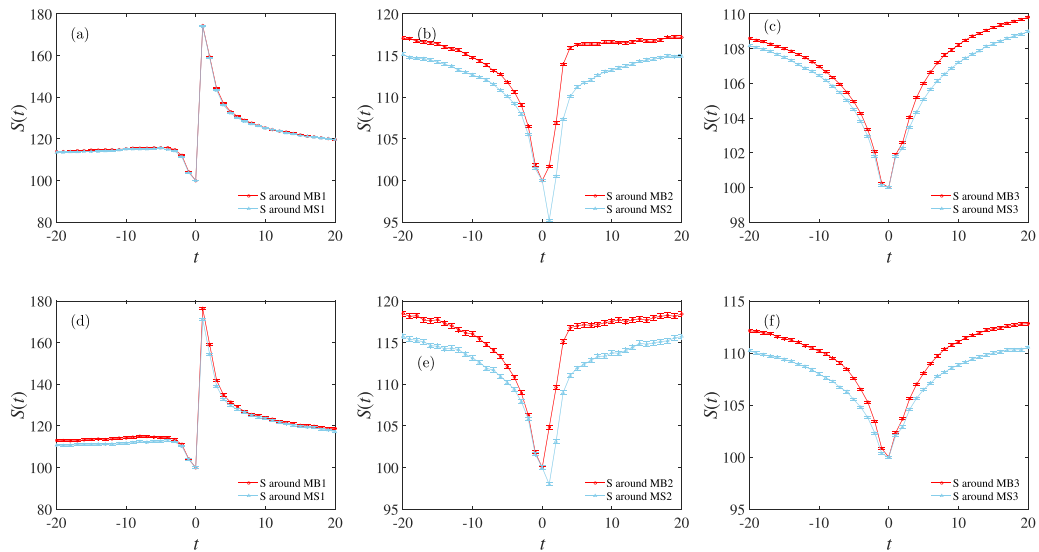


Figure 3. LOB resiliency behavior of bid-ask spread around different types of effective market orders for A-share stocks (top) and B-share stocks (bottom). The MB1 orders and the MS1 orders are in the left panel, the MB2 orders and MS2 orders in the middle panel, and the MB3 orders and MS3 orders in the right panel. Time $t = 0$ corresponds to the status of the LOB right before the arrival of an effective market order.

In the following, the A-share stocks and B-share stocks will be computed separately. All the same type of order events of 20 A-share stocks (10 B-share stocks) will be added up to the average procedure, no matter they belong to the same stock or not. One may consider whether there exist cross-sectional differences among stocks and non-stationeries in time. We give robust checks in section 4.

3. LOB resiliency analysis

3.1. Resiliency of bid-ask spread

Figure 3 illustrates the average resiliency behavior of the bid-ask spread before and after the six types of effective market orders with different aggressiveness. We also display the standard error bars in each plot (the same below). The standard error is calculated by dividing the standard deviation by the square root of number of samples that make up the mean. It reflects the difference between the sample average and the ensemble average. The first feature we can observe is that the resilience behaviors for the A-share stock and the B-share stock are very similar. By definition, the relative spread is 100 right before the effective market orders. We notice that the relative spread $S(0^-)$ is approximately minimal in almost all cases, which indicates that the submission of market orders is more likely when the liquidity is high. What is intriguing is the obvious difference between the resiliency behavior for different types of orders.

In the left panel of figure 3, we show the results for MB1 orders and MS1 orders. Before the microscopic liquidity shock, the bid-ask spread increases slightly and then

decreases. An effective market order of MB1/MS1 consumes at least all the orders on the opposite best and the bid-ask spread soars abruptly to a peak. On average, subsequent orders are less aggressive, be they limit orders or market orders. The spread narrows gradually and relaxes to its normal level after about 20 best limit updates. The evolutionary trajectories of bid-ask spread almost overlap around buy market orders and sell market orders.

In the middle panel of figure 3, we show the results for MB2 orders and MS2 orders. About 20 best limit updates before the microscopic liquidity shock, the bid-ask spread starts to decrease with an acceleration when approaching to $t = 0$. When the shock is from a buy order, the spread starts to resile and comes back to the normal level in a few updates. On the contrary, when the shock is from a sell order, the first subsequent update further narrows the spread and the resiliency begins since the second subsequent update. One may think that this phenomenon of buy-sell asymmetry was due to the asymmetry of the first price gap between the best price and the second best price. However, this is not the fact and the empirical results have shown that there is no obvious buy-sell asymmetry in the distribution of the first price gap [23]. Actually, it is simply due to the fact that the initial bid-ask spreads at $t = 0^-$ before the arrival of MS2 orders are on average larger than those before the arrival of MB2 orders, so that the spread has more space to narrow for MS2 orders. From table 2, we can confirm that the average initial bid-ask spread of MS2 orders is indeed higher than that of MB2 orders. Especially for SZ000858, the MS2 frequencies for $s = 0.02, s = 0.03$ and $s \geq 0.04$ are all higher than the MB2 frequencies. This justify our explanation.

In the right panel of figure 3, we show the results for MB3 orders and MS3 orders. The spread decreases before the microscopic liquidity shock and increases immediately after the shock. The spread at $t = 0$ is only slightly narrower than that at $t = -1$, which is due to the fact that the size of the order at $t = 0^+$ is no larger than the standing volume on the opposite best. The resilience speed after orders of MB3/MS3 is slower than those in the middle panel. This observation is not surprising because the first price gap between the best price and the second best price after orders of MB2/MS2 is much larger than the price gap after the orders of MB3/MS3 [24, 25].

Figure 4 shows the evolution of bid-ask spread around market orders for different spreads at $t = 0^-$. Because the average spread is between 0.01 CNY (1 tick) and 0.02 CNY (2 ticks), $S(t)$ decreases before the market orders at $t = 0$ and increases afterwards, as shown in the left column. For larger $s(0^-)$, the bid-ask spread increases first and then decreases. For $s(0^-) = 0.02$, the market orders further widen the spread and the maximum spread is at $t = 1^-$. For $s(0^-) = 0.03$, the market orders at $t = 0$ have minor impacts on the bid-ask spread and $s(0^-) \approx s(1^-)$. For $s(0^-) \geq 0.04$, the maximum spread is at $t = s(0^-)$ and the market orders usually narrow the spread, indicating that under this case, the greater falls of spread caused by MB2/MS2 orders dominate the spread expansions caused by MB1/MS1 orders on average, even though the absolute number of MB2/MS2 orders is smaller than that of MB1/MS1 orders.

3.2. Resiliency of LOB depth

Figure 5 shows the average resiliency behavior of depth at the best quotes around the six types of effective market orders with different aggressiveness. Both the A-share stock and the B-share stock display similar resilience behaviors. We can find that the same

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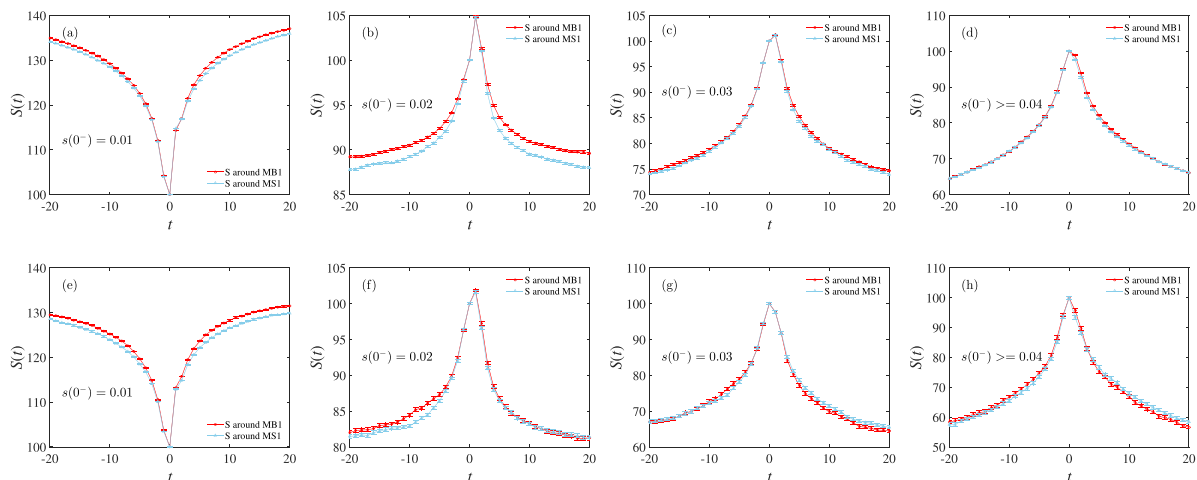


Figure 4. LOB resiliency behavior of bid-ask spread around effective market orders with different initial spreads at $t = 0^-$ for A-share stocks (top) and B-share stocks (bottom). The plots in the four columns from left to right correspond to $s(0^-) = 0.01$, $s(0^-) = 0.02$, $s(0^-) = 0.03$ and $s(0^-) \geq 0.04$. Time $t = 0$ corresponds to the status of the LOB right before the arrival of an effective market order.

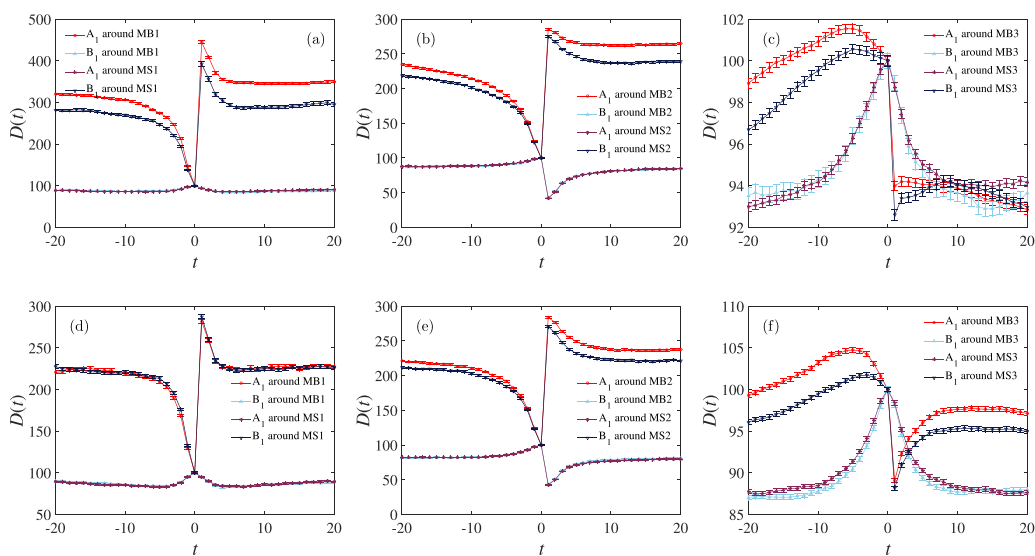


Figure 5. LOB resiliency behavior of depth at the best bid and the best ask around different types of effective market orders for A-share stocks (top) and B-share stocks (bottom). The MB1 orders and the MS1 orders are in the left panel, the MB2 orders and MS2 orders in the middle panel, and the MB3 orders and MS3 orders in the right panel. Time $t = 0$ corresponds to the status of the LOB right before the arrival of an effective market order.

side depth increases and the opposite side depth decreases before time $t = 0$ in almost all cases, which means that effective market orders are more likely to take place when the same side depth is high and the opposite side depth is low. This is due to the consistence of market orders. Like the analysis of bid-ask spread above, what is interesting is the obvious difference between the resiliency behavior for different types of orders.

In the first column of figure 5, we show the results for MB1 and MS1 orders. An effective market order of MB1 or MS1 penetrates at least one price level on the opposite order book and the neighboring level bears the new depth at the best. Empirical analysis has shown that the shape function of limit order book increases first and then decreases with respect to the distance of the price level to the best price for the Chinese stocks [26], which has been also observed in other markets [27–33]. Hence, the new depth on the opposite side right after the market order of MB1 or MS1 is markedly higher than the normal value, while the same-side depth does not change. After the shock, the depths on the same side and the opposite side will reverse to their normal values within about 20 best limit updates.

In the middle column of figure 5, we show the results for MB2 and MS2 orders. These effective market orders also lead to over-resiliency of the opposite depth. This is because the depth on the second best level before the shock becomes the depth on the best price level after the shock. More interestingly, MB2 and MS2 orders shocks can cause over-resiliency at the same side depth. This is because the unexecuted part of the order will reside on the order book forming the new same-side best. The depths at both sides relax to their normal levels after about 20 best limit updates.

In the right column of figure 5, we show the results for MB3 and MS3 orders. The patterns around these two types of orders are generally symmetric. Immediately after $t = 0$, the opposite side depth shows a sharp decline, which reflects the liquidity consumption by an effective market order of MB3 or MS3. For A-share stocks, after the shock of MB3 orders, the opposite side depth continues to decline; while after the shock of MS3 orders, the opposite side depth starts its resiliency slowly. However, this asymmetry does not appear for B-share stocks, that is, the opposite side depths show a reverse pattern consistently after either a MB3 order's shock or a MS3 order's shock. As for the same side depths, they all gradually reverse to their normal levels like other cases.

Figure 6 shows the evolution of depth around effective market orders for different spreads at $t = 0^-$. For the opposite side depths in each plot, they show the combined effects of over-resiliency (MB1, MS1, MB2 or MS2) and sharp declining (MB3 or MS3). However, these combined effects are different for different spreads at $t = 0^-$. For $s(0^-) = 0.01$, they show full-resiliency or partial-resiliency immediately after the effective market orders and then present flat trends. For $s(0^-) > 0.01$, they show over-resiliency immediately after the effective market orders and then present relaxation trends. For the same side depths in each plot, when $s(0^-) = 0.01$, the market orders cause sharp decline; when $s(0^-) > 0.01$, the market orders cause gradually decline. This indicates that orders of MB2 and MS2 are more likely submitted when $s(0^-) = 0.01$, which is consistent with table 2. We should also notice that, the phenomenon that effective market orders take place when the same side depth is high and the opposite side depth is low, can get the best expression when the initial spread is 0.01. In other words, the first column of figure 6 is significantly different from the others, which is also confirmed by figure 4 for spreads and figure 8 for intensity.

3.3. Resiliency of order intensity

The resiliency behavior of bid-ask spread/depth analyzed above can be attributed to the order flow resiliency essentially. For example, is the bid-ask spread resiliency mainly due to the placement of buy limit orders or sell limit orders in the spread?

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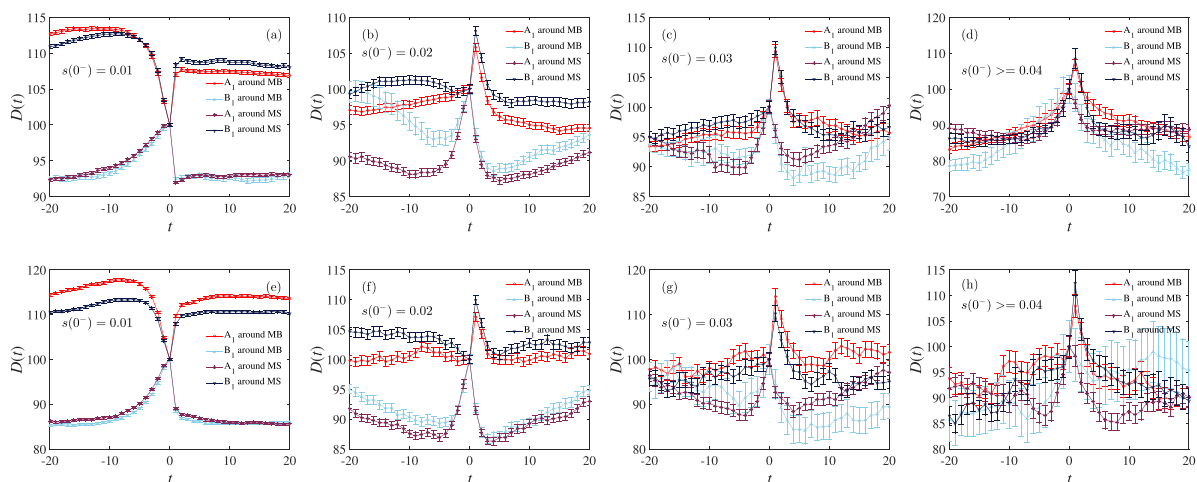


Figure 6. LOB resiliency behavior of depth at the best bid and the best ask around effective market orders with different initial spread at $t = 0^-$ for A-share stocks (top) and B-share stocks (bottom). The plots in the four columns from left to right correspond to $s(0^-) = 0.01$, $s(0^-) = 0.02$, $s(0^-) = 0.03$ and $s(0^-) \geq 0.04$. Time $t = 0$ corresponds to the status of the LOB right before the arrival of an effective market order.

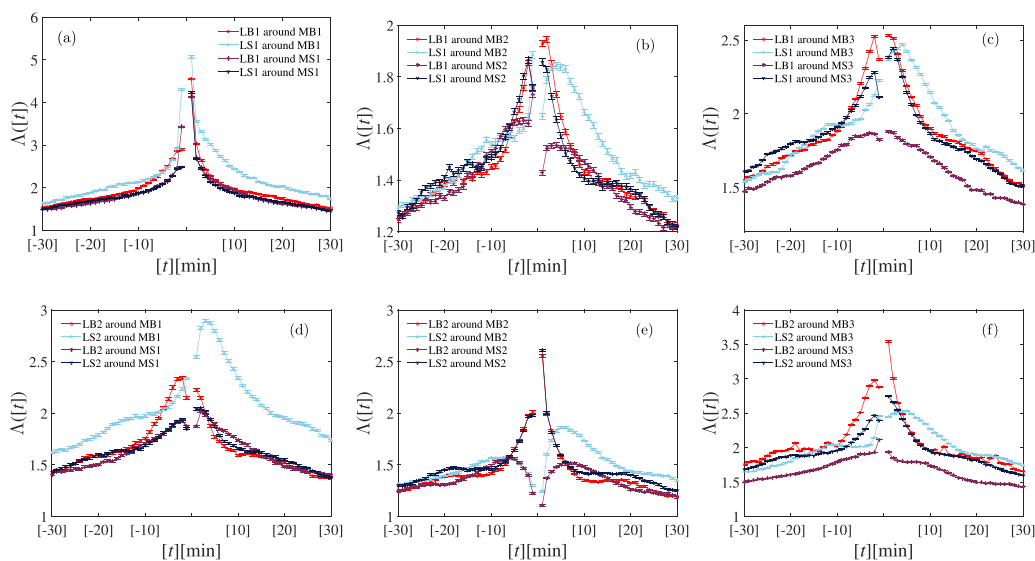


Figure 7. Intensity of limit orders for A-share stocks around effective market orders of different types: MB1/MS1 in left panel, MB2/MS2 in middle panel, MB3/MS3 in right panel. (a)–(c) Intensity of limit orders in the spread (LB1/LS1) around effective market orders. (d)–(f) Intensity of limit orders at the best quotes (LB2/LS2) around effective market orders.

Is the depth resiliency consistent with the intensity of limit orders placed at the best price level? Here we investigate the evolution of limit order intensity around different types of effective market orders to answer these questions. The empirical results for A-share stocks are illustrated in figure 7 for the intensity of limit orders placed in the spread (LB1 and LS1) and limit orders at the best price (LB2 and LS2). The results are very similar for B-share stocks we investigated.

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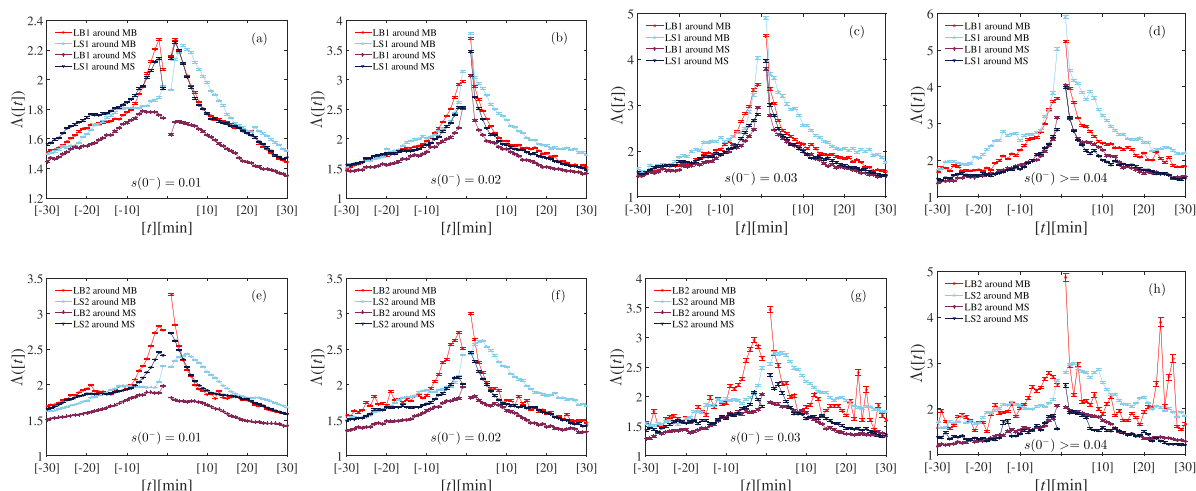


Figure 8. Impact of initial bid-ask spread on the intensity of limit orders around effective market orders for A-share stocks. The initial spreads for the four columns are $s(0^-) = 0.01$, $s(0^-) = 0.02$, $s(0^-) = 0.03$ and $s(0^-) \geq 0.04$, respectively. (a)–(d) Intensity of limit orders in the spread (LB1/LS1) around effective market orders. (e)–(h) Intensity of limit orders at the best quotes (LB2/LS2) around effective market orders. Time $t = 0$ corresponds to the transaction caused by the effective market order.

Figure 7(a) presents the evolution of intensity of limit orders placed in the spread (LB1 and LS1) around MB1 or MS1 orders, while figure 7(d) presents the evolution of intensity of limit orders placed at the best price (LB2 and LS2) around MB1 or MS1 orders. A MB1 order appears usually when the sell limit order intensity increases rapidly and is remarkably higher than the buy limit order intensity. This observation indicates that the most aggressive liquidity takers enter the market when there are more liquidity providers on the opposite side. After the arrival of a MB1 order, the bid-ask spread widens [24]. More limit orders are placed in one or two minutes and the limit order intensity decreases gradually afterwards to its average level within 30 min. Moreover, the intensities of LS1 (and LS2) orders are higher than those of LB1 (and LB2) orders in the after-period of MB1 orders' shock. This means that, sell limit orders contribute more to the resiliency of spread and LOB depth than buy limit orders. Hence the price reversal behavior is dominant after MB1 orders. In addition, the intensity of LB1 orders increases more than LS1 orders right after the MB1 orders, which can be probably viewed as an indicator of herding behaviors among traders. However, around MS1 orders, the intensities of LB1 (and LB2) orders show no obvious difference with the intensities of LS1 (and LS2) orders. This indicates that the most aggressive market sell orders cannot excite more limit buy orders. This can also be reflected by the fact that the best bid depth around MS1 is lower than the best ask depth around MB1.

In figures 7(b) and (e), we present respectively the evolution of intensity of limit orders placed in the spread (LB1 and LS1) and at the best price (LB2 and LS2) around MB2 or MS2 orders. Both intensities of LB1 and LB2 orders increase before MB2 orders. They continue increasing in the first minute after MB2 orders and then decay to the average level within about 30 min, which implies a herding behavior of buyers, similar as LB1 and LB2 orders after MB1 orders shown in figures 7(a) and (d).

The patterns of sell limit orders around MB2 orders are very different. Before MB2 orders, the intensity of LS1 orders increases, while the intensity of LS2 orders increases first and starts to decrease about 10 min before MB2 orders. After MB2 orders, the intensity of LS1 and LS2 orders decreases immediately in the first minute and then increases in the subsequent a few minutes before decaying to the average level. This phenomenon indicates that buy limit orders contribute more to the resiliency of spread than sell limit orders after MB2 orders and the price continues to rise. It is also worth noting that, comparing the intensity of limit orders at at best quotes shown in figure 7(e) with the depth at the best quotes after MB2/MS2 orders in figure 5(b), the limit order intensity play an obvious effect on the bid-ask depth balance since more limit orders are placed to the side with lower depth. The patterns of limit order intensity around MS2 orders are similar.

In figures 7(c) and (f), we illustrate respectively the evolution of intensity of limit orders placed in the spread (LB1 and LS1) and at the best price (LB2 and LS2) around MB3 or MS3 orders. Before MB3 orders, the intensities of buy and sell limit orders increase continuously and buy limit orders have higher intensity than sell limit orders. After the entering of MB3 orders, the intensities of different types of limit orders increase slightly and then decay to the average level within 30 min. In the first a few minute after MB3 orders, buy limit orders have higher intensity than sell limit orders, indicating again that buy limit orders contribute more to the resiliency of spread than sell limit orders and the price continues to rise. The observations for limit order intensity around MS3 orders are qualitatively similar. However, the intensity difference between sell limit orders and buy limit orders after MS3 orders is larger than the intensity difference between buy limit orders and sell limit orders after MB3 orders, suggesting that traders are more sensitive to bad news and have stronger herding behaviors.

Another obvious feature in figure 7 is the asymmetrical stimulus caused by buy and sell market orders. We can find that, the intensities of limit sell orders after market buy orders' shock are always higher than the intensities of limit buy orders after market sell orders' shock, no matter which aggressive level the shock or the limit order belong to. This indicates that the LOB recover faster when it is shocked by market buy orders than by market sell orders.

Plots (a)–(d) of figure 8 show the evolution of intensity of limit orders in the spread (LB1 and LS1) around effective market orders with different initial spreads. Plots (a) and (d) also confirm that market buy orders excite more LS1 orders than LB1 orders, indicating that the price reversal behavior is dominant. However, for market sell orders, the situation is slightly more complicated. For $s(0^-) \geq 0.02$, the intensities of LB1 orders has no obvious different with those of LS1 orders after market sell orders' shock, which is similar with figure 7(a). However, for $s(0^-) = 0.01$, the intensities of LS1 show obvious higher than those of LB1 orders after market sell orders' shock. This indicates the price continuous behavior is dominant. We find that the corresponding intensity curves in figure 8(a) and in figure 7(c) are quite similar, indicating that the price continuation behavior after MS3 orders is especially significant when the initial spread is 0.01 CNY. In addition, the asymmetrical stimulus discussed above is also observed in Plots (a)–(d).

Similar analysis can be performed on the limit orders placed at the best quotes (LB2 and LS2). Plots (e)–(h) of figure 8 show the intensity of limit orders at the best quotes

around effective market orders with different initial spreads. The four plots are roughly the same. Specifically, effective buy market orders attract more buy limit orders at best bid (LB2), while effective sell market orders attract more sell limit orders at best ask (LS2). These observations can be used to explain the price continuation behavior after less-aggressive effective market orders (especially for MS3 orders). Indeed, the intensity curves in figures 8(e)–(h) and in figure 7(f) share similar patterns.

4. Robust checks

The above analyses take the market order event as independent sampling, so that the de-seasoned measures across stocks can be added up to the average process. In this section, we will check the existence of cross-sectional differences among stocks and non-stationeries in time.

Figure 9 shows the cross-sectional tests among 20 A-share stocks for LOB resiliency around effective market orders of different types. We first average the same type of market orders' shocks in one stock, then give average to the 20 A-share stocks. Therefore, the sampling number in computing the standard error is 20. Figures 9(a)–(c) display the resiliency of bid-ask spread, which have the same patterns as figures 3(a)–(c). Figures 9(d)–(f) display the resiliency of depth at the best quotes, which have the same patterns as figures 5(a)–(c). Figures 9(g)–(i) display the resiliency of intensities of limit orders in the spread (LB1/LS1), which have the same patterns as figures 7(a)–(c). Figures 9(j)–(l) display the resiliency of intensities of limit orders at the best quotes (LB2/LS2), which have the same patterns as figures 7(d)–(f). Hence, we can conclude that the resiliency analyses are robust since the standard errors coming from cross-sectional variability do not change the resiliency patterns.

Figure 10 shows the time-evolving tests among 12 months for LOB resiliency around effective market orders of different types. We first divide the same type of market orders of all stocks into 12 parts according to their arrival sequences, and average each part separately. Then we further give average to the 12 parts. Therefore, the sampling number in computing the standard error is 12. Figures 10(a)–(c) display the resiliency of bid-ask spread, which have the same patterns as figures 3(a)–(c). Figures 10(d)–(f) display the resiliency of depth at the best quotes, which have the same patterns as figures 5(a)–(c). Figures 10(g)–(i) display the resiliency of intensities of limit orders in the spread (LB1/LS1), which have the same patterns as figures 7(a)–(c). Figures 10(j)–(l) display the resiliency of intensities of limit orders at the best quotes (LB2/LS2), which have the same patterns as figures 7(d)–(f). Hence, we can conclude that the resiliency analyses are robust since the standard errors coming from non-stationary do not change the resiliency patterns.

It is interesting to compare the different sources of error, i.e. cross-sectional variability and non-stationarity. For spread and depth, the standard errors from cross-sectional variability (figures 9(a)–(f)) seem larger than those from non-stationarity (figures 10(a)–(f)). On the other hand, for limit order intensity, the standard errors from cross-sectional variability (figures 9(g)–(l)) seem smaller than those from non-stationarity (figures 10(g)–(l)). When considering these standard error bars, some of

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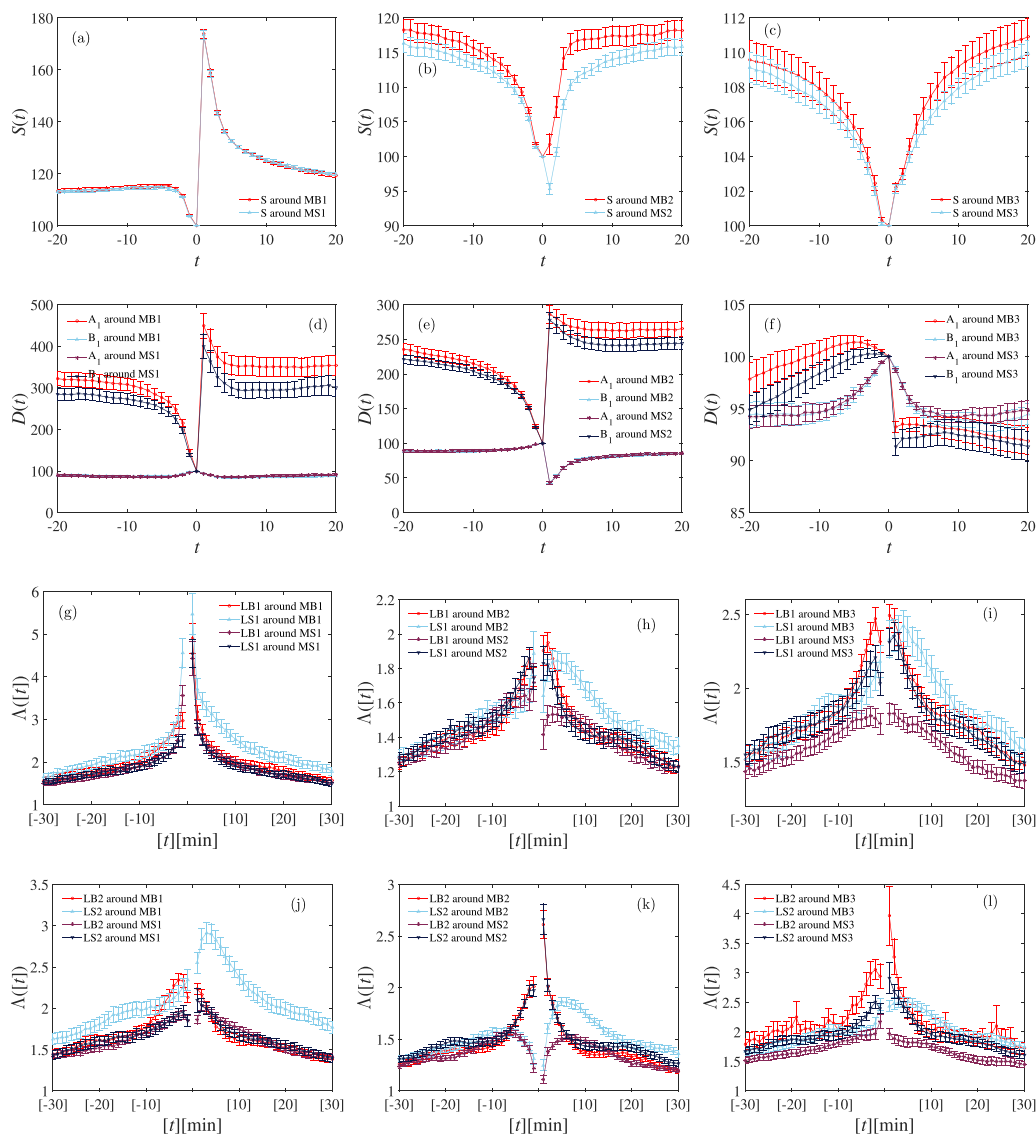


Figure 9. Cross-sectional differences among 20 A-share stocks for LOB resiliency around effective market orders of different types: MB1/MS1 in left panel, MB2/MS2 in middle panel, MB3/MS3 in right panel. (a)–(c) Bid-ask spread around effective market orders. (d)–(f) Depth at the best quotes around effective market orders. (g)–(i) Intensity of limit orders in the spread (LB1/LS1) around effective market orders. (j)–(l) Intensity of limit orders at the best quotes (LB2/LS2) around effective market orders.

the curves indeed overlap with each other. For example, figure 9(c) shows compatible curves and part of curves in figures 10(h)–(l) also show some degree of overlap. However, some significant asymmetries can still be found in both cross-sectional test and non-stationarity test. For example, an MS2 order shock further narrows the first subsequent spread on average and thus its resiliency curve lies obviously below the curve of MB2 (figures 9(b) and 10(b)). After an MB1 market order shock, the intensities of sell limit orders lie obviously higher than those of buy limit orders (figures 9(g) and (j) and 10(g) and (j)).

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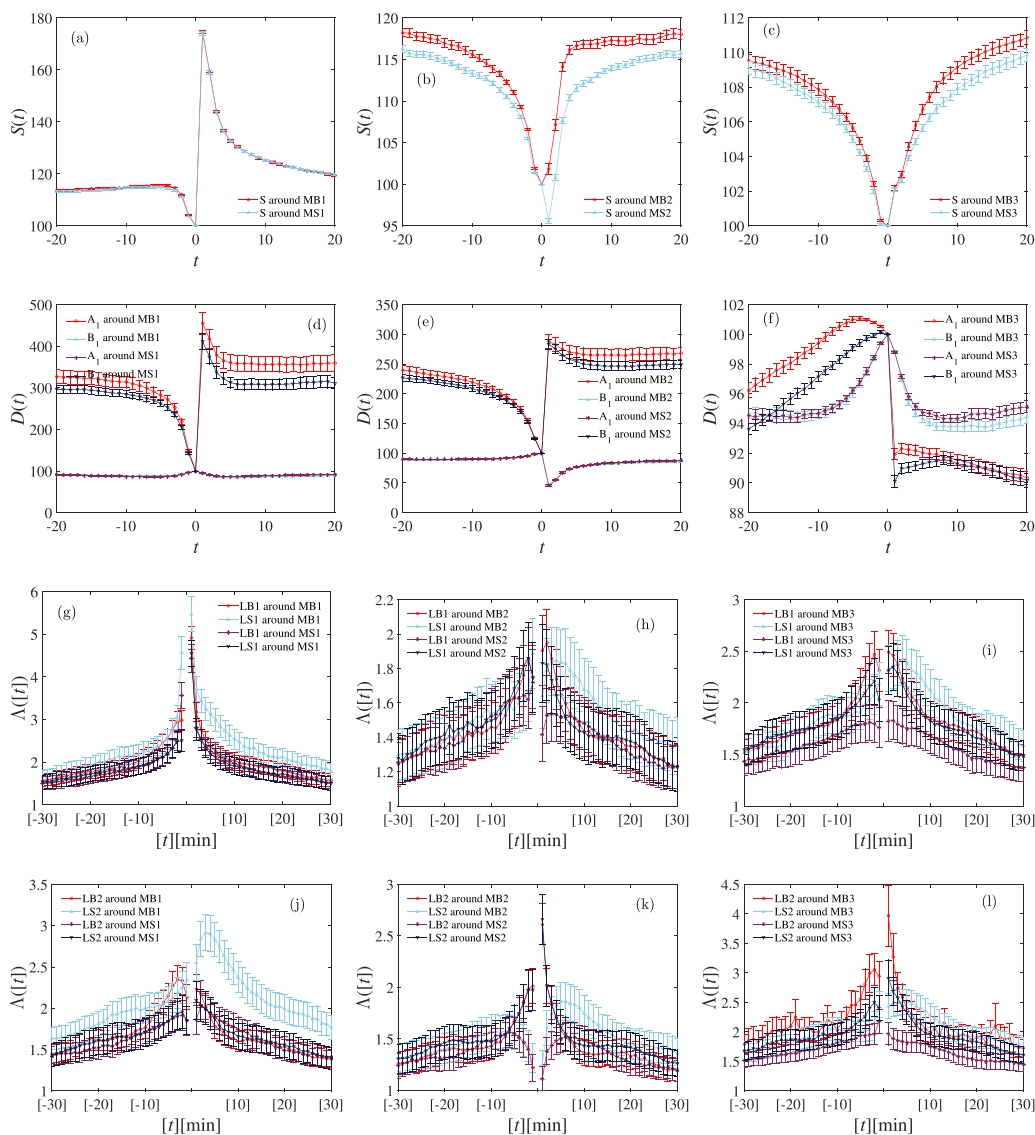


Figure 10. Time-evolving differences among 12 months for LOB resiliency around effective market orders of different types: MB1/MS1 in left panel, MB2/MS2 in middle panel, MB3/MS3 in right panel. (a)–(c) Bid-ask spread around effective market orders. (d)–(f) Depth at the best quotes around effective market orders. (g)–(i) Intensity of limit orders in the spread (LB1/LS1) around effective market orders. (j)–(l) Intensity of limit orders at the best quotes (LB2/LS2) around effective market orders.

5. Conclusions

In this paper, we quantify and compare the resiliency of limit-order book after the submission of different types of effective market orders, using ultrahigh frequency data sets from the Chinese stock market. The orders are classified by their aggressiveness and the resiliency is analyzed based on three dimensions, namely the bid-ask spread, the LOB depth, and the intensity of incoming orders. We adopt a traditional approach to filter the intra-day seasonality of these indicators and then take average of the same type

orders. Our results suggest that the evolutionary consistency between bid-ask spread/depth and order intensity.

First, the relative spread before the arrival of effective market orders is approximately minimal in almost all cases, which indicates that submitting market orders is more likely when the liquidity is high. The resiliency patterns of bid-ask spread show obvious diversity among different types of market orders. However, they all can return to the sample average within 20 best limit updates.

Second, effective market orders are prone to take place when the same side depth is high and the opposite side depth is low. This phenomenon is especially significant when the initial spread is 0.01 CNY (1 tick). After a market order shock, the LOB depth will also recover within about 20 best limit updates. Furthermore, the LOB resiliency behavior of depth is quite symmetric for MB1 order shock versus MS1 order shock, MB2 order shock versus MS2 order shock, and MB3 order shock versus MS3 order shock. There are some other related researches about the relation between probability to see an aggressive order and the imbalance of LOB [34–36].

Third, aggressive market orders do attract more placements of limit orders. After an aggressive buy (MB1) market order shock, sell limit orders contribute more than buy limit orders to the resiliency of LOB (figures 7(a) and (d)). In other words, the price resiliency behavior is dominant after MB1 orders. However, after a relatively less-aggressive market sell (MS3) order shock, limit sell orders contribute more to the resiliency of LOB than limit buy orders (figures 7(c) and (f)). This means that, after MS3 orders, the price continuation behavior is dominant. We also conclude that, the resiliency stimulus of buy-sell shock is asymmetrical. The intensities of limit sell orders after market buy orders' shock are always higher than the intensities of limit buy orders after market sell orders' shock (figure 7).

The analysis of LOB resiliency can be applied to improve the estimation of the transient or permanent price impact [37–40], to solve the optimal trade execution problem more precisely [41, 42], and to design and calibrate computational models for order-driven markets [43–46].

Acknowledgments

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