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Minireview

Promotion of cooperation induced by two-sided players in prisoner's dilemma game

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We examine how real-world individuals and companies can either reach an agreement or fail to reach an agreement after several stages of negotiation. We use a modified prisoner's dilemma game with two-sided players who can either cooperate or not cooperate with their neighbors. We find that the presence of even a small number of these two-sided players substantially promotes the cooperation because, unlike the rock–paper–scissors scenario, when the cooperators change to the non-cooperators to gain a payoff, they can turn to the two-sided players and continue negotiating. We find that the network structure influences the spread of strategies. Lattice and regular-random (RR) networks benefit the spread of both non-cooperation and two-sided strategies, but scale-free (SF) networks stop both strategies. We also find that the Erdös–Rényi (ER) network promotes the two-sided strategy and blocks the spread of non-cooperation. As the ER network density decreases, and the network degree is lowered the lifetime of non-cooperators increases. Our results expand our understanding of the role played by the two-sided strategy in the growth of the cooperative behavior in networks.

Contents

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1. Introduction

Both cooperation and non-cooperation (defection) are ubiquitous in biological and social systems, and in recent decades the evolution of cooperation has attracted considerable attention [1–5]. Examining the element of risk in interaction, Szabó and Hauert introduced a third strategic type with small fixed earnings [6]. Influential nodes affect the strategies of their neighbors but are not influenced by them [7]. Most participants want to win and also to set negotiating patterns, the measurement of which was introduced in Ref. [2]. Neighbor selection obeys a dynamic preferential rule, i.e., the more frequently a neighbor's strategy is adopted in previous rounds, the higher the probability it will be adopted in subsequent rounds [8]. A "tit-for-tat" cooperation in the first round encourages a participant to adopt their opponent's previous round strategy [9–12]. Extortive players gain the highest payoffs [13]. Cooperation is higher among participants with similar characteristics [14], and participants can receive payoffs from both neighbors and themselves [15]. The experimental results showed the onymity could promote cooperation behavior [16]. The rewarding of cooperative participants and the punishing of non-cooperative players were examined in Ref. [17–19], where they found that punishment was more effective than rewards. Heterogeneous punishment promote cooperation under limited conditions [20]. Because all participants share the same memory, which may be in part either true or false, this model cannot forecast the future using this common "understanding" [21]. Strategic behaviors and evolutionary game can be influenced by the multilayer networks structure [22,23]. The scale-free networks can provide a unifying framework for the evolution of cooperation [24], and a finding has motivated the research on many interactive networks [25], including those that co-evolved as the game evolved [26–28].

The previous studies promote only the partial cooperation. In real-world negotiations the cooperation allows both participants to receive a payoff and the non-cooperation allows neither to receive a payoff. Participants can either cooperate or not. Although at every stage of negotiations they select the strategy they deem most advantageous, they strongly tend to cooperate with cooperators and avoid the non-cooperators. Here, we introduce two-sided players and study the evolution of cooperation. Two-sided players choose either to cooperate or not in response to their opponent's strategy, and will choose to cooperate if their opponent also chooses to cooperate.

2. The model

We use an iterated prisoner's dilemma game, which is frequently used to study the evolution of the cooperation among selfish individuals [13,29,30]. Each player can choose between either cooperation (C), defection (D), or a two-sided strategy (T). In the donation PD game, the cooperators provide a benefit b to their partners at a cost c to themselves ($b > c = 1$) and the defectors neither provide benefits nor incur costs. Two-sided players gain benefit c with T and C partners. With two-sided players participating in the game, the payoff matrix is

$$\begin{array}{c|ccc} & \mathrm{C} & \mathrm{D} & \mathrm{T} \\ \hline \mathrm{C} & c & b & c \\ \mathrm{D} & b & 0 & 0 \\ \mathrm{T} & c & 0 & c, \end{array} \tag{1}$$

where the parameter b is the temptation to defect.

Initially each player holds a C or D strategy with equal probability. The players collect the gains according to the payoff matrix (2) when playing with their neighbors.

Using the approach in Ref. [25], and given the payoffs (E_i and E_j for players i and j, respectively) from the previous round, a player i randomly adopts the strategy of a neighbor j with a probability

$$W = \frac{1}{1 + \exp[-(E_j - E_i)/\kappa]}, \tag{2}$$

where κ is a normalization constant, i.e., the noise allows irrational choices. We set the value of κ to 0.1, similar to that in Refs. [8,15].

We iterate the model using a synchronous updating strategy. At each Monte Carlo (MC) time step, all players get payoffs from their neighbors and then simultaneously update their strategies according to Eq. (1). All simulations except those in Fig. 3 are performed in the systems with 100 × 100 players and, with the exception of Figs. 4 and 5, are averaged over 1000 realizations.

3. Simulation results

We begin with an initial state characterized by equal fractions of C and D participants and describe the initial percentages of the two-sided strategy p_t at step $t = 0$. The evolution eventually leads to a dynamic equilibrium state with small fluctuations of defector density around an average value.

Fig. 1 shows the players driven by payoffs [see Eq. (1)]. The fraction of defectors f_d decreases with p_t. There are about 0.007 defectors at step $t = 500$ when the initial fraction of two-sided players is $p_t = 0.001$. These defectors virtually disappear when the initial fraction of two-sided players $p_t \geq 0.002$ is increased.

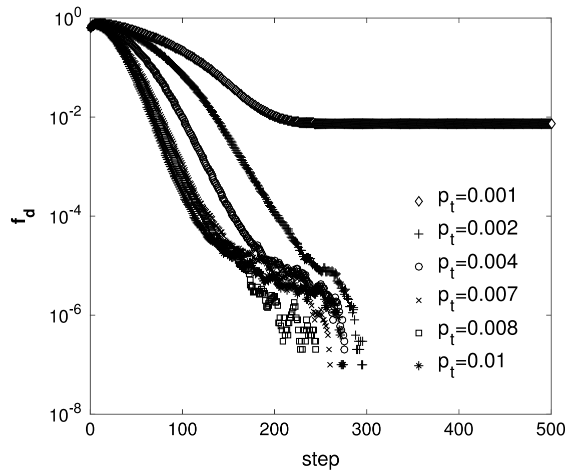

Fig. 1. (Color online) Stationary fractions of defection (f_d) in each step with different fractions of two-sided players at step = 0 (p_t) for $b = 1.01$.

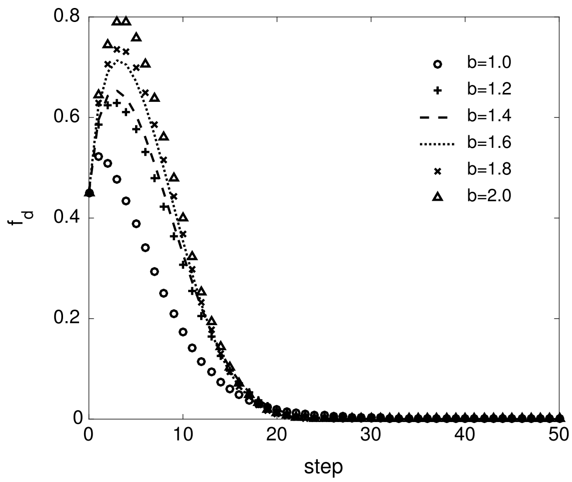

Fig. 2. (Color online) Average fractions of defectors, f_d, as a function of the temptation to defect b at the equilibrium state for $p_t = 0.1$.

Fig. 2 shows the impact of the temptation to defect b on our expanded PDG model. The fraction of defectors (f_d) first increases and then decreases step by step to about 0. At step 1, f_d grows to its highest value 0.52 and then gradually drops to 0 for $b = 1.0$. At step 3, f_d increases to 0.79 and then slowly decreases to 0 for $b = 2.0$. At step 3, f_d reaches its highest value and then slowly decreases to 0 for $1.1 < b < 2.0$. The larger the value of b is, the greater the value of f_d is at each step. As temptation increases, the number of cooperators changing their strategy to defection increases. At the end of the game, nearly all the defectors have left.

Fig. 3 shows the frequency of defectors for different sizes of lattice network. The stationary fractions of defectors (f_d) are approximately 0.147, 0.05, 0.01 and 0.0006 in the $50 \times 50, 70 \times 70, 90 \times 90$ and 110×110 lattice networks, respectively. The value of f_d is about 0 for the large size of lattice network ($n > 110 \times 110$). The subfigure in Fig. 3 shows that the fraction of T players disappears in a stable state for different lattice sizes.

We next study how the numbers of cooperators, defectors, and two-sided players changes over time. Fig. 4 shows the snapshots of C, D, and T in the equilibrium state when $p_t = 0.1$ and $b = 1.01$. Initially T participants are randomly distributed with $p_t = 0.1$, and C and D are randomly located with an equal probability 0.45 [Fig. 4(a)]. At step 1 when the cooperators are attacked by the defectors for a payoff, the fraction of D is greater than that of C. The number of two-sided players increases slowly [Fig. 4(b)]. The clusters of T continue to expand and the D clusters shrink at step 5 because D cluster players cannot receive payoffs from each other [Fig. 4(c)]. The two-sided players can connect whereas after a few time steps the T clusters occupy the main area of the lattice network [Fig. 4(d)]. The D clusters then rapidly shrink [Fig. 4(e)], and the C clusters increase smoothly until they reach a stable state [Fig. 4(f)].

Fig. 5 shows the time series of the fractions of the cooperators f_c, the defectors f_d, and the two-sided players f_t. The f_d values first increase to the highest value of 0.54, which corresponds to Fig. 4(b), then decreases to 0, as shown in Fig. 4(f). In

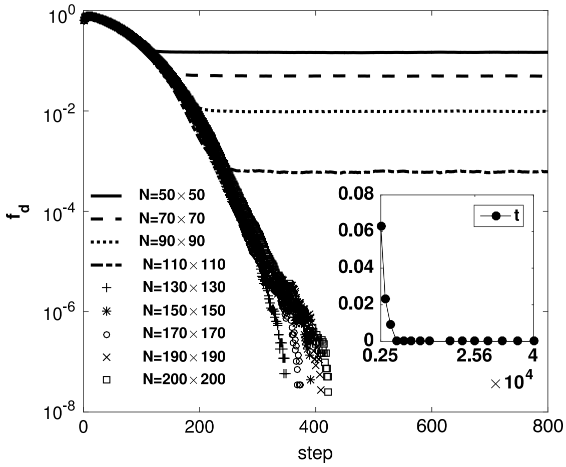

Fig. 3. (Color online) The fractions of defector (f_d) in each step vs different sizes of lattice networks for $p_t = 0.001$ and $b = 1.01$. The subfigure shows that the fraction of T disappears as a function of lattice network size.

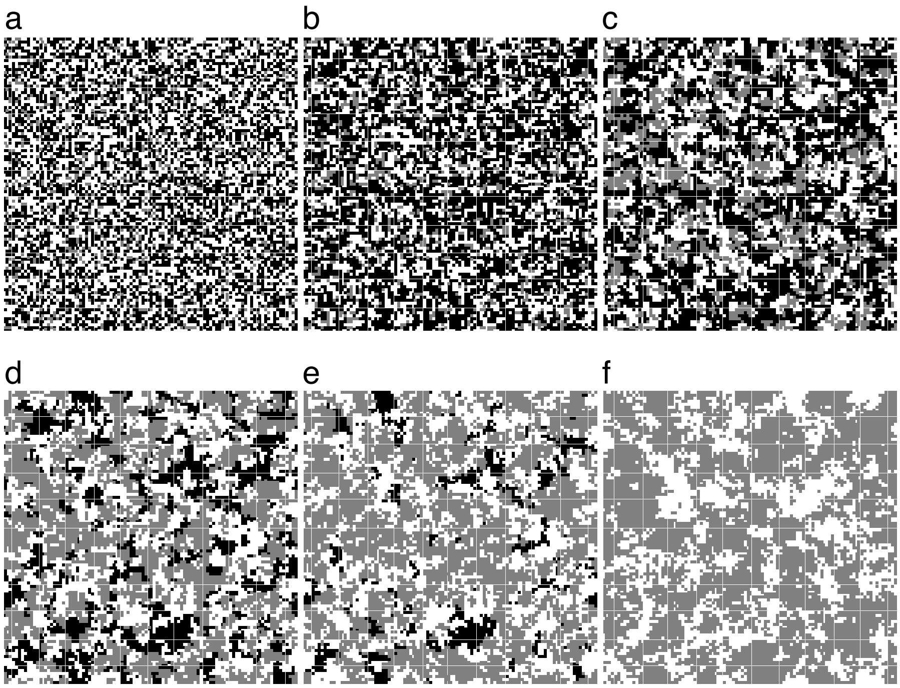

Fig. 4. (Color online) Typical spatial distributions of the cooperators (white), the defectors (black), and two-sided players (gray) on the square lattice by means of synchronous strategy updating. (a) step = 0, (b) step = 1, (c) step = 5, (d) step = 10, (e) step = 15, and (f) step = 60. Parameter values in the cases are $p_t = 0.1$ and $b = 1.01$.

contrast, f_c first decreases to the lowest value of about 0.34, and then grows to a stationary state at the value 0.39, which is consistent with C in Fig. 4, i.e., it first increases and then decreases. f_t gradually increases to 0.6 as shown in Fig. 4(f).

Fig. 6 shows the effect of network structure and compares the step-by-step defector frequency for different network structures. Note that the defector frequency f_d first increases and then decreases for all four types of networks. The highest value of f_d is about 0.72 in RR networks while $f_d = 0.68$ in lattice network. The values of f_d are 0.62, 0.54, and 0.53 in ER, SF with $\gamma = 2.5$, and $\gamma = 3.0$, respectively. The value of f_d rapidly declines to about 0 in lattice and RR networks. Thus lattice and RR networks are beneficial to both defectors and two-sided players, but in ER networks f_d rapidly decreases to a low value and becomes stationary, which means that ER networks prefer the defectors over two-sided players. In contrast, in SF networks the fraction of defectors smoothly decreases, and the spread of defection and two-sided strategies are resistant.

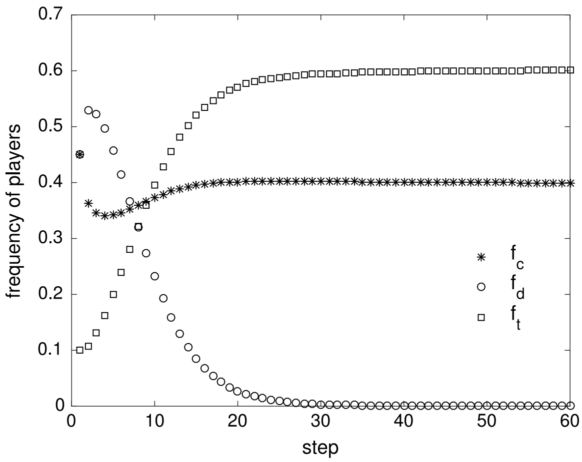

Fig. 5. (Color online) The fractions of the cooperator f_c (star), the defector f_d (circle), and the two-sided player f_t (square) in each step for special value of $p_t = 0.1$ and $b = 1.01$.

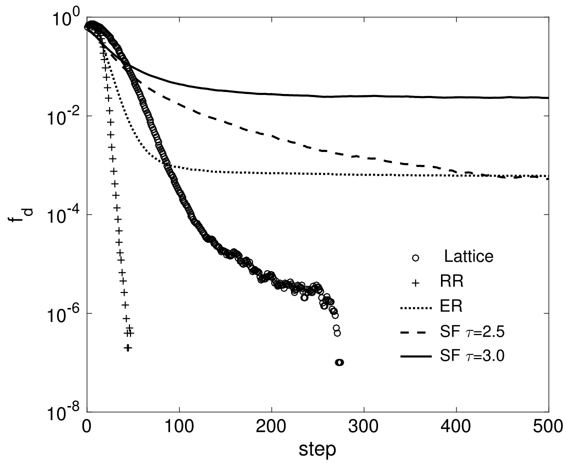

Fig. 6. (Color online) The fractions of the defector f_d in each step for the special values of $p_t = 0.01$ and $b = 1.01$ in Erdös–Rényi (ER) network, scale-free (SF) network, regular-random (RR) network and lattice network.

The sparsity of the network affects the expanded PDG. Fig. 7 shows that increasing the fraction of removed edges in lattice networks causes the defectors to slowly decrease. If the average degree of an ER network is increased, the defectors persist in fewer steps. The final fraction of defectors ($f_d = 0.001$) in a lattice network with a fraction of removing edges $r = 0.3$ and $r = 0.35$ is approximately equal to that in an ER network with a average degree $\langle k \rangle = 5$.

4. Analysis and discussion

In the expanded PDG performed on networks, even a small number of two-sided players cause defectors to leave the game. The larger the fraction of T players at $step = 0$ (p_t) is, the more quickly the D players leave the game. The defectors exist in a small social network, and the smaller the size of a lattice network is, the larger the number of defectors in the stationary state is. The fraction of defectors is limited by the benefit b. The f_d rapidly increases to the highest value, then declines to 0. Initially, the D value expands and the C value shrinks, but after a few steps D smoothly decreases and T increases. This is the case because, although the cooperators may first turn to the defectors for a good payoff, two-sided players can stop C players from turning to D players and can cause D players to become T players. Alternatively, D collapses and T dominates. Because two-sided players can switch strategies when facing different players, and they enhance cooperation.

Defectors are more robust in SF networks than those in other types of network. The defector strategy can more quickly increase in lattice and RR networks than those in ER and SF networks, i.e., lattice and RR networks allow the defectors and two-sided strategies (the value of f_d first rapidly increases and then decreases). Although SF networks do not encourage the spread of D (although initially it slightly increases) and T (it slowly declines), ER networks encourage the two-sided strategy

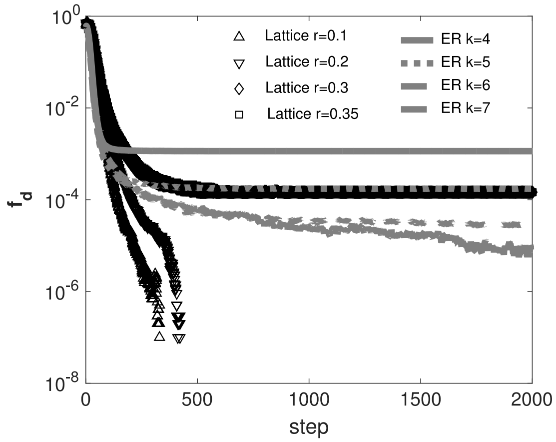

Fig. 7. (Color online) The ratios of the defector f_d in each step for the special values of $p_t = 0.01$ and $b = 1.01$ in lattice networks with removing the edge (symbols) and ER network with different degrees k (lines).

(f_d quickly decreases) but not the defectors (f_d grows only slightly). Network sparsity also affects the spreading of defectors. Dense networks can more quickly expel defectors than sparse networks.

5. Conclusions

We have introduced two-sided players into the original PD game. The defectors inhibit the cooperators, while the cooperators and two-sided players not. Initially the number of defectors rapidly increases, the number of cooperators rapidly decreases, and the number of two-sided players gradually increases. For the defectors to get more benefit than cooperators, most of the cooperators become defectors, but after reaching the maximum, the number of defectors rapidly decreases. At the same time, the number of two-sided players rapidly increases. Because the defectors cannot benefit from each other, almost all defectors become two-sided players. In addition, the network structure affects the spread of the strategies [31–35]. Lattice and RR networks encourage the expansion of both defectors and two-sided players, but SF networks disallow their expansion. ER networks promote the two-sided strategy, but block the defection strategy. In sparse networks, the defectors continue to exist for a longer period of time.

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References

[1] M.A. Nowak, Five rules for the evolution of cooperation, Science 314 (5805) (2006) 1560–1563.
[2] M.A. Nowak, K. Sigmund, Evolution of indirect reciprocity by image scoring, Nature 393 (6685) (1998) 573–577.
[3] H. Ohtsuki, C. Hauert, E. Lieberman, M.A. Nowak, A simple rule for the evolution of cooperation on graphs and social networks, Nature 441 (7092) (2006) 502–505.
[4] M.A. Nowak, R.M. May, Evolutionary games and spatial chaos, Nature 359 (6398) (1992) 826–829.
[5] M. Perc, A. Szolnoki, Coevolutionary games - A mini review, Biosystems 99 (2) (2010) 109–125.
[6] G. Szabó, C. Hauert, Evolutionary prisoner's dilemma games with voluntary participation, Phys. Rev. E 66 (6) (2002) 062903.
[7] B.J. Kim, A. Trusina, P. Holme, P. Minnhagen, J.S. Chung, M. Choi, Dynamic instabilities induced by asymmetric influence: Prisoner's dilemma game in small-world networks, Phys. Rev. E 66 (2) (2002) 021907.
[8] Z.-X. Wu, X.-J. Xu, Z.-G. Huang, S.-J. Wang, Y.-H. Wang, Evolutionary prisoner's dilemma game with dynamic preferential selection, Phys. Rev. E 74 (2) (2006) 021107.
[9] L.A. Imhof, D. Fudenberg, M.A. Nowak, Evolutionary cycles of cooperation and defection, Proc. Natl. Acad. Sci. USA 102 (31) (2005) 10797–10800.
[10] M. Nowak, K. Sigmund, A strategy of win-stay, lose-shift that outperforms tit-for-tat in the Prisoner's Dilemma game, Nature 364 (6432) (1993) 56.
[11] M. Milinski, Tit for tat in sticklebacks and the evolution of cooperation, Nature 325 (6103) (1987) 434–435.
[12] M.A. Nowak, K. Sigmund, Tit for tat in heterogeneous populations, Nature 355 (6357) (1992) 250.
[13] C. Hilbe, M.A. Nowak, K. Sigmund, Evolution of extortion in iterated prisoners dilemma games, Proc. Natl. Acad. Sci. 110 (17) (2013) 6913–6918.
[14] R.L. Riolo, M.D. Cohen, R. Axelrod, Evolution of cooperation without reciprocity, Nature 414 (6862) (2001) 441–443.
[15] G. Szabó, C. Tőke, Evolutionary prisoner's dilemma game on a square lattice, Phys. Rev. E 58 (1) (1998).
[16] Z. Wang, M. Jusup, R.-W. Wang, L. Shi, I. Yoh, M. Yamir, K. Jürgen, Onymity promotes cooperation in social dilemma experiments, Sci. Adv. 3 (3) (2017) e1601444.
[17] K. Sigmund, C. Hauert, M.A. Nowak, Reward and punishment, Proc. Natl. Acad. Sci. 98 (19) (2001) 10757–10762.

[18] J. O'Doherty, M.L. Kringelbach, E.T. Rolls, J. Hornak, C. Andrews, Abstract reward and punishment representations in the human orbitofrontal cortex, Nature Neurosci. 4 (1) (2001) 95–102.
[19] A. Szolnoki, M. Perc, Reward and cooperation in the spatial public goods game, Europhys. Lett. 92 (3) (2010) 38003.
[20] M. Perc, A. Szolnoki, A double-edged sword: Benefits and pitfalls of heterogeneous punishment in evolutionary inspection games, Sci. Rep. 5 (11027) (2015).
[21] A. Cavagna, Irrelevance of memory in the minority game, Phys. Rev. E 59 (4) (1999) R3783.
[22] Z. Wang, A. Szolnoki, M. Perc, Interdependent network reciprocity in evolutionary games, 2013. arXiv preprint arXiv:1308.1947.
[23] Z. Wang, L. Wang, A. Szolnoki, M. Perc, Evolutionary games on multilayer networks: a colloquium, 2015. arXiv preprint arXiv:1504.04359.
[24] F.C. Santos, J.M. Pacheco, Scale-free networks provide a unifying framework for the emergence of cooperation, Phys. Rev. Lett. 95 (9) (2005) 098104.
[25] G. Szabó, G. Fath, Evolutionary games on graphs, Phys. Rep. 446 (4) (2007) 97–216.
[26] M.G. Zimmermann, V.M. Eguíluz, M. San Miguel, Coevolution of dynamical states and interactions in dynamic networks, Phys. Rev. E 69 (6) (2004) 065102.
[27] J.M. Pacheco, A. Traulsen, M.A. Nowak, Coevolution of strategy and structure in complex networks with dynamical linking, Phys. Rev. Lett. 97 (25) (2006) 258103.
[28] A. Szolnoki, M. Perc, Evolution of extortion in structured populations, Phys. Rev. E 89 (2) (2014) 022804.
[29] Y.J. Kim, M. Roh, S.-W. Son, Network structures between strategies in iterated prisoners dilemma games, J. Korean Phys. Soc. 64 (3) (2014) 341–345.
[30] W.H. Press, F.J. Dyson, Iterated prisoners dilemma contains strategies that dominate any evolutionary opponent, Proc. Natl. Acad. Sci. 109 (26) (2012) 10409–10413.
[31] S.A. Levin, R.T. Paine, Disturbance, patch formation, and community structure, Proc. Natl. Acad. Sci. 71 (7) (1974) 2744–2747.
[32] E. Lieberman, C. Hauert, M.A. Nowak, Evolutionary dynamics on graphs, Nature 433 (7023) (2005) 312–316.
[33] B. Adlam, M.A. Nowak, Universality of fixation probabilities in randomly structured populations, 2014. arXiv preprint arXiv:1407.2580.
[34] T. Antal, S. Redner, V. Sood, Evolutionary dynamics on degree-heterogeneous graphs, Phys. Rev. Lett. 96 (18) (2006) 188104.
[35] M. Frean, P.B. Rainey, A. Traulsen, The effect of population structure on the rate of evolution, Proc. R. Soc. B 280 (1762) (2013) 20130211.