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Time-varying lead–lag structure between the crude oil spot and futures markets

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HIGHLIGHTS

- The lead–lag structure between the crude oil spot and futures markets is investigated at three different time frequencies.
- Symmetric thermal optimal path and self-consistent test methods are applied to determine the lead–lag relationship.
- There exists an alternate lead–lag structure instead of a dominance between the two markets.

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ABSTRACT

The linkages between crude oil spot and futures markets are widely studied in previous literatures. Most of these conventional methods are static, linear and parametric. In order to overcome this difficulty, we employ the nonparametric and nonlinear symmetric thermal optimal path (TOPS) method to determine the time-dependent lead–lag relationship between two time series. Based on the daily, weekly, monthly spot and futures returns for maturities of one, two, three and four months of the West Texas Intermediate (WTI) crude oil benchmark, we apply the TOPS method together with self-consistent test to investigate the time-varying lead–lag dependences between spot and futures markets from 1987 to 2017. We find that, for total three data sets with different frequencies, the TOPS paths indicate an alternate lead–lag structure instead of a dominance between spot and futures markets in most of the time periods. For the daily data, the lead–lag relationship between the two markets exists within one day except on some specific days. Moreover, without considering the units of time, on average, the lead–lag steps between the two markets are diminishing with the increasing frequency. Roughly speaking, these periods with strong lead–lag signals coincide with major influential changes in the oil and stock markets and the geopolitics, which can reflect real economic, financial and geopolitical regime changes. All in all, the results show that the lead–lag relationship between the spot and futures oil markets exists only temporarily.

1. Introduction

It is a widely accepted notion that the crude oil spot and futures markets play an extremely crucial role in the economic and financial systems and our modern industrial society [1,2]. The price discovery is significant in understanding the

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interaction patterns between the spot and futures markets [3]. Theoretically speaking, futures prices should neither lead nor lag the spot prices from the perspective of the Efficient Markets Hypothesis (EMH) [4,5]. Nevertheless, financial markets are complex and exist frictions. Thus, there usually exist diversified lead–lag relationships among financial markets [3,6–10]. Hence, the controversial lead–lag relationship between the spot and futures markets forms an enduring area of research [8]. Besides several new techniques [9,11,12], the combination of Granger causality test [13], cointegration test and vector error correction model [14–16] are conventionally adopted to investigate lead–lag relationship in economic or financial markets. Using different methodologies, researchers have reported diverse and inconsistent results in analyzing the interaction relationship among futures and spot prices in various markets and different time periods.

Extensive studies conducted on the empirical relationship between the spot and futures markets demonstrate the view that futures prices lead spot prices. Concretely, Bopp and Sitzer [17] found near-term futures prices added information to the heating oil cash prices forecasting process. Schwarz and Szakmary [18] analyzed the lead–lag relationship between spot and futures prices of the light sweet crude oil, No. 2 heating oil and unleaded gasoline from 1985 to 1991. They strongly suggest that futures dominate in price discovery in all three petroleum product markets. Likewise, Ng and Pirrong [19] have investigated the price dynamics in refined petroleum spot and futures markets, favoring the standpoint that futures market facilitates the flow of information to the spot market. Silvapulle and Moosa [20] examined the lead–lag relationship between the spot and futures prices of WTI crude oil using a sample of daily data. The linear causality test suggests that futures prices lead spot prices. Chiou-Wei et al. [21] revealed that prices react first in the futures market. By using 5-min data, Chan [6] presented strong evidence that the futures market leads the cash index market. Similarly, by using 10-min observations from June 1996 to June 1997, Brooks et al. [22] demonstrated a short-run interaction patterns that lagged changes in the index futures prices can help to predict changes in the FTSE 100 index prices. Further, Kang et al. [23] explored the lead–lag relations among the KOSPI200 spot market, the KOSPI200 futures market, and the KOSPI200 options market employing 5-min data, from 1 October 2001 to 30 December 2002. Their results indicate that the KOSPI200 futures and options markets lead the KOSPI200 spot market by up to 10 min in terms of returns. Analogously, using 1-min high-frequency returns, Wang et al. [3] studied the lead–lag dependence between the CSI 300 index spot and futures markets from 2010 to 2014. Their findings suggested that there exists a price discovery in the Chinese futures market.

In contrast, Quan [24] indicated that the crude oil spot price generally leads the futures price in incorporating new information, while the crude oil futures price does not play a very important role in the price discovery process. Lien et al. [11] identified the structural changes in the Nikkei spot index and futures price adopting a nonparametric genetic programming approach. Their works show that major market changes originated from the spot market and spread over to the futures market. By using monthly observations of WTI crude oil spot and futures prices, Moosa and Al-Loughani et al. [25] demonstrated that futures prices are not efficient forecasters of spot prices. Ivanov et al. [26] analyzed the ETF, futures and spot markets of different indexes in the United States from 2002 to 2011, indicating that the spot market rather than the futures market leads the price discovery. Chen and Gau [27] find that, after the minimum tick size in the stock market decreases, the contribution of the spot market to price discovery increases. Based on the intraday tick-by-tick observations, from April 4, 2005 to July 29, 2005, the spot market is found to consistently lead the price discovery process for both Euro and Japanese Yen exchange rates during the sample period [28]. Moreover, empirical evidence for Poland shows that the introduction of index futures trading does not destabilize the spot market [29]. Likewise, lagged changes in the spot market plays a more dominant role in the China and Thailand markets [30,31]. Additionally, there exist literatures demonstrating a bidirectional relationship or evidence against a unidirectional relationship between the spot and futures markets [8,16,32–36].

Many scholars indicated that the direction of causality is sensitive to the choice of the sample period [8,20,33,37,38]. For instance, by separating almost 22 years of daily WTI crude oil futures and spot price data into three periods based on certain important events, Huang et al. [38] revealed that the lead–lag relationship depends on the sign and magnitude of the basis and exist difference in the three subperiods. Correspondingly, an increasing number of research unveils that the interaction between the spot and futures prices including oil prices is nonlinear and time-varying [20,33,38]. Specifically, Silvapulle and Moosa [20] uncovered a bidirectional relationship between the spot and futures prices by adopting the nonlinear causality test. Bekiros and Diks [33] have investigated the nonlinear causality between the daily spot and futures prices of the WTI crude oil. Their findings indicate that if the nonlinear effects are accounted for, the pattern of leads and lags changes over time. Moreover, they found volatility effects may partly account for nonlinear causality. Recently, Balcilar et al. [8] using a Markov-switching vector error correction model and the daily spot and futures prices of WTI crude oil from 1986 to 2013, find that the causal links (unidirectional causality, bidirectional causality and non-causality) between the spot and futures oil markets are strongly time-varying and exist only temporarily. In addition, the mixed time-varying lead–lag relationship between WTI crude oil spot and futures markets can also be tracked in other studies [25,37,39].

Considering the evidence on both the nonlinear and time-varying lead–lag relationship between the spot and futures markets, we re-examine the dynamic lead–lag relationship between WTI crude oil spot and futures markets using an improvement (TOPS) [9] of the novel thermal optimal path (TOP) technique [12,40,41] for the joint analysis of two time series. Compared with TOP (with time-forward weights) method, TOPS (with time-reversed symmetric weights) method presents greater consistency and stability which was confirmed with abundant synthetic tests by Meng et al. [9]. Contrast to other popular methods, TOPS/TOP is a non-parametric approach and does not require the stationarity of time series. In particular, the TOPS method is able to efficiently unveil nonlinear correlations of time series based on a time-dependent lead–lag structure [9]. Correspondingly, combining TOPS method with self-consistent test, Meng et al. [9] uncovered the special time-varying lead–lag structures of house price and monetary policy of the United Kingdom and United States from 1991 to 2011. They found that, for both countries, the TOPS paths unveil three common phases and indicate that interest rate changes are lagging behind house price index changes until the crisis in 2006–2007.

In general, there exist three important crude oil markets, including the WTI, the European Brent and the Organization of Petroleum Exporting Countries (OPEC). Meanwhile, the prices in the three markets are related to one another, so any of the three markets can indicate what happens at the international oil markets. Currently, we concentrate on the time-varying lead–lag relationship between the spot and futures markets of WTI crude oil which is used as an indicator of world oil prices and is the underlying commodity of the New York Mercantile Exchange's (NYMEX) oil futures contracts.

The paper is organized as follows. Section 2 summarizes the original TOP method, together with the improved TOPS method. Section 3 describes the data used and presents preliminary analysis. Section 4 presents applications of TOPS method to the identification of the time-varying lead–lag structures between spot and futures markets of WTI crude oil. Section 5 discusses and summarizes.

2. Method

2.1. The symmetric thermal optimal path method

The TOPS method is an improvement of the TOP method [12,40,41], which is proposed as a novel method to quantify the dynamical evolution of lead–lag structures between two time series [9]. The idea of TOPS method is as follows. Suppose we have two standardized time series $X(t_1) : t_1 = 0, \ldots, N-1$ and $Y(t_2) : t_2 = 0, \ldots, N-1$. Note that the TOPS method also applies to situations where two time series have different lengths. We first form an $N \times N$ distance matrix $E_{X,Y}$ that allows us to compare the distances systematically between all the values of $X(t_1)$ with $Y(t_2)$ along the time axis. The elements of the distance matrix $E_{X,Y}$ are defined as

$$\epsilon(t_1, t_2) = |X(t_1) - Y(t_2)| \ . \tag{1}$$

The dependence structure between the two time series is obtained by searching for a one-to-one mapping

$$t_2 = \phi(t_1) \tag{2}$$

between the times $X(t_1)$ and $Y(t_2)$ such that the two time series are closest in some sense. In other words, we have intuitively

$$\phi(t_1) = \min_{t_2}\{\epsilon(t_1, t_2)\}. \tag{3}$$

To eliminate unreasonable large jumps or contradicting causality, Sornette and Zhou [12] replace the local minimization by a global minimization

$$\min_{\{\phi(t_1), t_1=0,1,\ldots,N-1\}} E := \sum_{t_1=0}^{N-1} |X(t_1) - Y(\phi(t_1))| \tag{4}$$

under the constraint of continuity expressed in discrete time

$$0 \le \phi(t_1 + 1) - \phi(t_1) \le 1 \ . \tag{5}$$

which imposes that the mapping $t_1 \to t_2 = \phi(t_1)$ should be continuous in the continuous time limit.

Sornette and Zhou [12] solve Eq. (4) for the TOP method accurately. They transform the coordinates (t_1, t_2) to (t, x) as follows

$$\begin{cases} t = t_2 + t_1 \\ x = t_2 - t_1, \end{cases} \tag{6}$$

and determine the optimal thermal path $\langle x(t) \rangle$ by

$$\langle x(t) \rangle = \sum_x x W(t, x)/W(t), \tag{7}$$

where $W(t, x)$ is the local weight factor and

$$W(t) = \sum_x W(t, x). \tag{8}$$

Here, $W(t, x)/W(t)$ is the probability for a path to be at position x at time t. To maintain the direction of time required by 'causality', a feasible path arriving at $(t_1 + 1, t_2 + 1)$ can come from $(t_1 + 1, t_2)$ vertically, $(t_1, t_2 + 1)$ horizontally, or (t_1, t_2) diagonally. Therefore, the local weights at (t, x) can be determined in a recursive way as follow,

$$W(t + 1, x) = [W(t, x - 1) + W(t, x + 1) + W(t - 1, x)]e^{-\epsilon(t+1,x)/T}, \tag{9}$$

where T is a parameter controlling the impact of noise.

Meng et al. [9] notice that the probability for a path to be present on a given node should be independent on whether the path is determined recursively from left to right (past to future) or from right to left (future to past). The breaking of time-reversed symmetry in the TOP method can be interpreted as the different numbers of paths contributing to the probability attributed to a given node when calculated from left to right compared from the construction of the probability on that same node from right to left [9]. To remedy this issue, they propose the TOPS method by employing a time-reversed invariant node weight process. They perform the TOP method from left to right to obtain $\overrightarrow{W}(t,x)$ and from right to left to obtain $\overleftarrow{W}(t,x)$ and the TOPS path is modified to

$$\langle x(t)\rangle = \sum_x x \frac{\overrightarrow{W}(t,x)/\overrightarrow{W}(t) + \overleftarrow{W}(t,x)/\overleftarrow{W}(t)}{2}, \tag{10}$$

where the arrow $\rightarrow$ denotes that the recursive weight process is along the time-forward direction and the arrow $\leftarrow$ denotes that the recursive weight process is along the time-backward direction.

In addition, it is worth noting that neither the TOP method nor the TOPS method address the fundamental philosophical and epistemological question of the genuine causality links; rather, they attempt to detect a dynamics lead–lag dependence relationship between two time series $X(t_1)$ and $Y(t_2)$ [9,10,12]. Actually, even the Granger causality does not address this subtle question [42,43].

2.2. Self-consistent test of the lead–lag structure

We here describe the self-consistent test for the lead–lag structure $\langle x(t)\rangle$ identified by the TOPS method. The underlying logic of the test is that, if the lead–lag path $\langle x(t)\rangle$ is significant, synchronizing the two time series $X(t)$ and $Y(t)$ using the time-varying $\langle x(t)\rangle$ should lead to a statistically significant correlation, or at least lead to a statistically higher correlation than for the non-synchronizing case [9,10]. In other words, $X(t-\langle x(t)\rangle)$ and $Y(t)$ should be synchronized and exhibit a strong linear dependence. It leads to the following regression

$$Y(t) = c + aX(t-\langle x(t)\rangle) + \varepsilon(t), \tag{11}$$

where the coefficient a should be significantly different from 0 for a statistically significant dependence [9,10].

3. Data description and preliminary analysis

Our analysis is based on the daily, weekly and monthly price time series for the WTI crude oil spot and futures traded in NYMEX, which are freely available at the web site of the US Energy Information Administration. For convenience, the spot price of WTI was denoted by WTIS thenceforth. Explanatorily, the WTI futures prices are for maturities of one, two, three and four months, which are quoted for delivering a specified quantity of WTI at a specified time in the future. The NYMEX oil futures market exposes crucial price information to buyers and sellers around the world, although few NYMEX crude oil futures contracts are executed for physical delivery [33]. Similarly, we denote the prices of four WTI crude oil futures with different maturities as WTIF1, WTIF2, WTIF3, and WTIF4, respectively.

The data sets cover the period from April 23, 1987 to October 10, 2017 in daily frequency, from April 17, 1987 to October 6, 2017 in weekly frequency and from March 15, 1987 to September 15, 2017 in monthly frequency. In total, the observations for daily, weekly and monthly spot and futures oil prices time series are 7639, 1591 and 314, respectively. For the subsequent analysis, logarithmic returns are used for the analysis of the daily, weekly and monthly data sets. The returns are defined as continuously compounded returns as follows:

$$r(t) = \ln[P(t)/P(t-1)] \tag{12}$$

where $P(t)$ is the asset price (WTIS or WTIF1, WTIF2, WTIF3, WTIF4) at time t. We shall perform the TOPS analysis on the stationary returns series of WTI crude oil spot and futures.

Fig. 1 illustrates the evolution of the price and return of the WTI crude oil spot and futures. As shown in Fig. 1, there exist abundantly large negative returns throughout the sample period, which implies the occurrence of regime shifts during the specific period [8,44]. These large fluctuations in prices or returns are corresponding to several significant economic, financial and geopolitical events, such as the Gulf war of 1990–1991, the East Asian crisis in 1997–1998, the 9–11 attack of 2001, the Iraq war in 2003, the 2007–2009 demand hike and a series of decisions made by OPEC in the following years. One can find events that disrupt supply or increase uncertainty about future oil supplies and tend to drive the prices up. Moreover, volatility clustering is easily observed from return series in Fig. 1.

We apply the Augmented Dickey–Fuller (henceforth ADF) unit root test referred in [45] to the crude oil spot and futures price series to investigate their stationarity. For the ADF tests, we select the lag length k, using the Bayesian Information Criterion (BIC) with the maximum lag length set to 30. Moreover, the ADF tests we apply here have two different model variants in the regression equation: one contains a constant, while another contains a constant and a linear trend. The results are presented in Table 1. The p-value reported in Table 1 for the ADF tests indicate that the null hypothesis of a unit root cannot be rejected at the 1% significance level for all the price series with different sample frequencies and can

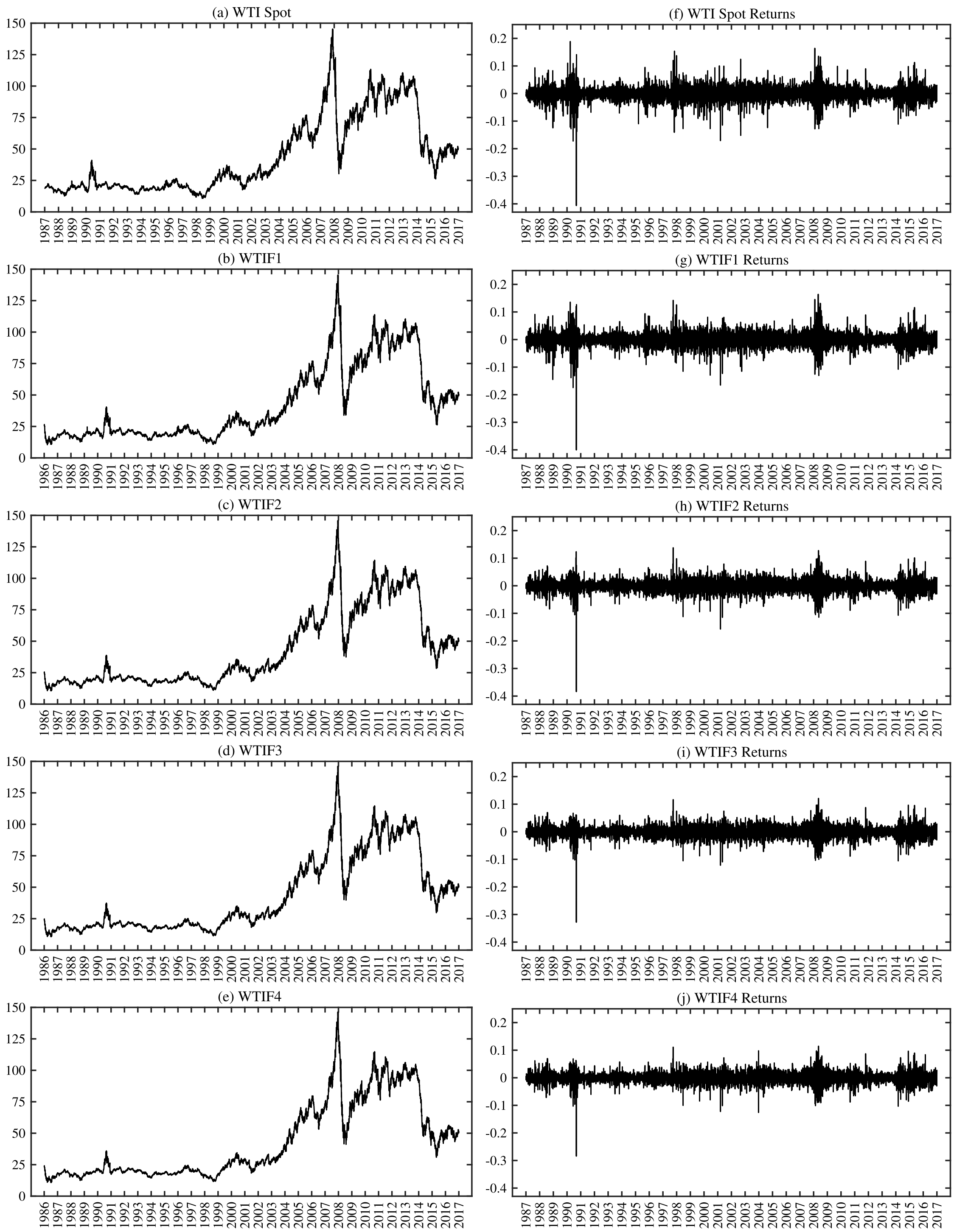

Fig. 1. The evolution of the price and return of the WTI crude oil spot and futures. Notes: In the left column (a–e), we plot the daily WTI spot and futures prices from April 23, 1987 to October 10, 2017. In the right column (f–j), we plot the log returns of the corresponding crude oil prices in the left panels.

be rejected at the 1% significance level for all the return series, regardless of whether or not the linear trend is included in the deterministic component. Hence, we analyze the return time series in the rest of this work.

Table 1
Descriptive statistics for the WTI crude oil spot and future price series and returns.

	Daily					Weekly					Monthly				
	WTIS	WTIF1	WTIF2	WTIF3	WTIF4	WTIS	WTIF1	WTIF2	WTIF3	WTIF4	WTIS	WTIF1	WTIF2	WTIF3	WTIF4
Panel A: Log prices															
Observations	7639	7639	7639	7639	7639	1591	1591	1591	1591	1591	314	314	314	314	314
$\text{ADF}_{p-\text{value}}$ [b]	0.411	0.426	0.499	0.536	0.560	0.493	0.502	0.533	0.562	0.586	0.491	0.495	0.526	0.547	0.564
$\text{ADF}_{p-\text{value}}$ [c]	0.218	0.263	0.416	0.499	0.549	0.407	0.438	0.510	0.577	0.634	0.438	0.446	0.506	0.544	0.580
Panel B: Log returns															
$\text{ADF}_{p-\text{value}}$ [b]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\text{ADF}_{p-\text{value}}$ [c]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean $\times 10^2$	0.013	0.013	0.013	0.014	0.014	0.064	0.064	0.065	0.066	0.067	0.266	0.266	0.270	0.273	0.276
Maximum	0.189	0.164	0.138	0.121	0.115	0.251	0.198	0.167	0.152	0.138	0.214	0.215	0.194	0.182	0.170
Minimum	−0.406	−0.400	−0.384	−0.328	−0.284	−0.192	−0.190	−0.210	−0.184	−0.174	−0.332	−0.312	−0.293	−0.291	−0.289
Std Dev	0.025	0.024	0.021	0.020	0.019	0.042	0.041	0.037	0.035	0.033	0.082	0.081	0.076	0.072	0.069
Skewness	−0.694	−0.719	−0.868	−0.727	−0.610	−0.161	−0.290	−0.392	−0.403	−0.413	−0.661	−0.626	−0.675	−0.716	−0.743
Kurtosis	17.441	17.498	19.565	15.189	12.533	5.908	5.449	5.430	5.292	5.310	4.497	4.319	4.231	4.333	4.484
$\text{JB}_{p-\text{value}}$ [a]	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Panel C: Correlation matrix of returns															
WTIS	1.000					1.000					1.000				
WTIF1	0.902	1.000				0.964	1.000				0.999	1.000			
WTIF2	0.879	0.952	1.000			0.940	0.975	1.000			0.989	0.992	1.000		
WTIF3	0.869	0.934	0.992	1.000		0.920	0.952	0.994	1.000		0.978	0.982	0.997	1.000	
WTIF4	0.857	0.917	0.980	0.993	1.000	0.904	0.934	0.984	0.997	1.000	0.968	0.972	0.991	0.998	1.000

Notes:
[a]JB is the Jarque–Berra test of normality, which is distributed as $\chi^2(2)$, and the $\text{JB}_{p-\text{value}}$ is the associated p-value.
[b]ADF is the Augmented Dickey–Fuller test of unit root, which contains a constant in regression equation, and the $\text{ADF}_{p-\text{value}}$ is the associated p-value.
[c]ADF is the Augmented Dickey–Fuller test of unit root, which contains a constant and a linear trend in regression equation, and the $\text{ADF}_{p-\text{value}}$ is the associated p-value.

The descriptive statistics for the daily, weekly and monthly log returns of WTI crude oil are depicted in Table 1. For simplicity, we can regard WTIS as WTIF0, that is, the spot prices are treated as futures prices with the maturity M being 0. At each sampling frequency, the mean return increases with the maturity M. The maximum returns decrease with M, while the minimum return increases with M. It means that the prices fluctuate less for larger M. It is consistent with the observation that the standard deviation becomes smaller for larger M. The returns are negatively skewed and the skewness shows an increasing trend with respect to M, except for the daily data. The kurtosis is greater than 3 and no clear trend is observed. Therefore, the Jarque–Bera normality tests indicate that none of the return series can be assumed to be normally distributed at the 1% significance level. The cross correlations between different return series are very large. The correlation coefficient deceases if the difference of the two maturities becomes large. The results at different sampling frequencies are qualitatively the same. Nevertheless, there are also quantitative differences. With the decrease of the sample frequency, the means, standard deviations, skewnesses and Pearson correlation coefficients of returns become larger, while the kurtosis decreases rapidly.

4. Time-varying lead–lag patterns between crude oil spot and futures markets

Let us now explore the time-varying lead–lag relationship between the WTIS and WTIF returns using the TOPS method. After generating the return time series $r(t)$, we then standardize all the return series $r(t)$ of the WTI futures and the underlying spot market as follows:

$$R(t) = \frac{r(t) - \bar{r}}{\sigma}. \tag{13}$$

where $\bar{r}$ and σ are the mean and standard deviation of logarithmic return series $r(t)$. This procedure ensures the comparability of the return series for the TOPS analysis. In addition, we denote the standardized returns of WTI futures using $X(t_1)$ and the standardized returns of WTI spot using $Y(t_2)$.

4.1. Daily WTIS and WTIF

Fig. 2 shows the average optimal thermal path $\langle x(t) \rangle$ between the normalized returns $R(t)$ of daily WTIF and WTIS conducted by the TOPS analysis, together with the self-consistency test presented in the previous section, which is well captured by the following relationship

$$R_{\text{WTIS}}(t) = c + aR_{\text{WTIF}}(t - \langle x(t) \rangle) + \varepsilon(t). \tag{14}$$

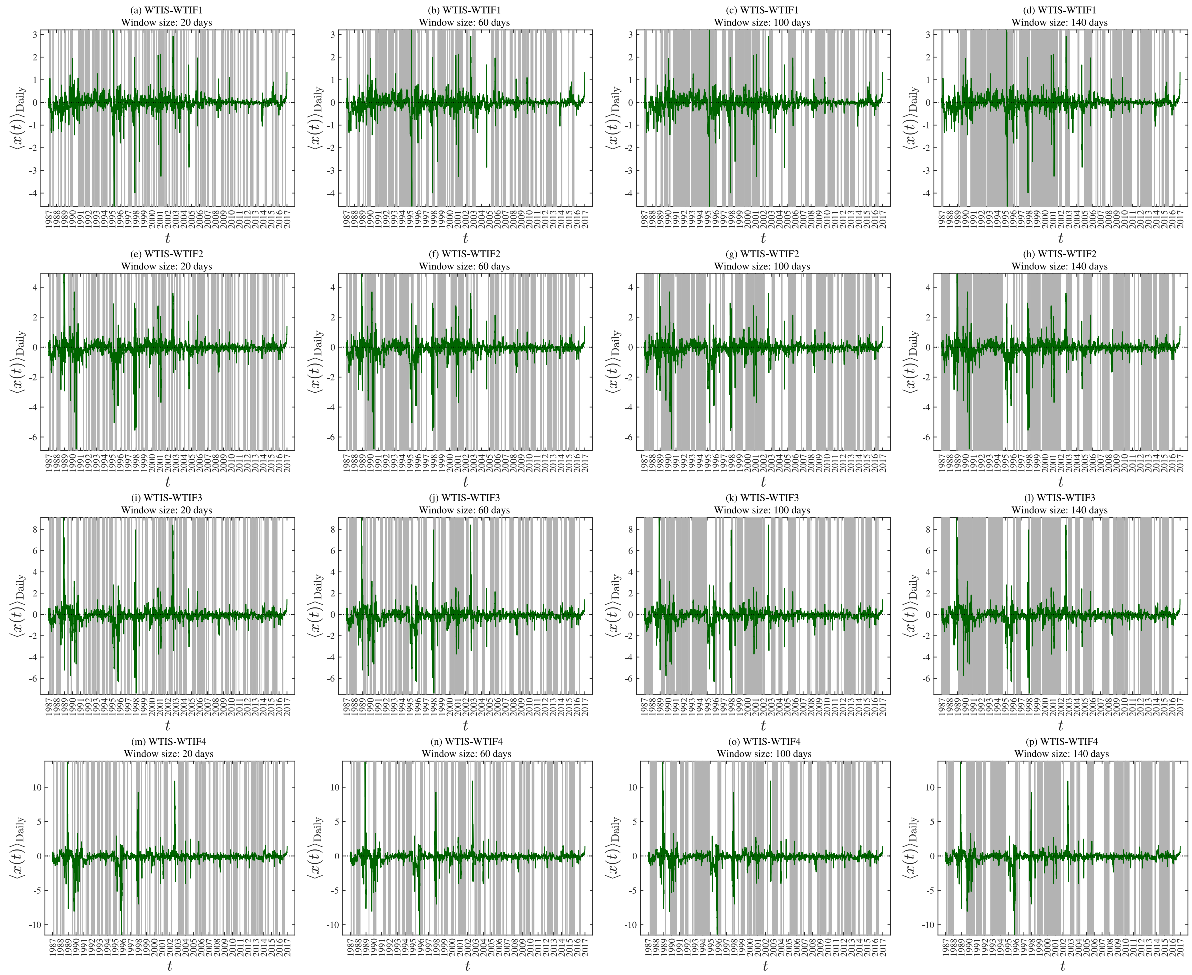

Fig. 2. TOPS analysis of the normalized returns $R(t)$ of daily WTIF and WTIS. The analysis is implemented at $T = 2$, using the distance definition ϵ_{-}. Each row shows the result for a pair of "WTIF vs. WTIS" series. The pairs WTIF1 vs. WTIS, WTIF2 vs. WTIS, WTIF3 vs. WTIS, WTIF4 vs. WTIS correspond respectively to the first, second, third, fourth rows. Each green line is the resulting TOPS path $\langle x(t)\rangle$, which is chosen as the one with lowest free energy among 41 × 41 paths of different starting points ($t_1 = i_1, t_2 = i_2$) and ending points ($t_1 = N - i_1, t_2 = N - i_2$) for $i_1, i_2 = 0, 1, 2, \ldots, 30$. The case when $\langle x(t)\rangle > 0$ indicates that WTIF returns are preceding WTIS returns at time t, and vice versa. The self-consistent test is implemented within moving windows with sizes of 20 to 140 days, corresponding to columns 20 to 140 respectively. The domains in gray indicate the times when the consistency test is significant at the 5% level.

For each TOPS path of a pair of time series (WTIF vs. WTIS), we carry out the self-consistent test defined by Eq. (14) in moving windows with the sizes ranging from 20 to 140 days at a step of 40 days. Following Meng's research [9], one can easily find that the optimal path with $T = 2$ have the best performance to capture the lead–lag structure of two time series. We set T to be 2 for our analysis while very similar results are obtained for $T = 0.5$, 1 and 1.5. We take 41 starting points and 41 ending points for the TOPS analyses of WTIF and WTIS. A positive (resp. negative) $\langle x(t)\rangle$ indicates that the returns of WTI futures lead (resp. lag) the returns of WTI spot. The overall information is that all the four average optimal thermal paths follow a similar time-vary lead–lag relationship. With few exceptions, $\langle x(t)\rangle$ lies within the interval $[-1, 1]$, suggesting a lead–lag relationship between the WTIF and the WTIS within one day. Moreover, the maximum of $\langle x(t)\rangle$ becomes larger with the increasing maturities for WTI futures. The price discovery process seems to be bidirectional, with both positive and negative values of $\langle x(t)\rangle$ existing during the whole period, thus it is hard to recognize which causality relationship plays a predominant role. Furthermore, each consistency test is a regression with time window of 40 days obeying Eq. (14). We can observe that, on most days, the consistency test is significant, with a deviating from 0. The gray shades in all plots indicate that the consistency test is significant at the 5% significance level (p-value <0.05). In other words, if a period possesses very strong lead–lag signals, there should exist many significant moving windows overlapping in its neighborhood. Concerning the local structure of $\langle x(t)\rangle$, one can find the values near special economic, financial and geopolitical events always correspond to large fluctuations, such as the Gulf war of 1990–1991, the East Asian crisis in 1997–1998, the 9–11 attack of 2001, the Iraq war in 2003, the 2007–2009 demand hike and

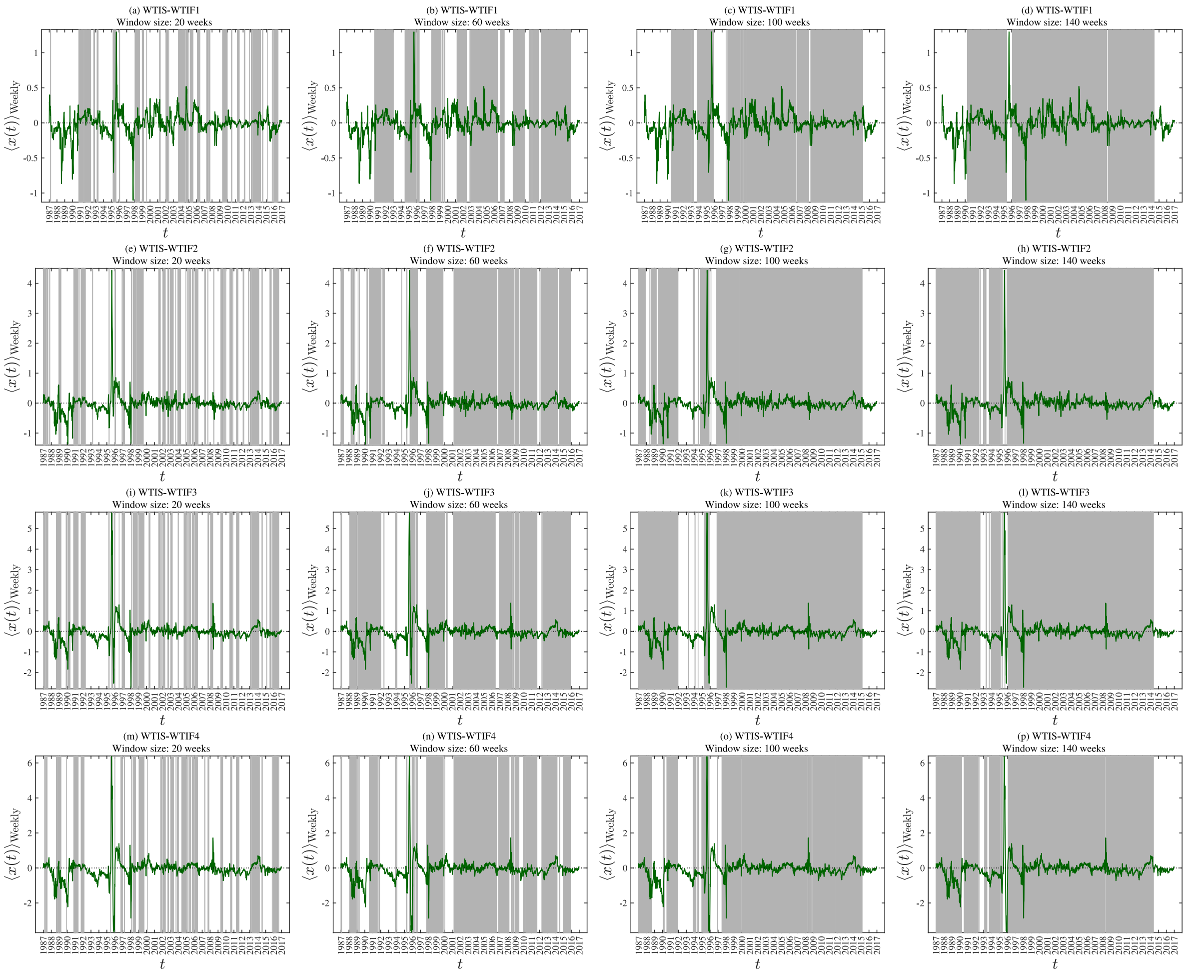

Fig. 3. TOPS analysis of the normalized returns $R(t)$ of weekly WTIF and WTIS. The analysis is implemented at $T = 2$, using the distance definition ϵ_-. Each row shows the result for a pair of "WTIF vs. WTIS" series. The pairs WTIF1 vs. WTIS, WTIF2 vs. WTIS, WTIF3 vs. WTIS, WTIF4 vs. WTIS correspond respectively to the first, second, third, fourth rows. Each green line is the resulting TOPS path $\langle x(t)\rangle$, which is chosen as the one with lowest free energy among 41×41 paths of different starting points $(t_1 = i_1, t_2 = i_2)$ and ending points $(t_1 = N - i_1, t_2 = N - i_2)$ for $i_1, i_2 = 0, 1, 2, \ldots, 30$. The case when $\langle x(t)\rangle > 0$ indicates that WTIF returns are preceding WTIS returns at time t, and vice versa. The self-consistent test is implemented within moving windows with sizes of 20 to 140 weeks, corresponding to columns 20 to 140 respectively. The domains in gray indicate the times when the consistency test is significant at the 5% level.

a series of decisions made by OPEC in the following years. Another interesting phenomenon that should be mentioned happens at the second half of $\langle x(t)\rangle$, which shows a smaller fluctuation than the first half. The small fluctuations indicate a remarkable improvement of efficiency of the WTI market.

4.2. Weekly WTIS and WTIF

Fig. 3 presents the average optimal thermal path between the normalized returns $R(t)$ of weekly WTIF and WTIS. Although there are some differences between the daily and weekly $\langle x(t)\rangle$ curve, the range of weekly $\langle x(t)\rangle$ does not change much. Therefore, one can easily find the overwhelming majority of $\langle x(t)\rangle$ lies within the interval $[-1, 1]$ by observing Fig. 3. Four curves of $\langle x(t)\rangle$ exhibit extremely similar behavior except for the maximum, which are consistent with the situation in Fig. 2. The weekly $\langle x(t)\rangle$ are closer to zero, which can be observed obviously at the second half of the $\langle x(t)\rangle$ curve in Fig. 3. This can be associated with the fact that the normalized returns $R(t)$ of weekly WTIF and WTIS track each other well. Affected by the significant economic, financial and geopolitical events at several fixed periods (Gulf war of 1990–1991, the East Asian crisis in 1997–1998 and so on), the weekly $\langle x(t)\rangle$ curves show an alternate lead–lag structure with relatively large fluctuations in these periods. In addition, the self-consistent test exhibits remarkable significance at the second half of $\langle x(t)\rangle$ when the size of moving windows are greater than 60 weeks. It is worth noting that the infamous and widespread global financial crisis in 2007–2009 did not cause obviously positive or negative oscillations of $\langle x(t)\rangle$.

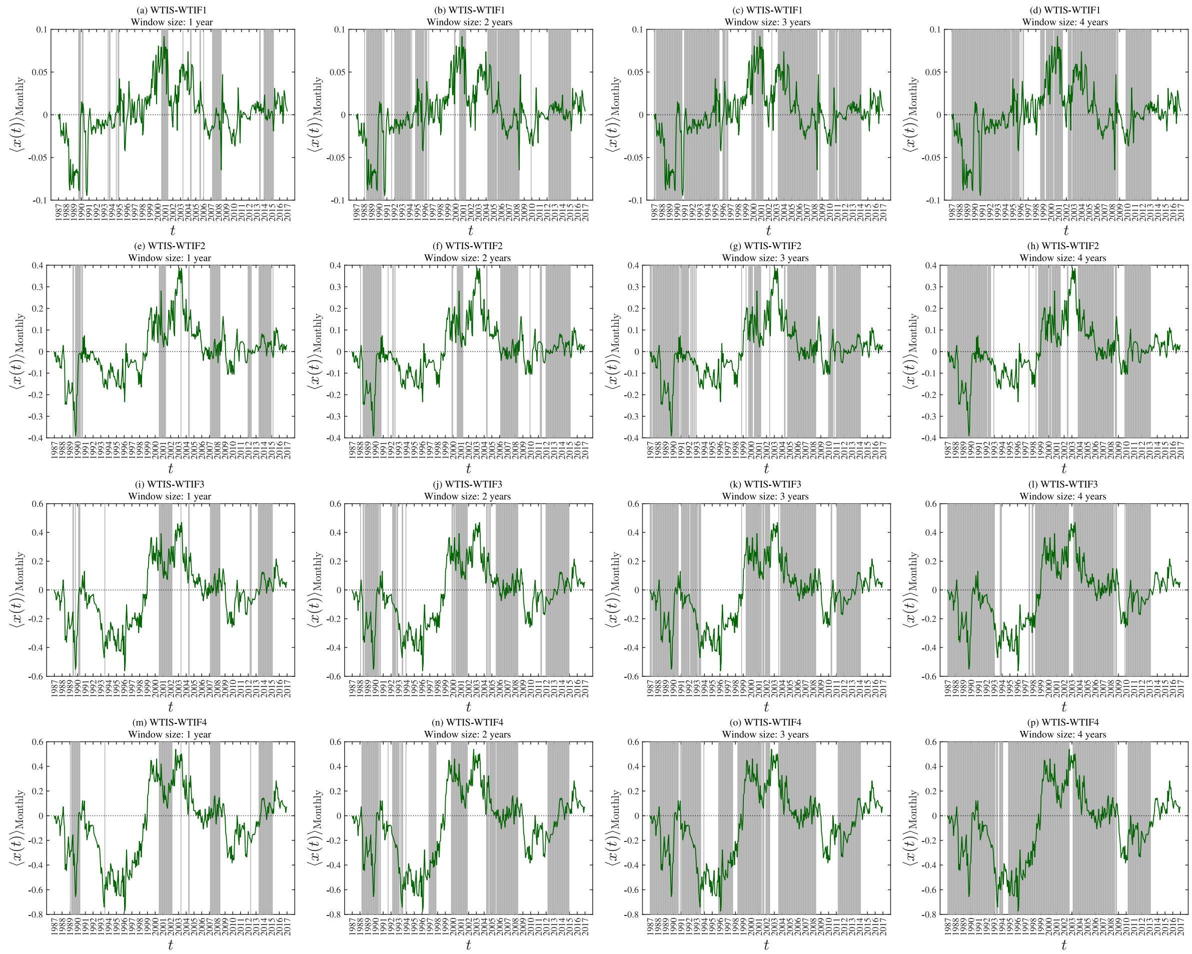

Fig. 4. TOPS analysis of the normalized returns $R(t)$ of monthly WTIF and WTIS. The analysis is implemented at $T = 2$, using the distance definition ϵ_-. Each row shows the result for a pair of "WTIF vs. WTIS" series. The pairs WTIF1 vs. WTIS, WTIF2 vs. WTIS, WTIF3 vs. WTIS, WTIF4 vs. WTIS correspond respectively to the first, second, third, fourth rows. Each green line is the resulting TOPS path $\langle x(t)\rangle$, which is chosen as the one with lowest free energy among 41×41 paths of different starting points $(t_1 = i_1, t_2 = i_2)$ and ending points $(t_1 = N - i_1, t_2 = N - i_2)$ for $i_1, i_2 = 0, 1, 2, \ldots, 30$. The case when $\langle x(t)\rangle > 0$ indicates that WTIF returns are preceding WTIS returns at time t, and vice versa. The self-consistent test is implemented within moving windows with sizes of 1 to 4 years, corresponding to columns 1 to 4 respectively. The domains in gray indicate the times when the consistency test is significant at the 5% level.

4.3. Monthly WTIS and WTIF

Similar to the daily and weekly cases, as illustrated in Fig. 4, the TOPS paths between the monthly returns of WTIF and WTIS present an uncomplicated lead–lag signals from 1987 to 2017. Overall, we notice that the four curves given in Fig. 4 are quite analogous, which corresponds to all pairs of the WTI futures returns and spot returns at one to four month maturities. Likewise, the maximum of the monthly $\langle x(t)\rangle$ increases with maturity. Correspondingly, the average time lag paths throughout the sample period are mostly in the range of $-0.5 \leq \langle x(t)\rangle \leq 0.5$, indicating a small lead–lag time step in price discovery, which can be easily observed in Fig. 4. Interestingly, the TOPS paths presented in Fig. 4 have three regime-switch points which also showed an imperfect match with the significant economic, financial and geopolitical events. Except for some local inconsistency, it can be roughly observed that the WTI spot returns lead the WTI futures returns from 1987 to 1999, then the leading relationship turns into the opposite direction from 1999 to 2007, while the TOPS curves $\langle x(t)\rangle$ become alternate lead–lag structures after 2007. Additionally, the self-consistent tests shown in Fig. 4 give more insignificant a values when compared to Figs. 2 and 3. This is probably due to the fact that the monthly data contain mostly aggregated information on a longer time scale compared to the relatively high frequency data.

5. Discussion and conclusion

In the study, we investigated the time-dependent lead–lag relationship between the WTI crude oil futures and WTI crude oil spot adopting the TOPS method, which possessed more consistent and stable performance comparing with the

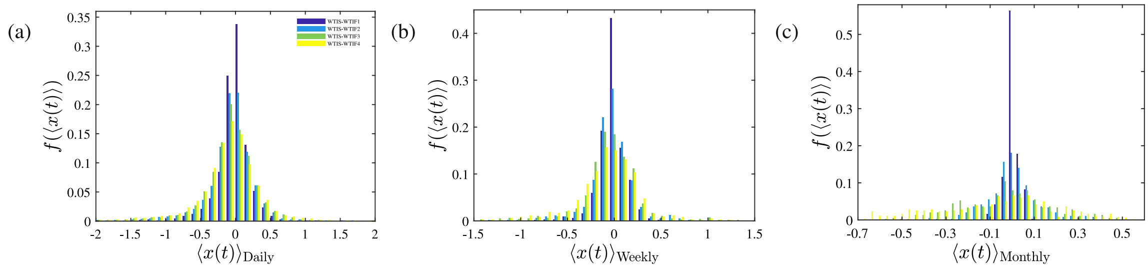

Fig. 5. Histogram of TOPS paths $\langle x(t) \rangle$ at $T = 2$. (a) Probability distribution of daily $\langle x(t) \rangle$. (b) Probability distribution of weekly $\langle x(t) \rangle$. (c) Probability distribution of monthly $\langle x(t) \rangle$.

TOP method [9]. The datasets used in this research cover daily, weekly and monthly futures and spot prices from 1987 to 2017, which witness many significant diversified events relevant to fluctuation of oil prices.

Further analysis is carried out for the distribution of $\langle x(t) \rangle$ using histogram. Fig. 5(a) shows the symmetric probability distribution of daily $\langle x(t) \rangle$ with a center near 0. More specifically, the percentages of positive values of $\langle x(t) \rangle$ are 51.76%, 43.94%, 39.65%, 38.54% for the four WTI futures time series, respectively, which from an economic point of view, represent the probabilities that WTI futures market leads the WTI spot market. Additionally, Fig. 5(b) and (c) present the similar distribution of weekly and monthly $\langle x(t) \rangle$ compared with the daily $\langle x(t) \rangle$. Correspondingly, the percentages of positive values of weekly $\langle x(t) \rangle$ are 45.56%, 42.54%, 41.37%, 39.99% for the four WTI futures time series, respectively. The percentages of positive values of monthly $\langle x(t) \rangle$ are 53.70%, 49.32%, 43.84%, 43.01% for the four WTI futures time series, respectively. What is more, we find the percentages of positive values of $\langle x(t) \rangle$ become smaller with the maturity of WTI futures get longer.

For all of the daily, weekly and monthly data sets, the dynamic patterns of $\langle x(t) \rangle$ between the WTI crude oil spot returns and each of the WTI crude oil futures returns evolve quite similarly throughout the sample period. Our empirical analysis using the TOPS method demonstrates that the lead–lag dependence between the WTI futures market and the spot market in the past few years is volatile and exhibit obviously temporal characteristics. In other words, neither of the two markets plays a predominant role during the price discovery process. In addition, our study enhances the existing literature by providing a time-dependent resolution, together with reliable statistical test results, concerning the analysis of correlations between the WTI crude oil futures and spot time series. In summary, our study revealed both the long-run (weekly frequency and monthly frequency) and short-run (daily frequency) lead–lag relationships between the WTI futures and WTI spot markets, showing the alternate lead–lag structure. Therefore, our findings roughly accord with the weak-form Efficient Market Theory in crude oil market [4,5].

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