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Scale invariance and universality: organizing principles in complex systems

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Abstract

This paper is a brief summary of a talk that was designed to address the question of whether two of the pillars of the field of phase transitions and critical phenomena – scale invariance and universality – can be useful in guiding research on a broad class of complex phenomena. We shall see that while scale invariance has been tested for many years, universality is relatively more rarely discussed. In particular, we shall develop a heuristic argument that serves to make more plausible the universality hypothesis in both thermal critical phenomena and percolation phenomena, and suggest that this argument could be developed into a possible coherent approach to understanding the ubiquity of scale invariance and universality in a wide range of complex systems.

1. Introduction

Empirical evidence has been mounting that supports the intriguing possibility that a number of systems arising in disciplines as diverse as physics, biology, ecology, and economics may have certain quantitative features that are intriguingly similar. These properties can be conveniently grouped under the headings of scale invariance and universality. By scale invariance is meant a hierarchical organization that results in power-law behavior over a wide range of values of some control parameter such as species size, heartbeat interval, or firm size, and the exponent of this power law is a number characterizing the system. By universality is meant a tendency for the set of exponents found for diverse systems to partition themselves into distinct "universality classes", with the property that all systems falling into the same universality class have

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the same exponent – suggesting that there are features in common among the underlying microscopic mechanisms responsible for the observed scale invariant behavior. The principles of scale invariance and universality have been found to hold empirically for a number of complex systems, and this number grows as more experiments are performed and more data are analyzed. Thus far, a conceptual framework has not been found that incorporates these empirical facts. It is not entirely clear from which direction one should approach the problem, but perhaps one can gain insight by learning from the paradigm of critical phenomena. The great conceptual advances in understanding critical phenomena arose only after a lengthy period of gathering empirical facts. Then these facts began to be put together into a phenomenological framework, using the principles of scale invariance and universality. Finally, after some years, Wilson's renormalization group was developed to provide a theoretical underpinning to the experimental facts. The field of complex phenomena is, with some exceptions, still in the period in which empirical facts are being uncovered. Theoretical developments exist, but no unified coherent theory has emerged that describes in a satisfactory fashion all – or even a large fraction of – the empirical facts. In this talk, we briefly review some of the reasons to believe that a theoretical framework could be not too far off.

2. Scale invariance of heartbeat intervals: "Healthy disorder"

Let us start with a question. If you were going to describe a disease, say heart disease, which word would you reach for – order or disorder? Until very recently, nearly every physician would answer "disorder". Now, increasing numbers would not. Researchers are observing that an "ordered" sequence of heartbeat intervals very often indicates the presence of heart disease, and that a "disordered" sequence of heartbeat intervals is a pretty clear indication of a healthy heart.

How could this be true? For a vivid analogy, recall the infamous Tacoma Narrows Bridge that once connected mainland Washington with the Olympic peninsula.[1] One day it suddenly collapsed after developing a remarkably "ordered" sway in response to a strong wind. Physics students learn the explanation for this catastrophe: the bridge, like most objects, has a small number of characteristic vibration frequencies, and one day the wind was exactly the strength required to excite one of them. The bridge responded by vibrating at this characteristic frequency so strongly that it fractured the supports holding it together. The cure for this "diseased bridge" was a design that is capable of responding to many different vibration scales in an approximately equal fashion, instead of responding to one frequency excessively. Surprisingly, scientists are finding that the Tacoma Narrows bridge is perhaps a useful metaphor for many complex systems arising in biology [1], ecology [2], and even economics [3–5].

[1] The bridge collapsed on November 7, 1940 at approximately 11:00 a.m. and had been open to traffic for only a few months. The reader is invited to view historical film footage which shows in 250 frames (10 s!) the maximum torsional motion shortly before failure of this immense structure [http://cee.carleton.ca/Exhibits/Tacoma_Narrows/].

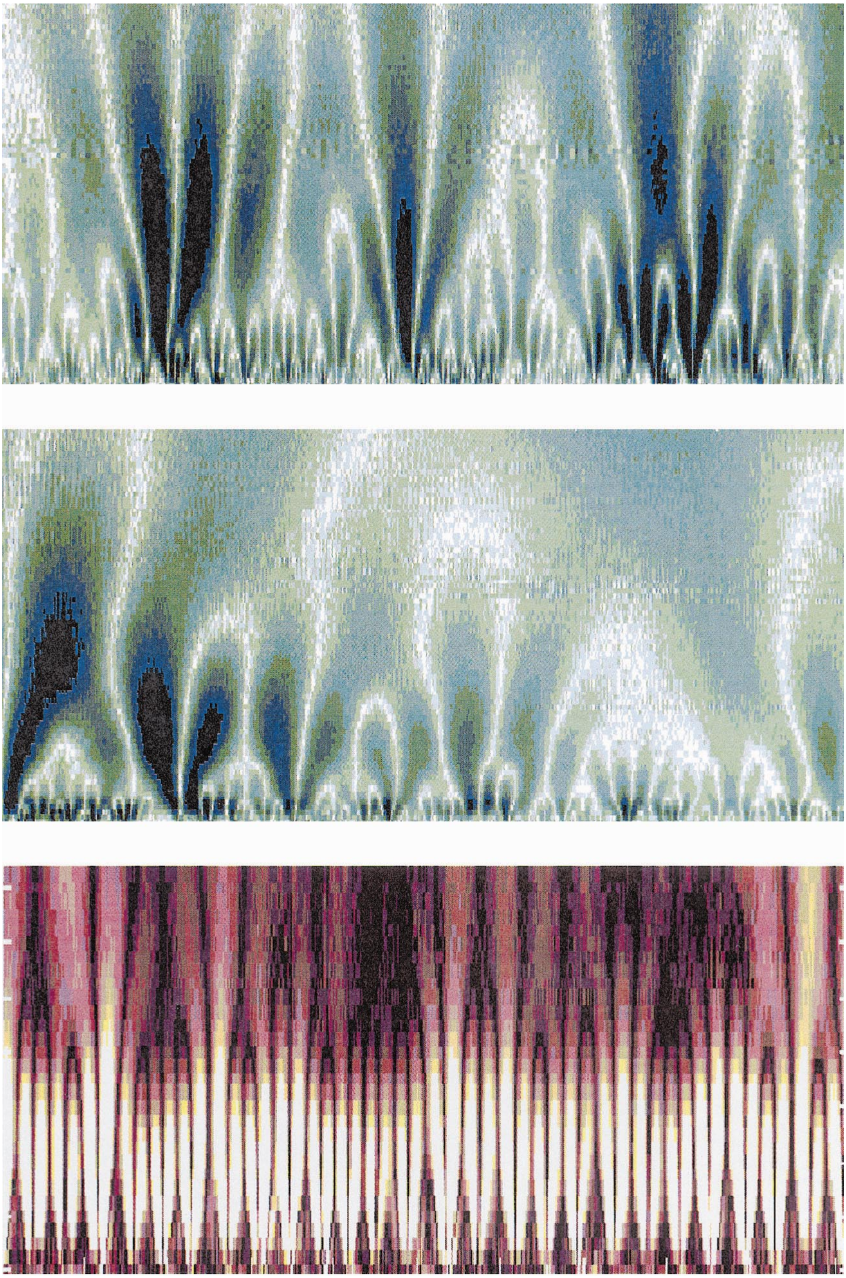

Fig. 1. *Top*: Color coded analysis of a recording of 2000 heartbeats from a healthy subject. The x-axis represents time and the y-axis indicates the analysis scale, with large scales at the top. The brighter colors indicate larger values of the heartbeat fluctuations. This analysis, called wavelet analysis, uncovers a hierarchical scale invariance of heartbeat fluctuations. *Middle*: Magnification of the central ten percent of the top panel comprising only 200 beats. By comparing the top and middle figures, can you identify which 10% is magnified? Can you see that the statistical patterns of the original heartbeat analysis and the magnification resemble each other? This statistical resemblance is called scale invariance. *Bottom*: Analysis of a recording of 2000 heartbeats from a subject with a diseased heart. Can you see that the rich range of scales characteristic of the healthy subject is now lost, and the pattern has in fact approximately the same properties for all scales studied? This figure is courtesy of P.Ch. Ivanov.

Consider again the heart. If the cardiovascular system were designed to be as orderly as the Tacoma Narrows bridge, then the heart would be susceptible to the analog for the heart of the wind-induced resonances that destroyed the bridge. It is now becoming accepted that organisms respond to influences that cover a wide range of scales, and so can immunize themselves against the damage that can result from too strong a response to an external influence at any one scale and responding less to influences at other scales. Thus, the theme of the disease then becomes the "loss of healthy disorder" or, as it is often called, the "loss of complexity".

Scientists are now challenged to characterize and understand this healthy disorder. It is becoming apparent that this disorder occurs on a huge range of scales, and that the contribution of one scale is related to the contribution of another scale by a quantitative relation called a power law, characterized by a unique exponent (e.g., the unique exponent of the power law that graphs as a parabola is 2). Research on this phenomenon, called scale invariance [6], was perhaps most intense and productive during the 1960s; it was carried out primarily by practitioners in the interdisciplinary field of phase transitions and critical phenomena [7], where one studies the fluctuations on all scales not of a healthy bridge or a healthy heart, but of a system of interacting particles near the system's critical point. This "in animate" system is a paradigm for many complex systems in nature, both inanimate and animate, that also display statistical properties on a huge range of length or time scales.

The fluctuations of a healthy heartbeat also exhibit scale invariance [8]. We see, as shown in Fig. 1, that this complex object can have the same statistical properties and hence "look almost the same" at many different scales of observation [9,10]. Of course, the actual object is different, but since its *statistical* properties are the same, one cannot readily distinguish the original complex object from a magnification of a part of it.

3. Scale invariance in other complex systems

The different scaling properties of healthy and diseased hearts have their counterparts in a wide range of complex systems. Just as the most likely way a human can perish is via a cardiovascular incident, the most likely way the biosphere could perish is through the extinction of vital species. Accordingly, many scientists study the growth and shrinkage of populations of various species, and recently it was found that when the population of a given bird species grows and shrinks, the pattern of changes is scale invariant [2].

Indeed, it is beginning to appear that a number of ecological phenomena obey regular laws which are scale invariant, and that these laws are universal in the sense that they do not depend on details concerning the actual species. Perhaps we are moving closer to understanding the wider implications of the scientific concepts behind the prescient intuition of Darwin, when he wrote near the end of *Origin of Species*:

> It is interesting to contemplate an entangled bank, clothed with many plants of many kinds, with birds singing on the bushes, with various insects flitting about, and

with worms crawling through the damp earth, and to reflect that these elaborately constructed forms, so different from each other, and dependent on each other in so complex a manner, have all been produced by laws acting around us.

Today, Darwin's "entangled bank" encompasses large geographic regions inhabited by people and other living species, regions that are too important to be neglected by serious science. The entangled bank metaphor may not be unrelated to the economic infrastructure of the globe. Indeed, a third way that scale invariance and universality impacts society has to do with the economy. Researchers have found new and surprising results by applying concepts and methods of scale invariance and universality to the economy. The economy is perhaps the most complex of all complex systems. A very small piece of "bad news" in a remote market may trigger a very large response in financial indices all over the globe. The societal impact of such economic fluctuations can be devastating. Privately, economists will confirm that the probability of such an "economic earthquake" is not entirely negligible – that a sudden and disastrous "phase transition" could occur from the present healthy state of our economy to a new state of a completely devastated economy. A noteworthy example of the societal devastation caused by economic earthquakes is the collapse of the German economy following World War I, which directly contributed to the rise of Hitler. Another example is the recent "devaluation" in Indonesia that has contributed to the starvation of many of Indonesia's poor.

In the case of economics, unlike other complex systems, virtually every economic transaction has been recorded – somewhere. The challenge is to obtain the needed data and to analyze them in such as way as to reveal the underlying principles. Remarkably, one finds that if one makes a histogram of price changes for any stock (the analog of the Gutenberg – Richter histogram of earthquake magnitude) this histogram is very close to a power law [11–14]. This discovery suggests that large shocks are related in a scale-invariant fashion to smaller, commonplace, economic fluctuations – i.e., large shocks and everyday economic fluctuations are basically different manifestations of the same phenomenon. The greatest societal impact occurs when "the big one" occurs, whether it be a geophysical earthquake or an economic earthquake. Hence, scaling concepts make it possible for scientists to understand these rare but catastrophic events through appropriately designed research focused on everyday phenomena.

4. Universality in complex systems

This scale invariance in complex systems is matched by an equally remarkable phenomenon of universality. For example, the functional form of the histograms describing how organizations grow or shrink from year to year appear to be identical whether one studies the growth statistics of business firms [15,16], countries [17], or university research budgets [18]. When one studies firm growth in diverse countries, one finds similar statistical properties even though the economies of the countries are radically different [19].

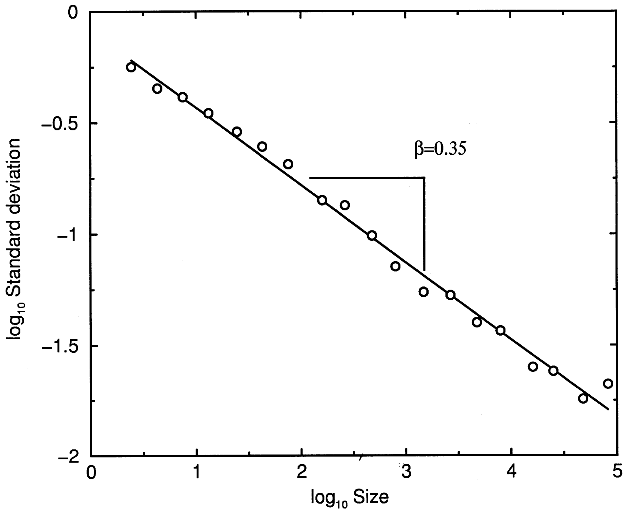

Fig. 2. Log–log plot showing the dependence of the standard deviation of a family of histograms for the growth rates of a group of bird species of similar size. The linearity implies a power law relation, and the slope of the regression fit gives the exponent β, which has a value ($\beta \approx 1/3$) roughly twice as large as that found for firm growth and country growth ($\beta \approx 1/6$) and slighly larger than found for university research budgets ($\beta \approx 1/4$). This figure is courtesy of T.H. Keitt.

Perhaps more surprising is that one finds when studying the growth of bird populations histograms with the same "tent" shape as found for the economics examples [2]. The fashion in which the standard deviation of the histogram decreases with size (Fig. 2) is qualitatively the same as for the economy examples (a power law), but quantitatively different (the exponent for the birds problem is roughly twice as large as found for the economy examples).

This "universality" is also found for the scaling laws uncovered in finance studies; e.g., when one compares countries as different as the USA and Norway, one finds similar scaling of price changes [20]. We are hoping to develop a theoretical framework within which to understand these intriguing examples of universality.

Why does universality arise in these complex systems? To develop some feeling for the concept of universality, consider the field of critical point phenomena. The history of twentieth century physics has been marked by many paradigm shifts – as reviewed in the opening lecture of Professor P.C. Hohenberg (see also the minireview [21]). One of these is the way in which we think of phase transitions, where the macroscopic state of an entire system suddenly changes when a microscopic tuning parameter crosses a threshold value.

Suppose we hold up a bar magnet. We know it is a ferromagnet because it is capable of picking up thumbtacks, the number of which defines an "order parameter" M. As we heat this system, M decreases and eventually, at a certain critical temperature T_c, it reaches zero: no more thumbtacks remain! In fact, the transition is remarkably sharp, since M approaches zero at T_c with infinite slope. Such singular behavior is an example of a "critical phenomenon." Critical phenomena are by no means limited to the order parameter. For example, response functions such as the specific heat and the isothermal susceptibility become infinite at the critical point.

One prototype example used to understand such critical phenomena is the Ising model of magnetism, which describes a collection of spins as objects that can exist in only two alignments, up or down, with the rule that whenever neighboring spins are in the same alignment, the energy is lowered by a small amount J. At zero temperature, all spins will be in the same alignment; the system is in a configuration of "absolutely perfect order". At infinite temperature, the above rule has no effect and a spin is likely to be in either alignment; the system is in a configuration of "absolutely perfect disorder". At some intermediate critical temperature T_c, the system is on the knife-edge of just barely ordering.

One reason for the interest in such critical phenomena is *scale invariance*: the fluctuations that exist near T_c occur on all possible length and time scales. A second reason for our interest is called *universality*: the striking similarity in behavior near the critical point among systems that are quite different from each other far from the critical point. A celebrated example is the Lee-Yang "lattice-gas" analogy between the behavior of a single-axis ferromagnet and a simple fluid, near their respective critical points. Even the numerical values of the critical-point exponents describing the quantitative nature of the singularities are identical for large groups of apparently diverse physical systems.

We have been seeking to develop a a more comprehensive understanding of the recently-emerging examples of scaling and universality that occur in a wide range of complex systems. Specifically, we are working to achieve some understanding of why it is that power laws arise, and in particular to develop a number of heuristic explanations for the existence of power laws and universality in models of magnetism.

Why would understanding the origin of power laws point the way to understanding their ubiquity in complex systems? One possible answer concerns the way in which correlations spread throughout a system comprised of subunits. The puzzle is to understand how can these inter-dependences give rise not to exponential functions – as ones intuition would suggest – but rather to the power laws characteristic of critical phenomena.

The paradox is simply stated: the probability that two spins are aligned is unity only at $T=0$, and our intuition tells us that for $T>0$ the correlation $C(r)$ between subunits separated by a distance r must decay exponentially with r – for the same reason the value of money stored in ones mattress decays exponentially with time (each year it loses a constant fraction of its worth). Thus, we might expect that $C(r) \sim e^{-r/\xi}$, where ξ, the correlation length, is the characteristic length scale above which the correlation function is negligibly small. Experiments and also calculations on mathematical models confirm that correlations do indeed decay exponentially. However, if the system is right at its critical point, then the rapid exponential decay magically turns into a much less rapid long-range power-law decay of the form $C(r) \sim 1/r^A$, which is of infinite range.

How can correlations actually propagate an infinite distance, without requiring a series of amplification stations all along the way? We can understand such "infinite-range propagation" as arising from the huge multiplicity of interaction paths that connect two spins if $d > 1$ (if $d = 1$, there is no multiplicity of interaction paths, and spins order only at $T = 0$).

For any $T > T_c$, the correlation between two spins along each of the interaction paths that connect them *decreases* exponentially with the length of the path. On the other hand, the number of such interaction paths *increases* exponentially, with a characteristic length that is temperature independent, depending primarily on the lattice dimension. This exponential increase is multiplied by a "gently decaying" power law that is negligible except for the very special point, the critical point.

Consider a fixed temperature T_1 far above the critical point, so that ξ is small, and consider two spins separated by a distance r which is larger than ξ. The exponentially decaying correlations along each interaction path connecting these two spins is so severe that it cannot be overcome by the exponentially growing number of interaction paths between the two spins. Hence, at T_1 the exponential decrease in correlation along each path wins the competition between the two exponentials, and we anticipate that the net correlation $C(r)$ falls off exponentially with the distance r. Consider now the same two spins at a fixed temperature T_2 far below the critical point. Now the exponentially decaying correlation along each interaction path connecting these two spins is insufficiently severe to overcome the exponentially growing number of interaction paths between the two spins. Thus, at T_2 the exponential increase in the number of interaction paths wins the competition. Clearly there must exist some intermediate temperature in between T_1 and T_2 where the the two exponentials just balance, and this temperature is the critical temperature T_c. Right at the critical point, the gentle power-law correction factor in the number of interaction paths, previously negligible in comparison with the exponential function that it multiplies, emerges as the victor in this stand-off between the two warring exponential effects. Further, this geometric exponent giving the correction to the number of paths joining two points depends only on the dimension of the system. As a result, two spins in a system right at this knife-edge of criticality are well correlated even at arbitrarily large separation.

One element of current research is to explore the degree to which this heuristic "explanation" of scale invariance (power laws) and universality in the simple Ising model can be extended to other complex systems, and can be made more mathematically precise. Preliminary work suggests that, indeed, it can be carried over to percolation, where the role of temperature T is played by the bond occupation probability p. The Ising model can be expressed in the geometrical language of percolation by focusing on the clusters of aligned spins that form. It has not escaped our attention that possibly one reason that diverse systems in such different fields as physics, biology, and ecology have quantitative features in common may relate to the fact that the complex interactions characterizing these systems could be mapped onto some geometric sysem, so that scaling and universality features of other complex systems may ultimately be understood – just as the Ising model can now be so interpreted – in terms of the connectivity of geometrical objects.

We conclude by thanking all our collaborators and colleagues from whom we learned a great deal. These include the researchers and faculty visitors to our research group with whom we have enjoyed the pleasure of scientific collaboration. Those whose research provided the basis of this lecture summary include, in addition to the co-authors, S.V. Buldyrev, D. Canning, P. Cizeau, X. Gabaix, A.L. Goldberger, S. Havlin, R.N. Mantegna, C.-K. Peng, M.A. Salinger, and M.H.R. Stanley. This work was supported in part by grants from the National Science Foundation and by the NIH/National Center for Research Resources (grant P41 13622).

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