Comparison between response dynamics in transition economies and developed economies

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In developed economies, the sign of the price increment influences the volatility in an asymmetric fashion—negative increments tend to result in larger volatility (increments with larger magnitudes), while positive increments result in smaller volatility. We explore whether this asymmetry extends from developed economies to European transition economies and, if so, how such asymmetry changes over time as these transition economies develop and mature. We analyze eleven European transition economies and compare the results with those obtained by analyzing U.S. market indices. Specifically, we calculate parameters that quantify both the volatility asymmetry and the strength of its dependence on prior increments. We find that, like their developed economy counterparts, almost all transition economy indices exhibit a significant volatility asymmetry, and the parameter γ characterizing asymmetry fluctuates more over time for transition economies. We also investigate how the association between volatility and volatility asymmetry varies by type of market. We test the hypothesis of a negative correlation between volatility and volatility asymmetry. We find that, for developed economies, γ experiences local minima during (i) "Black Monday" on October 19, 1987, (ii) the *dot-com* bubble crash in 2002, and (iii) the 2007–2009 global crisis while for transition economies, γ experiences local maxima during times of economic crisis.

I. INTRODUCTION

The focus of econophysics is marrying economics to physics by importing concepts and techniques from the latter to the former. However, one must be cautious in doing so, since a number of generalizations that can be taken for granted in physics do not extend to economics.

(i) A physics law discovered in the U.S. presumably holds universally over the entire Earth. By contrast, few expect such country invariance to hold in economics, since economics laws tend to depend on the wealth level of a country. For example, the hypothesis of the weak form of market efficiency [1], which assumes that stock prices at any future time cannot be predicted, holds in many large developed markets [2], even as evidence of violation has been found in ten transition (developing) economies in Eastern and Central Europe [3,4]. Also, highly developed economies [5–8] and those of different levels of aggregation (continents) [9] display power-law probability distributions in their price fluctuations. However, analysis of the Indian National Stock Exchange may instead show exponential distributions [10].

(ii) There is no guarantee that economics laws are time-independent, even for countries of a given level of wealth. Just how time-dependent economics laws are remains under investigation.

We seek here to explore (i) the extent to which economics laws depend on both level of economic development and (ii) time.

II. BACKGROUND

Many complex systems exhibit temporal or spatial correlations that can be approximated by power-law scaling [11–18], and a range of stochastic models [19–23] have been proposed to explain this scale invariance. Recent studies have reported that power-law correlations in empirical data are often characterized by significant skewness or asymmetry in the distributions of increments. Examples include astrophysical data [24], genome sequences [25], respiratory dynamics [26], brain dynamics [27], heartbeat dynamics [28], turbulence [29], physical activities, finance [30–32], and geophysics weather data [33]. Besides power-law correlations in the increments, different complex systems exhibit power-law correlations in the absolute values of increments. Examples include finance [34–36], physiology [37], air temperature changes [38,39], and seismology [40,41]. Applications for this phenomenon are particularly salient in finance because the absolute values measure the level of financial risk.

The autoregressive conditionally heteroscedastic (ARCH) process models financial series with a time-dependent volatility (the standard deviation of price changes) [34]. The time dependence is captured by defining the volatility σ_t at a given time t to be dependent on the previous increments in the series. The question arises of whether σ_t depends not only on the magnitude of preceding increments but also on their sign. Commonly, stockholders may not react equally to bad news (negative price increments) as compared to good news (positive price increments). Many extensions of original ARCH process [34] and its generalization (GARCH [35]) have been subsequently defined in order to incorporate such "volatility asymmetry" [42,43] (see Sec. II), which have revealed such asymmetry in a variety of developed markets [34,43–48].

We ask how universal the phenomenon of volatility asymmetry for global markets is, particularly in transition economies, which often show different statistics than those of developed economies. For example, in contrast to the tendency of financial time series of developed markets to exhibit only short, exponentially decaying autocorrelations in price changes, financial time series of Central and Eastern European transition economies [3,4,49] and even some developed economies [50], exhibit long memory. If the volatility asymmetry exists in transition economies, is it persistent or does it change over time? What can we say about the persistence of volatility asymmetry in developed economies?

Here, we extend the study [48] of volatility asymmetry to Central and Eastern European transition economies using a generalization of the ARCH process, finding that most of the indices under investigation also display statistically significant volatility asymmetry. Surprisingly, we find that such asymmetry is far more pronounced during the 2007–2009 world financial crisis than for the preceding eight years, indicating a greater universality of asymmetric market response during times of economic adversity.

Here we investigate financial time series of index returns of eleven European transition economies of Central and Eastern Europe. We analyze eleven stock market indices—PX, BUX, WIG, RTS, SKSM, SVSM, CRO, NSEL30, TALSE, RIGSE, and PFTS—each corresponding to one of the eleven transition economies of the Czech Republic, Hungary, Poland, Russia, Slovakia, Slovenia, Croatia, Lithuania, Estonia, Latvia, and Ukraine, respectively. As a representative of developed economies, we consider the U.S. stock market. We analyze three financial indices: the S&P500, NYSE, and NASDAQ. All data are recorded daily. We define the relative price change (or "return")

$$R_t \equiv \log S(t + \Delta t) - \log S(t), \qquad (1)$$

where $S(t)$ is the stock price at time t and $\Delta t = 1$ corresponds to a time lag of one day. The increments used in time series are the returns after the average return is subtracted so the resulting series has a mean of 0.

To estimate the parameter γ quantifying the volatility asymmetry of a time series, we employ the maximum likelihood estimation (MLE) method to ascertain which parameter values optimize the probability of a stochastic process to reproduce the observed time series. We start by deriving a likelihood function that is an expression for the probability of observing a given sample of N known data points $(X_1, X_2, \ldots, X_N)$. We denote the probability of obtaining the ith data point X_i as $P(X_i)$. Then the probability L of obtaining our particular N data points is the product of the probability $P(X_i)$ to obtain each

$$L = \prod_{i=1}^{N} P(X_i). \qquad (2)$$

In the paper, we choose a Gaussian for $P(X_i)$ as most of studies do when MLE is employed, though we obtained qualitatively similar results for the Student's *t*-distribution.

III. MODEL

The most widely used volatility processes are based on the ARCH approach. The GARCH(1,1) and ARCH(n) processes, for example, have the volatility, or time-dependent standard deviation, expressed by the squares of the increments, a choice which necessarily loses information because it eliminates the ability to explore if the volatility has any dependence on the sign of an increment. In order to account for the possible asymmetric dependence on an increment's sign, different variants of GARCH processes have been proposed. Here we employ GJR GARCH(p,q) [42], a process that incorporates this asymmetry. In order to model long memory in volatility auto-correlations, the current volatility σ_t depends on p prior volatilities σ_{t-i} and q prior fluctuations ϵ_{t-i},

$$\epsilon_t \equiv R_t - \mu = \sigma_t \eta_t, \qquad (3)$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} (\alpha_i + \gamma T_{t-i}) \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2, \qquad (4)$$

where the return R_t is defined in Eq. (1), t denotes time, μ is the mean of the return R_t, σ_t is the volatility, η_t is random number chosen from a Gaussian distribution with a standard deviation of 1 and mean equal to 0. The coefficients α_i and β_i are determined by MLE and $T_t = 1$ if $\epsilon_{t-1} < 0$, $T_t = 0$ if $\epsilon_{t-1} \geq 0$. The parameter γ is expected to be positive, bad news (negative increments) increases volatility more than good news.

For the sake of simplicity, we follow the common practice by setting $p = q = 1$. For estimation of the parameters we use MLE to solve Eqs. (3) and (4) numerically. We analyze data using several numerical tools to estimate parameters: R, the software package Global Optimization for MATHEMATICA, and the publicly available GRETL package [51]. During the approximately 10.5-year period studied, we calculate for each of the 11 different stock indices of transition economies the parameters of the GJR GARCH(1,1) process of Eqs. (3) and (4).

IV. RESULTS

We present our results in Table I. Based on these results, with the exception of the Lithuanian NSEL30 index, all indices exhibit volatility persistence since the sum $\alpha + \beta$ is close to 1. In addition, we find the asymmetry parameter γ to be statistically significant to within two standard deviations for all indices except the Slovakian SKSM, Ukrainian PFTS, and Estonian TALSE. We find the largest asymmetry parameter γ for the Russian RTS and Lithuanian NSEL30 indices. For all the indices except SKSM (Slovakian) and TALSE (Estonian), we find that γ is positive. We also find that only for PFTS, NSEL30, RIGSE, TALSE and CRO it holds that $\alpha + \beta + \gamma/2 < 1$, implying that only for these indices the second moment of GJR(1,1) exists [52]. Similar behavior was found for some other markets [48].

Besides the procedure explained in Eqs. (3) and (4) with subtracting the average μ from return R_t in order to generate

TABLE I. Estimates of GJR GARCH(1,1) with standard errors in parenthesis.

Index	GARCH $\alpha_1+\beta_1$	α_1	β_1	γ	Log-likelihood
RTS	0.981	0.065 (0.005)	0.905 (0.006)	0.276 (0.034)	−5094
BUX	0.977	0.085 (0.008)	0.885 (0.010)	0.212 (0.030)	−4923
WIG	0.990	0.055 (0.005)	0.932 (0.005)	0.140 (0.035)	−4673
SKSM	0.996	0.037 (0.002)	0.959 (0.002)	−0.021(0.021)	−4318
SVSM	0.962	0.314 (0.016)	0.648 (0.014)	0.105 (0.018)	−2949
PX	0.979	0.116 (0.011)	0.846 (0.013)	0.234 (0.030)	−4520
PFTS	0.931	0.184 (0.009)	0.748 (0.009)	0.015 (0.012)	−5218
NSEL30	0.821	0.184 (0.016)	0.613 (0.021)	0.260 (0.030)	−3365
RIGSE	0.956	0.219 (0.013)	0.738 (0.012)	0.075 (0.022)	−3817
TALSE	0.999	0.098 (0.005)	0.910 (0.003)	−0.016 (0.013)	−3967
CRO	0.950	0.193 (0.014)	0.752 (0.016)	0.076 (0.022)	−2941

fluctuations ϵ_t with zero mean, we also apply the autoregressive moving average ARMA(1,1) model [53] on the time series of return R_t prior GJR GARCH process in order to extract serial correlations in R_t. The final results and conclusions when ARMA(1,1) is applied are very similar to those obtained with the procedure explained in Eqs. (3) and (4).

Next we ask whether the statistical properties concerning volatility asymmetry are homogeneous. For comparison, DNA chains are not homogeneous in correlations in that long-range correlations exist only in intron-containing genes, and not in intron-less genes [54]. Hence, we ask if these statistical properties are more pronounced, for example, during market crashes and economic crisis.

To answer this question, we split the entire period, 12/31/98–07/10/09, into two subperiods: a "control" period, 12/31/98–01/01/07, and a "crash" period, 01/01/07–07/10/09, chosen to coincide with the world financial crisis. Note that in December 2007 a recession began in the United States and in July 2009 it was announced that the recession may have ended. The recession was followed by the global financial crisis. For each of 11 different indices, and for each subperiod, we estimate the GJR GARCH(1,1) process of Eqs. (3) and (4) and present the results for $\alpha+\beta$ and γ in Table II. First we note that the parameter $\alpha+\beta$ changes little during these two subperiods. Four indices—RTS, BUX, PX, and NSEL30—for both subperiods exhibit significant volatility asymmetry. For five other indices—WIG, SVSM, SKSM, RIGSE, and CRO—the control subperiod is characterized by no statistically significant volatility asymmetry, while the crash period is characterized by statistically significant volatility asymmetry. Note that the TALSE and PFTS indices exhibit no volatility asymmetry in either subperiod. We also find that for all indices, except the Russian index, the asymmetry parameter γ estimated for the most recent ≈ 2.5 year period characterized by 2007–2009 global recession and severe market crash is larger than γ estimated for the previous less volatile eight-year period. Thus, the most recent ≈ 2.5 years of the 2007–2009 world financial crisis are characterized by larger and statistically more significant volatility asymmetry than the previous eight years.

Additionally, we compare the persistence of the autocorrelations in the transition economies to that in developed economies by comparing the sum of the parameters $\alpha+\beta$. The smaller this sum, the longer the characteristic lifetime of the auto-correlations. Table II shows that the parameter responsible for persistence in auto-correlations, except for the CRO (Croatia), PFTS (Ukraine), and NSEL30 (Lithuania) indices, does not change much for different subperiods. Note that the β parameter determines the weight applied to the previous volatility, whereas the α parameter determines the weight applied to the most recent news. In contrast to $\alpha+\beta$, the parameter γ, which controls the volatility asymmetry, changes substantially for different subperiods. In Fig. 1 we show how γ estimated annually (by using ≈ 252 daily returns) for different indices changes over time. Figure 1(a) shows the annual variation of γ for representative countries with statistically significant γ for both subperiods. In Fig. 1(b) we show γ vs year for countries with statistically significant γ only for the crash subperiod. In Fig. 1(b) we find that γ substantially changes from positive to negative values.

For reference, we also compare our results to those of well-developed markets, using the S&P 500, NASDAQ, and NYSE Composite indices. Applying our method to the S&P 500 for the most recent 20 years in one-year intervals, in Fig. 1(c) we find that γ varies over time, and is always positive.

TABLE II. For two subperiods 1998/12/31–2006/01/01 and 2007/01/01–2009/07/10 we estimate the GJR GARCH(1,1) process with standard errors in parenthesis.

| Index	$\alpha_1+\beta_1$	γ	$\alpha_1+\beta_1$	γ
RTS	0.946	0.332 (0.049)	0.986	0.259 (0.053)
BUX	0.962	0.177 (0.036)	0.975	0.246 (0.068)
WIG	0.992	0.026 (0.044)	0.959	0.465 (0.201)
SKSM	0.967	−0.075 (0.040)	0.999	0.139 (0.038)
SVSM	0.893	0.015 (0.024)	0.932	0.231 (0.045)
PX	0.957	0.203 (0.042)	0.975	0.257 (0.055)
PFTS	0.865	0.007 (0.017)	0.979	0.025 (0.028)
NSEL30	0.751	0.156 (0.053)	0.692	0.308 (0.061)
RIGSE	0.953	−0.020 (0.026)	0.927	0.275 (0.053)
TALSE	0.999	−0.035 (0.019)	0.999	0.042 (0.024)
CRO	0.788	−0.087 (0.040)	0.979	0.164 (0.041)
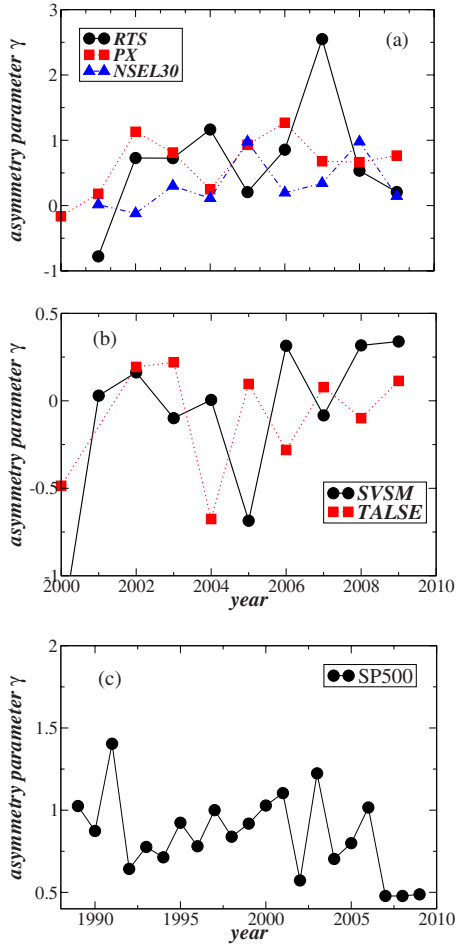
FIG. 1. (Color online) Changes of the volatility asymmetry parameter γ each year over the 20-year period 1989–2009. (a) For transition economies γ for both subperiods (crisis and control) changes over time. (b) The same, but for countries with statistically significant γ only for the crisis subperiod. The parameter γ substantially changes from positive to negative values. (c) As a representative for developed markets, we use the S&P 500 index. Over the last 20 years, γ values vary over time, but γ is always positive. The local minima for γ values we obtain during *dot-com* bubble crash and during the 2007–2009 global crisis.

As an interesting result we find that the smallest γ values occur in 2002 and 2007–2009 corresponding to the *dot-com* bubble crash and current global recession, respectively. We repeat our analysis, this time with two-year intervals on the S&P 500, and we also include the NASDAQ and NYSE Composite indices. Our results are shown in Fig. 2. We find the smallest γ values for 1982–1983 and 1988–1989 periods, proximal to the 1982 recession and Black Monday in 1987, respectively. Restricting ourselves to the last decade, the smallest γ values again occur in the time periods matching the *dot-com* crash and 2007–2009 recession. Due to sample variability, we expect the asymmetry parameter γ to change over time. We find, however, that for the well-known U.S. indices γ tends to decrease during economic crises.

For the two subperiods 12/31/1998–01/01/2006 (subscript 1 in Table III) and 01/01/2007–07–10–2010 (subscript 2 in Table III) we show the standard deviation and the GJR
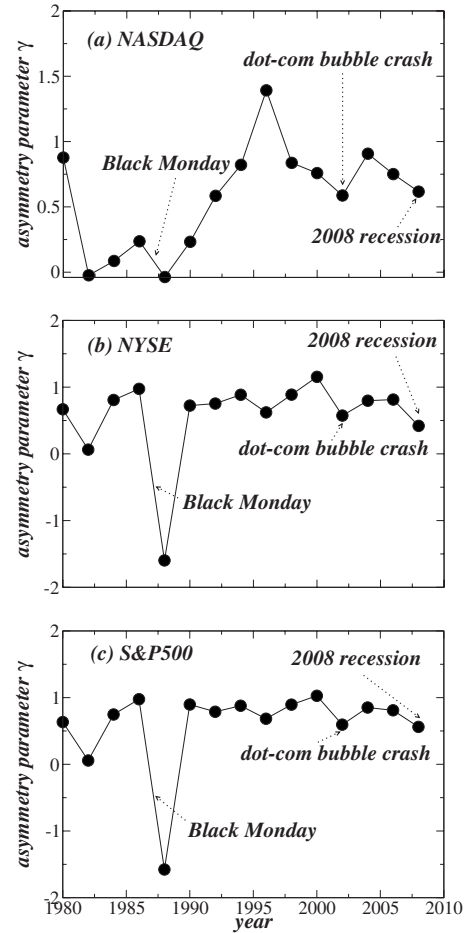

FIG. 2. Changes of the volatility asymmetry parameter γ, calculated every two years over the 27-year period 1980–2008 for three developed markets: (a) NASDAQ, (b) NYSE, and (c) S&P500. Note the local minima for γ values during Black Monday, the *dot-com* bubble crash, and during the 2007–2009 global crisis. The late-2000s recession began in the United States in December 2007.

GARCH(1,1) estimation for γ in Table I. Except for the Russian index (RTSI), in each of the eleven transition markets, an increase in standard deviation is followed by an increase in the volatility asymmetry parameter γ (the opposite of what is found for U.S. indices and the UK FTSE100 index).

Our results contradict the suggestion of Refs. [55,56] that a decrease in volatility implies a decrease in asymmetry. However, our work agrees with Ref. [57], where opposite results were found analyzing Asia-Pacific Stock Index Returns. Reference [57] found that high-volatility regimes (indicated by "fatter" tails returns) are associated with relatively low asymmetry. We therefore are in a position to confirm this finding for the leading U.S. financial indices. The negative association between volatility and asymmetry is obvious during both the *dot-com* bubble crash and the 2007–2009 global recession.

V. DISCUSSION AND CONCLUSIONS

By employing an ARCH-type process, we estimate the level of volatility asymmetry for eleven emerging markets in

TABLE III. For two subperiods 1998/12/31–2006/01/01 (subscript 1) and 2007/01/01–2009/07/10 (subscript 2) we show the standard deviation and the GJR GARCH(1,1) estimation.

Index	σ_1	σ_2	γ_1	γ_2
RTSI	1.469	3.002	0.3651	0.2547
BUX	1.467	2.155	0.1977	0.2278
WIG	1.357	1.723	0.0173	0.4994
SKSM	1.312	1.076	−0.0832	0.0972
SVSM	0.657	1.573	0.0026	0.4255
PX	1.242	2.236	0.2232	0.2782
PFTS	1.660	2.164	−0.050	0.0631
NSEL30	0.835	1.779	0.2337	0.3461
RIGSE	1.555	1.570	−0.0531	0.3251
TALSE	1.078	1.431	−0.0558	0.0891
CRO	1.130	2.060	−0.1078	0.2053
DOWJ	1.069	1.794	1.0170	0.4410
SP500	1.110	1.968	0.9802	0.4475
FTSE100	1.125	1.794	1.3776	0.4966

Central and Eastern Europe. Such volatility asymmetry is important in finance because the greater the volatility asymmetry, the greater the risk associated with owning a given stock during market crashes and economic crises. For each index, except the Russian index, we find that the level of volatility asymmetry is more pronounced during the 2007–2009 world financial crisis than for the preceding eight years. This result is in contrast with what we find for the S&P 500,

NASDAQ, and NYSE Composite indices, for which the smallest "local" γ values are immediately following Black Monday, during the 2002 *dot-com* bubble burst, and following the 2007–2009 global recession, implying negative association between volatility and volatility asymmetry, in contradiction to common economic belief. Thus, we find some elements of universality in the markets: both the developed markets and the majority of the transition markets display significant volatility asymmetry, but the effect of market crises on this asymmetry is qualitatively different for transition markets as compared to developed markets.

A variety of studies report that besides finance, different complex systems ranging from physiology [37] to seismology [40,41] generate time series of increments, the magnitudes of which are power-law correlated. Asymmetry in power-law magnitude correlations were first found in finance and more recently in physiology [58]. Hence, we can expect similar behavior in brain dynamics, seismology, hydrology and generally in physics phenomena where time series are studied. In these phenomena the present analysis may have the potential to be useful for diagnostic purposes. In physiology this type of analysis may prove important in distinguishing between diseased and healthy subjects.

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