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Fractionally integrated process for transition economics

Boris Podobnik[a,*], Dongfeng Fu[b], Timotej Jagric[c], Ivo Grosse[d], H. Eugene Stanley[b]

[a]Faculty of Civil Engineering, University of Rijeka, Rijeka, Croatia, Rudjer Boskovic Institute, Zagreb, Croatia
[b]Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA
[c]Faculty of Economics and Business, University of Maribor, Maribor, Slovenia
[d]Institute of Plant Genetics and Crop Plant Research (IPK), 06466 Gatersleben, Germany

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Abstract

We analyze the European transition economics and show that many time series of major indices exhibit (i) power-law correlations in their values, (ii) power-law correlations in their magnitudes and (iii) an asymmetric probability distribution. Applying the phase randomization procedure to these time series, we show that magnitude correlations completely vanish. We propose a stochastic model that can generate time series with features (i), (ii) and (iii), and we show by means of numerical simulations that this model is capable of reproducing these three features found in the empirical data.

Keywords: Fractionally integrated process; Detrended fluctuation analysis; Phase randomization

1. Introduction

The weak form of the law of market efficiency states that the present price of a stock impounds all of the information about past prices, and this implies that stock prices at any future time cannot be predicted. If the weak form of the law of market efficiency were not true, market analysts could make profit by interpreting charts of the past history of stock prices. Numerous studies of financial time series have supported the hypothesis of the weak form of the law of market efficiency for various stock markets [1–4] by demonstrating that the serial dependence in stock price changes is negligibly small and, hence, not sufficient for formulating trading rules that allow a profitable investment timing.

However, some studies have found evidence that there is a statistically significant long memory in time series of some of the individual US stocks listed on NYSE [5]. In contrast to the predominant behavior of financial time series of developed markets to exhibit only very short serial auto-correlations, financial time series of emerging markets exhibit a different behavior [6]. For example, a significantly long memory was found for weekly returns of a large number of Greek stocks [7] and for daily index returns recorded for six transition economies in east and central Europe [8].

*Corresponding author.
E-mail address: bp@sirius.phy.hr (B. Podobnik).

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2. Data analysis

We investigate financial time series of index returns of 10 European transition economies. We show that all time series exhibit (i) an asymmetric probability distribution, (ii) serial long-range auto-correlations [8], and (iii) serial long-range magnitude auto-correlations in agreement with financial time series commonly found in developed markets [9,10]. Finally, we show by means of numerical simulations that these properties can be encompassed by the stochastic processes recently proposed in Ref. [11].

We analyze the following 10 stock market indices PX50, BUX, WIG20, RTS, SAX, SBI, CROEMI, VILSE, TALSE and RICI corresponding to the following 10 transition economies of central and east Europe Czech Republic, Hungary, Poland, Russia, Slovakia, Slovenia, Croatia, Lithuania, Estonia and Latvia. All data are recorded daily, and we define the relative price changes of the logarithmized indices $S(t)$ by

$$R_t = \log S(t + \Delta t) - \log S(t), \tag{1}$$

where $\Delta t = 1$ corresponds to a time lag of one day. Basic statistics for the resulting time series $R(t)$ are presented in Table 1.

The skewness $\langle (x-\mu)^3 \rangle / \sigma^3$ is a measure of asymmetry, where μ is the expectation value of index x and σ is the standard deviation of index x. Table 1 shows that none of the index time series has a vanishing skewness, i.e., all of them have an asymmetric probability distribution. The five time series PX50, SBI, CROEMI, TALSE and RICI show a positive skewness, i.e., their probability distributions have a pronounced right tail, whereas the other five time series show a negative skewness, i.e., their probability distributions have a pronounced left tail. In Fig. 1, we show the probability distribution of the index returns of PX50, which exhibits a positive skewness.

In order to investigate to which degree the probability distributions are peaked (leptokurtic), we calculate the kurtosis defined as $\langle (x-\mu)^4 \rangle / \sigma^4$. For a Gaussian probability distribution, the kurtosis is equal to 3, for a probability distribution with more weight in the center and less weight in the tails, the kurtosis is smaller/greater than 3, and for a probability distribution with less weight in the center and more weight in the tails, the kurtosis is smaller/greater than 3. Table 1 shows that for none of the 10 index time series, the observed probability distribution are Gaussian. In order to test the null hypothesis that the observed probability distributions are Gaussian, we perform the Jarque–Bera test (chi-square with $\mathrm{df} = 2$). The reported p value is the probability that the Jarque–Bera statistic exceeds the observed value, and a small p value leads to the rejection of the null hypothesis that the observed probability distributions are Gaussian. Table 1 shows that we must reject the null hypothesis that the observed probability distributions are Gaussian for all of the 10 time series.

We employ the detrended fluctuation analysis (DFA) method [12] to analyze temporal auto-correlations. Following the DFA method, a given time series x_i of total length N is first mapped to the random walk

Table 1
Basic statistics of financial data

Country	*Rus*	*Hun*	*Pol*	*Slovak*	*Sloven*	*Czech*	*Lit*	*Lat*	*Est*	*Cro*
Index	*RTS*	*BUX*	*WIG*20	*SAX*	*SBI*	*PX*50	*VILSE*	*RICI*	*TALSE*	*CROEMI*
St. dev.	0.031	0.017	0.022	0.014	0.014	0.014	0.007	0.010	0.019	0.014
Skewness	−0.344	−0.865	−0.446	−0.409	0.416	1.342	−1.065	1.240	2.944	0.731
Kurtosis	7.96	17.69	11.97	9.48	25.34	17.18	26.89	22.46	47.80	12.51
Jarque–Bera	2342	30775	8476	3928	60087	22270	43803	167391	29459	5887
Probability	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
α_{R_t}	0.60	0.59	0.52	0.53	0.62	0.63	0.63	0.58	0.70	0.58
$\alpha_{\|R_t\|}$	0.79	0.80	0.84	0.66	0.74	0.86	0.69	0.65	0.80	0.70
$\alpha_{\tilde{R}_t}$	0.59	0.58	0.52	0.51	0.58	0.67	0.63	0.56	0.69	0.57
$\alpha_{\|\tilde{R}_t\|}$	0.51	0.50	0.45	0.53	0.51	0.47	0.54	0.55	0.53	0.52
Data points	2232	3373	2530	2204	2884	2567	1829	2025	183	1522

Besides skewness and kurtosis, which are the measures for asymmetry and "fatness" in the tails, also shown is Jarque–Bera test of normality. Also shown are DFA exponents for time series of indices and their magnitudes together with the corresponding values obtained after phase randomization.

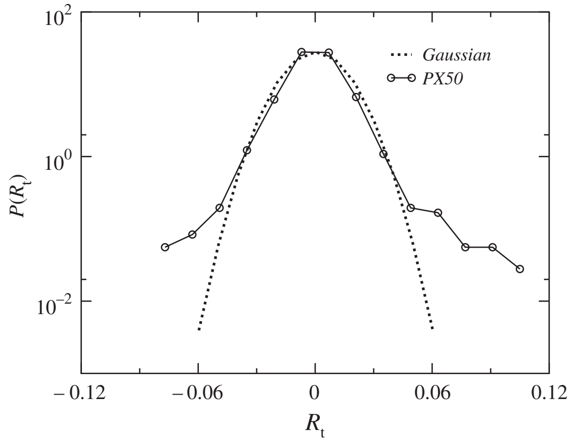

Fig. 1. Probability distribution of the daily rate of return R_t, $P(R_t)$, calculated for the PX50 index and Gaussian distribution with the same standard deviation as found for the PX50 index. Due to the visual differences in the tails of these two distributions, the kurtosis of $P(R_t)$ for the PX50 index is 17, which is much greater than 3, the kurtosis of $P(R_t)$ for the Gaussian probability distribution. Also, $P(R_t)$ for the PX50 index is positively skewed, in contrast to $P(R_t)$ for the Gaussian probability distribution which is symmetric.

$y(k) = \sum_{i=1}^{k}(x_i - \langle x_i \rangle)$, where $\langle x_i \rangle$ is the average of x_i. Second, the random walk $y(k)$ is divided into boxes of equal length n, and the local trend of $y(k)$ is calculated in each box by a least-square fit. Third, the random walk $y(k)$ is detrended by subtracting the local trend $y_n(k)$ in each box. And fourth, for a given box size n, the root-mean-square deviation is calculated by

$$F(n) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}[y(k) - y_n(k)]^2}. \tag{2}$$

The detrended fluctuation function $F(n)$ follows a scaling law $F(n) \propto n^{\alpha}$ if the time series x_i is power-law auto-correlated. A scaling exponent $\alpha > 0.5$ corresponds to time series with power-law correlations, $\alpha < 0.5$ corresponds to time series with power-law anti-correlations, and $\alpha = 0.5$ corresponds to time series with no or only short-range auto-correlations.

Table 1 shows the DFA scaling exponents α for the 10 time series R_t. The results indicate that there are at least two groups of markets. The first group is characterized by strong and medium long-range auto-correlations and includes indices of Estonia ($\alpha = 0.7$), Lithuania ($\alpha = 0.63$), Czech Republic ($\alpha = 0.63$), Slovenia ($\alpha = 0.62$), Russia ($\alpha = 0.6$), Hungary ($\alpha = 0.59$), Latvia ($\alpha = 0.58$), and Croatia ($\alpha = 0.58$). The second group is characterized by weak long-range auto-correlations and includes Poland ($\alpha = 0.56$) and Slovakia ($\alpha = 0.53$).

Next, we calculate the DFA scaling exponents for the time series of $|R_t|$. From Table 1 we see that for each time series (i) $|R_t|$ shows long-range auto-correlations, as found for almost all well-known market indices, and (ii) the DFA scaling exponent of $|R_t|$ is greater than the DFA scaling exponent of R_t.

In order to investigate to which degree the 10 time series exhibit linear and nonlinear properties [13,14], we apply a phase-randomization of the original time series, which changes (does not change) magnitude auto-correlations for a nonlinear (linear) process [15]. The phase-randomization procedure works as follows. First, one performs a Fourier transform of the original time series. Second, one randomizes the Fourier phases but keeps the Fourier amplitudes unchanged. Third, one performs an inverse Fourier transform and obtains the surrogate time series $\tilde{R}_t$.

Fig. 2 shows the detrended fluctuation functions $F(n)$ of the time series R_t and their magnitudes $|R_t|$ for the Russian index together with the phase-randomized surrogate time series $\tilde{R}_t$ and their magnitudes $|\tilde{R}_t|$. As expected, $\tilde{R}_t$ exhibits the same $F(n)$ curve as the original time series R_t [13]. In contrast, the magnitudes of the surrogate time series $|\tilde{R}_t|$ are uncorrelated ($\alpha_{|\tilde{R}|} = 0.5$), whereas the magnitudes of the original time series are

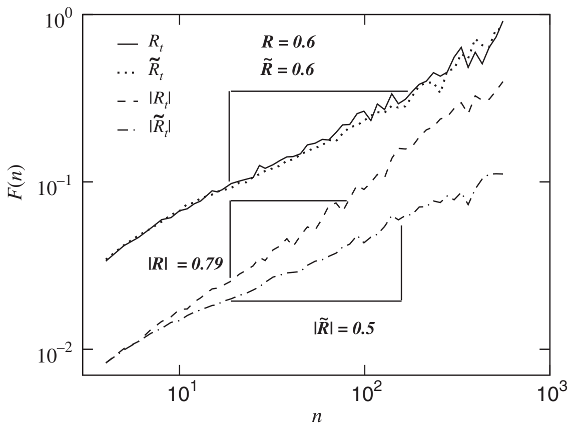

Fig. 2. DFA curves, $F(n)$, calculated for four time series of rate of return of Russian RTS index: times series R_t, time series obtained after phase randomization procedure $\tilde{R}_t$, and two magnitudes time series, $|R_t|$ and $|\tilde{R}_t|$. DFA exponents are $\alpha_R = 0.6$, $\alpha_{|R|} = 0.79$, and $\alpha_{\tilde{R}} = 0.6$, and $\alpha_{|\tilde{R}|} = 0.5$. Note that after phase randomization time series $|\tilde{R}_t|$ exhibits no auto-correlations.

power-law auto-correlated ($\alpha_{|R|} = 0.79$). We find the same behavior for all of the other 10 indices, i.e., auto-correlations in $|\tilde{R}_t|$ vanish by a phase randomization of the original time series R_t.

3. Model and simulation

To model time series R_t with an asymmetric probability distributions $P(R_t)$ and power-law auto-correlations in R_t and $|R_t|$, we employ the stochastic process defined in Ref. [11]:

$$R_i = \sum_{n=1}^{\infty} a_n(\rho_1)[R_{i-n} - \lambda|R_{i-n}|] + \sigma_i \eta_i, \tag{3}$$

$$\sigma_i = \sum_{n=1}^{\infty} a_n(\rho_2)\,\frac{|R_{i-n}|}{\langle |R_i| \rangle}, \tag{4}$$

$$a_n(\rho) = \rho\,\frac{\Gamma(n-\rho)}{\Gamma(1-\rho)\Gamma(1+n)}. \tag{5}$$

Apart of introduced asymmetry, the process is defined as a combination of two processes proposed in Refs. [16–18]. λ and $\rho_{1,2}$ in $(0, 0.5)$ are asymmetric and scaling free parameters, respectively. Γ denotes the Gamma function, and η_i denotes independently and identically distributed Gaussian variables with expectation value $\langle \eta_i \rangle = 0$ and variance $\langle \eta_i^2 \rangle = 1$. The weights $a_n(\rho)$ satisfy the constraint $\sum_{n=1}^{\infty} a_n(\rho) = 1$ [18], and by using the Stirling formula it can be shown that the weights scale as $a_n(\rho) \propto n^{1-\rho}$ for asymptotically large values of n.

For the process of Eqs. (3)–(5), we have shown in Ref. [15] that for the case $\lambda = 0$, $\rho_1 = \rho_2 = \rho$ ($\rho_{1,2} > 0.5$), the following two scaling relations $\alpha_R = 0.5 + \rho$ and $\alpha_{|R|} = 0.5 + \rho$ hold between two DFA exponents α_R and $\alpha_{|R|}$ and scaling parameter ρ. To model different auto-correlations exponents found for empirical time series and their magnitudes, we allow the parameters ρ_1 and ρ_2 in Eqs. (3)–(5) to be different. By numerical simulations [19] for model time series R_t where $\lambda = 0$, we find that the scaling relation $\alpha_R = 0.5 + \rho_1$ approximately holds regardless of value for ρ_2. $\lambda = 0$ we take due to small skewness in the empirical distributions. Hence, parameter ρ_2 does not affect auto-correlations in R_t. In contrast, for magnitude auto-correlations $|R_t|$ we find two different cases: (a) For the case $\rho_1 < \rho_2$ it holds $\alpha_{|R|} = 0.5 + \rho_2$ (see Fig. 3), while (b) for$\rho_1 > \rho_2$ we find no simple dependence, where both parameters ρ_1 and ρ_2 control auto-correlations in $|R_t|$ depending on range of ρ_1 (see Fig. 3).

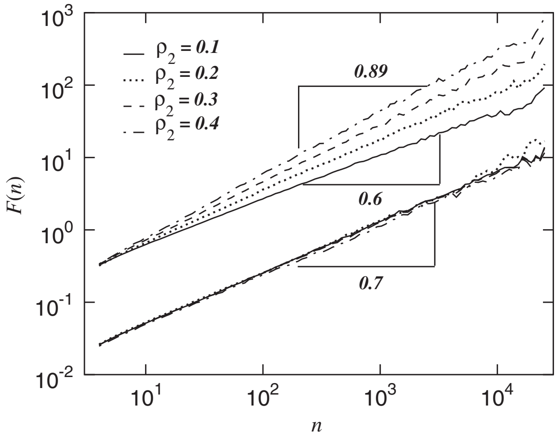

Fig. 3. DFA curves, $F(n)$, calculated for four times series of the process R_t (lower set) and $|R_t|$ (upper set) with fixed $\rho_1 = 0.2$ and varying ρ_2. DFA exponent for R_t, α_R, does not depend on parameter ρ_2. DFA exponent for $|R_t|$, $\alpha_{|R|}$, approximately satisfies $\alpha_{|R|} = 0.5 + \rho_2$. This does not hold for larger values of ρ_1 where $\alpha_{|R|}$ mainly depends on ρ_1.

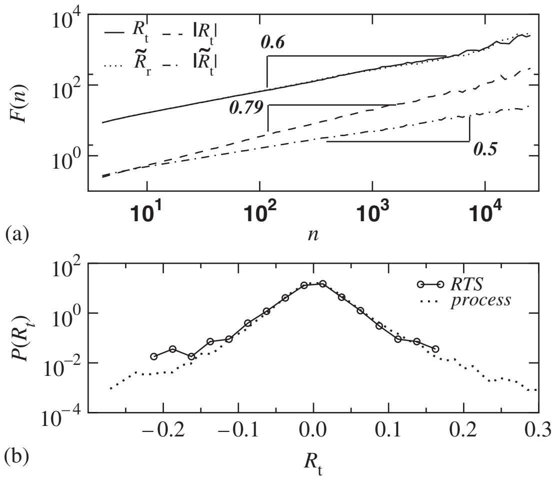

Fig. 4. DFA curves, $F(n)$, calculated for times series R_t for the process with $\rho_1 = 0.1$, $\rho_2 = 0.29$ and $\lambda = -0.1$ arbitrary chosen to model small skewness and corresponding time series $\tilde{R}_t$ obtained after phase randomization procedure, and for two magnitudes time series $|R_t|$ and $|\tilde{R}_t|$. DFA exponents are $\alpha_R = 0.6$, $\alpha_{|R|} = 0.79$ and $\alpha_{\tilde{R}} = 0.6$, $\alpha_{|\tilde{R}|} = 0.5$. We find that after phase randomization time series $|\tilde{R}_t|$ exhibits no auto-correlations. (b) Comparison of the probability distribution of daily return, $P(R_t)$, for the Russian index RTS and for the process. For the sake of comparison, $P(R_t)$ for the process is rescaled. Note that the differences in the tails are due to different lengths of empirical and model time scales.

To model empirical data characterized by different auto-correlations in R_t and $|R_t|$ where $\alpha_{|R|} > \alpha_R$ (see Table 1), we employ the process of Eqs. (3)–(5) where $\rho_1 < \rho_2$ (case (a)). For that case we simply take two parameters ρ_1 and ρ_2 from scaling relations $\alpha_R = 0.5 + \rho_1$ and $\alpha_{|R|} = 0.5 + \rho_2$, respectively.

Now we model the Russian index RTS, a representative index among transient economics. For the two parameters $\rho_1 = 0.1$ and $\rho_2 = 0.29$, set to model scaling exponent $\alpha_R = 0.6$ $\alpha_R = 0.79$ found in the data (see Table 1), we perform numerical simulations and in Fig. 4 we show the scaling function $F(n) \propto n^{\alpha}$ for both model time series R_t and $|R_t|$. We take $\lambda = 0.1$ a small value for λ to account for small skewness in the empirical distribution. For model time series we find (i) $\alpha_{|R|} > \alpha_R$ where $\alpha_R = 0.6$ and $\alpha_{|R|} = 0.79$ as we previously found in the empirical data; (ii) after performing the phase-randomization procedure, magnitude auto-correlations $|\tilde{R}_t|$ vanish, while auto-correlations in $\tilde{R}_t$ virtually remain unchanged compared to the original time series R_t, that is a behavior characteristic for empirical data. In Table 1 we see that the same scaling properties hold for all indices analyzed. We also find after rescale that the probability distribution $P(R_t)$ for the process fits the probability distribution of RTS index.

4. Conclusions

In conclusion, we analyze the long-range dependence in the capital markets of 10 transition economies in central and east Europe. Apart of Poland and Slovakia, all market indices analyzed exhibit (i) long-range dependence of power-law form. As expected, all market indices also show (ii) long-range dependence on the magnitudes. The probability distributions exhibit (iii) asymmetric behavior. These properties should be included in modeling of trading systems. For that reason, we propose a stochastic process specified by three parameters with properties as found in the empirical data. For data analyzed, model parameters are easily related with scaling exponents of empirical time series. The process can also be useful in modeling the US market since some US stocks exhibit long-range dependence [5].

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References

[1] E.F. Fama, J. Bus. 38 (1965) 34.
[2] C.W.J. Granger, O. Morgenstern, Spectral Analysis of New York Stock Market Prices, vol. 16, Kyklos, 1963.
[3] J.L. Sharma, R.E. Kennedy, J. Finan. Quant. Anal. 12 (3) (1977) 391.
[4] C. Hiemstra, J.D. Jones, J. Empirical Finance 4 (1997) 373.
[5] J.T. Barkoulas, C.F. Baum, Econ. Lett. 53 (1996) 253.
[6] J.H. Wright, Emerg. Market. Q. 5 (2001) 50.
[7] E. Panas, Appl. Finan. Econ. 38 (2001) 395.
[8] T. Jagric, B. Podobnik, M. Kolanovic, Eastern Eur. Econ. 43 (4) (2005) 85.
[9] S. Taylor, Modeling Financial Time Series, Wiley, New York, NY, 1994.
[10] Z. Ding, C.W.J. Granger, J. Econ. 73 (1996) 185.
[11] B. Podobnik, P.Ch. Ivanov, V. Jazbinsek, Z. Trontelj, H.E. Stanley, I. Gross, Phys. Rev. E 71 (2005) 025104.
[12] C.K. Peng, S.V. Buldyrev, S. Havlin, M. Simons, H.E. Stanley, A.L. Goldberger, Phys. Rev. E 49 (1994) 1685.
[13] J. Theiler, S. Eubank, A. Longtin, B. Galdrikian, J.D. Farmer, Physica D 58 (1992) 77.
[14] Y. Ashkenazy, P.Ch. Ivanov, S. Havlin, C.K. Peng, A.L. Goldberger, H.E. Stanley, Phys. Rev. Lett. 86 (2001) 1900.
[15] B. Podobnik, P.Ch. Ivanov, K. Biljakovic, D. Horvatic, H.E. Stanley, I. Grosse, Phys. Rev. E 72 (2005) 026121.
[16] C.W.J. Granger, R. Joyeux, J. Time Ser. Anal. 1 (1980) 15.
[17] J. Hosking, Biometrika 68 (1981) 165.
[18] C.W.J. Granger, Z. Ding, J. Econ. 73 (1996) 61.
[19] Instead of $n \to \infty$ in Eq. (1), we introduce a cutoff length $\ell = 20,000$, and we set $a_n(\rho) = 0$ for $n \geqslant \ell$.