## Direct Tests of the Aharony-Stauffer Argument

Aharony and Stauffer[1] (AS) have recently proposed an argument of considerable significance since it relates a dynamic exponent $d_w$ (the fractal dimension of a random walk) to a static exponent $d_f$ (the fractal dimension of the substrate on which the walk takes place) by the simple relation $d_w = d_f + 1$. Moreover, they identify a lower critical dimension for this problem, $d_f = 2$; for $d_f < 2$ the AS formula holds, and for $d_f > 2$ the Alexander-Orbach rule $d_w = \frac{3}{2} d_f$ may hold.

One purpose here is to test the key assumption underlying the AS idea, namely that the width $\Delta R$ scales as $R^0$, where $\Delta R$ is the width of the annulus on which the growth sites lie, and the number of growth sites scales as $G \sim R^{d_f - 1} \Delta R$.

To achieve this goal, we calculated the location of all the growth sites on the positive $x$ axis as the cluster grows up to size $s = 9000$. At regular intervals ($s = 500, 1000, \ldots, 9000$) we calculated the variance ($\Delta R$), and then averaged this quantity over 13 000 clusters. Our results for the $s$ dependence of $\Delta R$ [Fig. 1(a)] show clearly that $\Delta R$ is not constant; in fact it possibly scales as the cluster radius $R$. This result indicates growth sites are not on a single narrow annulus as assumed by AS. Of course, we cannot rule out the possibility of a fixed finite number of annuli each of zero width, in which case AS could still hold.

In addition to testing the basic *assumption* of the AS argument, we directly tested one of the main *predictions* of AS: how $G$ scales with $s$. AS predict $G \sim R^{d_G} \sim s^{d_G/d_f}$ with $d_G/d_f = 1 - 1/d_f$, while Leyvraz and Stanley[2] (LS) predict $d_G/d_f = \frac{1}{2}$. Since $1 - 1/d_f = \frac{43}{91} \cong 0.47$, the difference between the two predictions is not easy to detect. Accordingly, we plot $G/s^{d_G/d_f}$ for several trial values of $d_G/d_f$ in order to estimate the value that leads to the most convincing "plateau" for large $s$ [Fig. 1(b)]; we see that 0.49 is distinctly better than either 0.47 (AS) or 0.50 (LS).

In summary, we have tested both the assumption and the prediction of the AS argument. At present our data do not seem to be in support of either, but it is certainly possible that our data could cross over to the AS prediction for much larger system sizes. Since 150 h of time on an IBM3081 were used in the present work, it is not likely that we can resolve this in the immediate future. We emphasize that our data in Fig. 1(b) are consistent with the AS proposal that $d = 2$ is below the lower critical dimension for the Alexander-Orbach rule, an idea that was independently proposed in a totally different context of the random superconducting network.[3]

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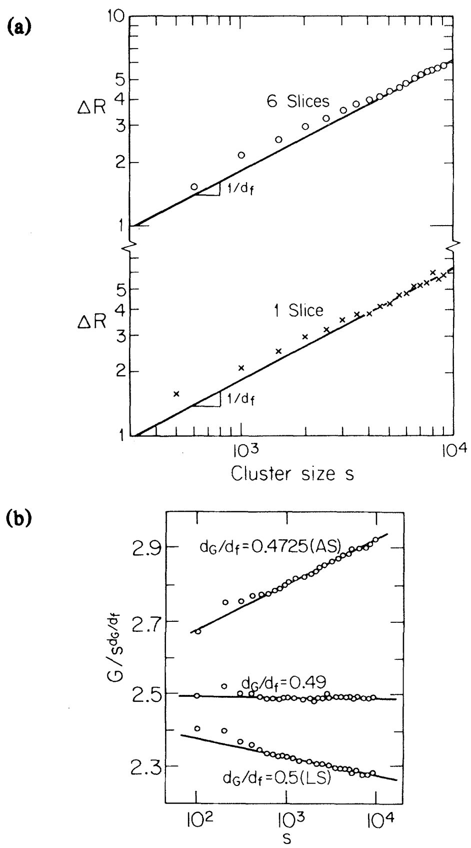


FIG. 1. (a) Dependence on cluster mass $s$ of the variance $\Delta R$ for growth sites on the positive $x$ axis. The line shown has slope $1/d_f = \frac{43}{91}$. Data shown are: (crosses) one slice, the positive $x$ axis and (circles) the average over six slices of the fractal with $x > 0$ at the constant values of $y$ given by $y = 0, 1, 2, 3, 4, 5$. (b) $G/s^{d_G/d_f}$ for various $d_G/d_f$, comparing the predictions of AS and LS.

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[3] A. Coniglio and H. E. Stanley, Phys. Rev. Lett. **52**, 1068 (1984).

 