ONSET OF HELICAL ORDER

J.F. NICOLL, G.F. TUTHILL, T.S. CHANG
Physics Department, Massachusetts Institute of Technology, Cambridge, Mass. 02139, USA

and

H.E. STANLEY
Physics Department, Massachusetts Institute of Technology, Cambridge, Mass. and Physics Department, Boston University, Boston, Mass. 02215, USA

Renormalization group methods are used to describe systems which model critical phenomena at the onset of helical order. This onset is marked by a change in the "bare propagator" used in perturbation theory from a k^2-dependence to a more general form. We consider systems which in the non-helical region exhibit $\mathcal{O}$ simultaneously critical phases. Results are given to first order in an ϵ-expansion. For the isotropic case of k^{2L} dependence and $\mathcal{O} = 2$, we give η to first order in $1/n$ for $d_- \leq d \leq d_+$ where $d_\pm$ are upper and lower borderline dimensions.

In 1959 Yoshimori [1], Villain [2], and Kaplan [3] independently proposed that for materials with certain forms of competing exchange interactions there could exist a ground state spin configuration in which the mean value of the order parameter varied periodically in space with a characteristic wavelength that depended on the exchange interactions and in general was not commensurate with the lattice constants of the material. Since their work, many materials have been found to display such helicoidal ordering [4].

The original theoretical work was concerned with ground state spin configurations, and was carried out in the molecular field approximation. Very recently Hornreich et al. [5] have used renormalization group methods to study phenomena associated with the onset of helicoidal ordering. This occurs at a specific point–termed a Lifshitz point [6]–in a phase diagram [fig. 1] in which temperature is plotted against some parameter p which may be conveniently thought of as a ratio of competing exchange interactions.

The onset of helical order can be incorporated into a Landau–Ginsberg model by including higher order derivatives of the magnetization. In particular, when the thermodynamic potential is written in terms of the Fourier transform of the magnetization, powers of the wave-vector k other than k^2 will occur. The usual uniform $k = 0$ phase will be thermodynamically favored as long as the k^2-term dominates for $k \to 0$. If the coefficient of the k^2-term can be made to vanish,

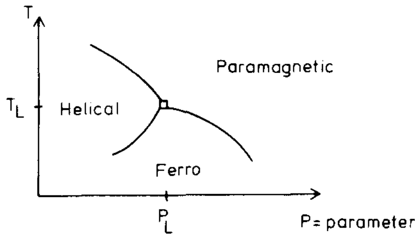

Fig. 1. Schematic phase diagram, indicating the occurrence of a Lifshitz point (T_L, p_L).

then the phases are still uniform but will have drastically altered correlation functions. If the coefficient of k^2 can be made negative, the free energy minimum will be achieved by a phase with non-zero k–a helical phase. Thus, even within the uniform phase region, the onset of helical order can be identified with the change in character of the derivatives in the thermodynamic potential, or equivalently, the bare propagator.

The simplest case is the "isotropic Lifshitz point" for which the bare propagator is given by

$$G^{-1} = (k^2)^L. \tag{1}$$

A more general situation is the anisotropic Lifshitz point with

$$G^{-1} = \sum_i k_i^{2L_i}, \tag{2}$$

with each k_i a d_i-dimensional vector; the propagator exponents L_i need not be integers.

We consider systems which in the uniform region encompass $\mathcal{O}$ simultaneously critical phases of the sort described in [7]. The case $\mathcal{O}=2$, $L_i=\{1,2\}$ was considered in ref. 5. We use renormalization group methods [8] to extend this to arbitrary $\mathcal{O}$ and $\{L_i\}$. The upper borderline dimension d_+ (above which mean-field exponents are correct) and the lower borderline dimension d_- (at which infrared divergences commence) depend on $\mathcal{O}$ and $\{d_i, L_i\}$. Thus the universality class is now presumably determined by $\mathcal{O}$, $\{d_i, L_i\}$ and n, where n is the spin symmetry; e.g. $\gamma = \gamma(d_i, n, \mathcal{O}, L_i)$.

In what follows we shall summarize our results, treating first the isotropic Lifshitz point and then the anisotropic Lifshitz point [8].

Case I. Isotropic Lifshitz point

A. ($\mathcal{O}=2$) critical points
1. Upper and lower borderline dimensionalities $d_\pm(L)$ (fig. 2)

$$d_+(L) = 4L, \qquad d_-(L) = 2L. \tag{3a,b}$$

Note that for $L>1$, the lower borderline dimension is greater than 3. Thus, the isotropic case is principally of academic interest.

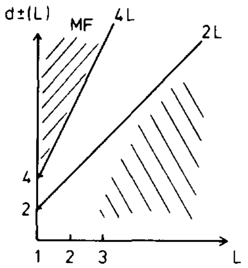

Fig. 2. Dependence of upper and lower critical dimensionalities $d_\pm(L)$ upon Lifshitz character L for the case of an isotropic Lifshitz point.

2. Exponents

Critical exponents were calculated to lowest order in

$$\epsilon \equiv d_+ - d = 4L - d, \tag{4}$$

and for η to lowest order in $1/n$. The result for the exponent η describing the decay of correlations at the Lifshitz point,

$$\eta(d, n, \mathcal{O}=2, L)$$

$$= \frac{(-1)^{L+1}}{n}\left[\frac{(4L-d)\sin\frac{1}{2}\pi(4L-d)}{\frac{1}{2}\pi}\right]\frac{1}{L} \times \frac{\Gamma(d-2L)\Gamma(2L)}{\Gamma(\frac{1}{2}d+L)\Gamma(\frac{1}{2}d-L)}, \tag{5}$$

is particularly interesting since it is an oscillatory function of d with $L+1$ zeros (cf. fig. 3).

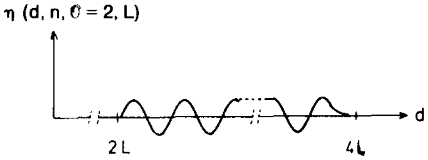

Fig. 3. Dependence upon lattice dimensionality d of the critical exponent η for an $\mathcal{O}=2$ critical point for the case of an isotropic Lifshitz point of Lifshitz character L.

3. Scaling laws

The various critical exponents are predicted to be related to one another by 2-exponent and 3-exponent scaling laws that are formally analogous to the 2-exponent and 3-exponent scaling laws relating the familiar $L=1$ exponents of an ordinary critical point. Thus, for example, we find for general L that

$$2-\alpha_L = d\nu_L, \tag{6a}$$

$$\gamma_L = (2L-\eta_L)\nu_L, \tag{6b}$$

$$\delta_L = \left(\frac{d}{2L-\eta_L}+1\right)\Big/\left(\frac{d}{2L-\eta_L}-1\right). \tag{6c}$$

Since eq. (6) contains some unfamiliar expressions, it is worthwhile to note that the usual thermodynamic scaling relations are maintained, e.g.

$$\delta_L = (2-\alpha_L+\gamma_L)/(2-\alpha_L-\gamma_L). \tag{7}$$

B. Critical point of arbitrary $\mathcal{O}$
1. Upper and lower borderline dimensionalities $d_\pm(\mathcal{O}, L)$

The generalization of eqs. (3) for arbitrary $\mathcal{O}$ is

$$d_+(\mathcal{O}, L) = 2L\mathcal{O}/(\mathcal{O}-1), \qquad d_-(\mathcal{O}, L) = 2L. \tag{8a,b}$$

Thus the lower borderline dimension d_- is independent of $\mathcal{O}$, while the upper borderline dimension decreases with $\mathcal{O}$ and approaches d_- as $\mathcal{O}$ approaches infinity.

2. Exponents

As for the case $\mathcal{O} = 2$, critical exponents were calculated to lowest order in

$$\epsilon \equiv (\mathcal{O} - 1)(d_+ - d) = 2L\mathcal{O} - d(\mathcal{O} - 1). \qquad (9)$$

3. Scaling laws

For each value of $\mathcal{O}$, there are a family of scaling laws relating the appropriate exponents.

Case II. Uniaxial Lifshitz anisotropy

Next we treat the physically interesting case in which one of the components of the d-dimensional wave vector $\boldsymbol{k} \equiv (k_1, k_2, \ldots, k_d)$ is raised to the power $2L_1$ so that eq. (1) becomes

$$G^{-1} = + k_1^{2L_1} + k_2^2 + k_3^2 + \ldots + k_d^2. \qquad (10)$$

1. Upper and lower borderline dimensionalities $d_\pm(\mathcal{O}, L_1)$

$$d_+(\mathcal{O}, L_1) = (3\mathcal{O} - 1)/(\mathcal{O} - 1) - 1/L_1 \qquad (11a)$$

$$d_-(\mathcal{O}, L_1) = 3 - 1/L_1. \qquad (11b)$$

Thus, $d_- < 3 \leq d_+$ for $\mathcal{O} \leq 2L_1 + 1$.

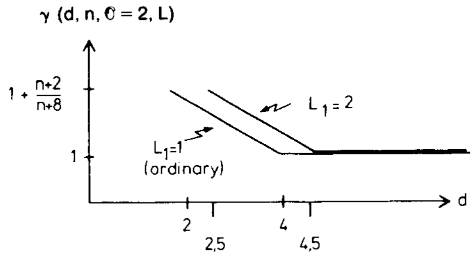

Fig. 4. Dependence upon d of the susceptibility critical exponent γ for the case of uniaxial Lifshitz anisotropy. Curves for anisotropy parameter L_1 greater than two are similarly shifted to the left, with the upper and lower borderline dimensionalities being given by eq. (11) with $\mathcal{O} = 2$.

2. Exponents

Critical exponents were calculated to lowest order in

$$\epsilon \equiv (\mathcal{O} - 1)(d_+ - d) = (3\mathcal{O} - 1) - (\mathcal{O} - 1)(d + 1/L_1).$$

The results bear many resemblances to the results for the case of an ordinary critical point ($L_1 = 1$). For example, fig. 4 shows the dependence on d of the critical exponent $\gamma(d, n, \mathcal{O} = 2, L_1)$ for the susceptibility.

3. Scaling laws

The scaling laws corresponding to eqs. (6) for an isotropic Lifshitz point are

$$2 - \alpha_L = \nu_{L_1} + (d - 1)\nu_\perp, \qquad (13a)$$

$$\gamma = (2L_1 - \eta_{L_1})\nu_{L_1} = (2 - \eta_\perp)\nu_\perp, \qquad (13b)$$

$$\delta = \left(\frac{1}{2L_1 - \eta_{L_1}} + \frac{d-1}{2 - \eta_\perp}\right) \Big/ \left(\frac{1}{2L_1 - \eta_{L_1}} - \frac{d-1}{2 - \eta_\perp}\right), \qquad (13c)$$

where the exponents with a subscript L_1 denote the behavior of correlations between spins joined by a vector whose components lie entirely along the "1" direction, and "⊥"denotes directions perpendicular to "1".

In [8], the general anisotropic case is considered and explicit expressions are given for all the exponents.

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