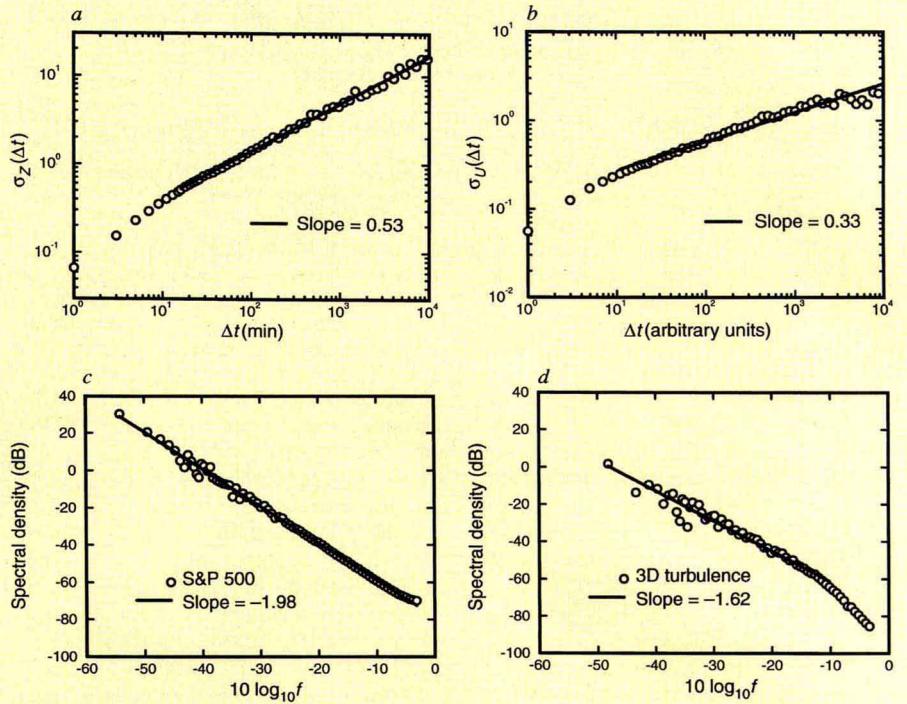


FIG. 1a, Standard deviation $\sigma_Z(\Delta t)$ of the probability distribution $P(Z)$ characterizing the price changes $Z_{\Delta t}(t)$ plotted double logarithmically as a function of Δt for the S&P 500 time series. After a time interval of superdiffusive behaviour ($0 < \Delta t \leq 15$ minutes), diffusive behaviour close to the one expected for a random process with independent identically distributed increments occurs; the measured diffusion exponent 0.53 is very close to the theoretical (uncorrelated) value $1/2$. b, Standard deviation $\sigma_U(\Delta t)$ of the probability distribution $P(U)$ characterizing the velocity changes $U_{\Delta t}(t)$ plotted double logarithmically as a function of Δt for the velocity-difference time series in turbulence. Data recorded in the atmosphere at a Taylor microscale Reynolds number R_λ of the order of 1,500 (data provided by K. R. Sreenivasan). After a time interval of superdiffusive behaviour ($0 < \Delta t \leq 10$), a diffusive behaviour close to the one expected for a fluid in the inertial range is observed (the measured diffusion exponent 0.33 is close to the theoretical (anti-correlated) value $1/3$). c, Spectral density of the S&P 500 time series for the time period 1984–87 representative of the 6-year time period 1984–89 (an investigation performed for the time period 1986–89 gives a curve overlapping with the figure shown). The $1/f^2$ power-law behaviour expected for a random process with independent increments is observed over a frequency interval of more than 4 orders of magnitude. d, Spectral density of the velocity time series of a three-dimensional fully developed turbulent fluid. The $1/f^{5/3}$ inertial range (low frequency) and the dissipative range (high frequency) are clearly observed.



of a price in foreign exchange markets (A. Arneodo *et al.*, preprint cond-mat/9607120 at <http://xxx.lanl.gov>).

Features observed in economic data that are not explained in terms of the TLF model are the time dependence of the scale factor parameter γ of the TLF distribution, which shows a fluctuating behaviour with bursts of activity localized in limited intervals of time, and the long-

range correlations observed in the time evolution of γ and in the variance of price changes (two quantities related to what is called 'volatility' in the economics literature) (ref. 5).

Can we reconcile the known intuitive parallels between finance and turbulence with the fact that quantitatively the two phenomena are quite different? This quantitative difference might disappear

for a turbulent system in an abstract space of non-integral dimensionality. Theoretical studies^{6,7} show that for $d \approx 2.05$ there exists (under the Taylor hypothesis) an uncorrelated turbulent behaviour characterized by a spectral density $S(f) \propto f^{-2}$, just as for the S&P 500. Further study is required to test if a 2.05-dimensional turbulence could be really consistent with the stochastic properties of market data and with the main assumption of mathematical finance that is that no arbitrage is possible in an efficient market⁸.

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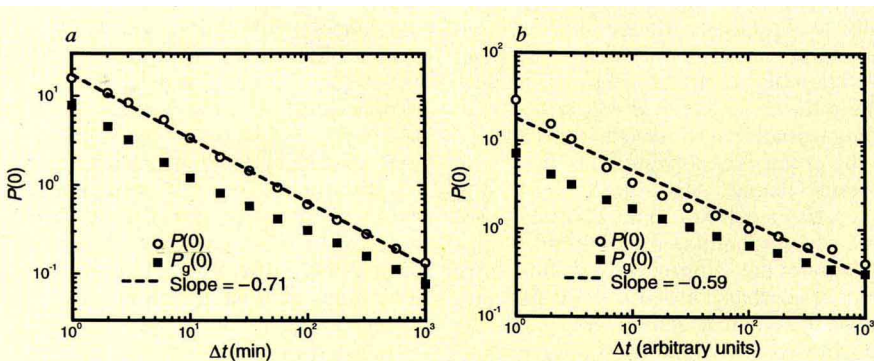


FIG. 2a, Probability of return to the origin $P(Z=0)$ for the S&P 500 index (circles) and $P_g(Z=0) = 1/\sqrt{2\pi}\sigma(\Delta t)$ (filled squares) as functions of the time sampling interval Δt . $P_g(Z=0)$ is the probability of return to the origin expected for a gaussian stochastic process determined by measuring the standard deviation $\sigma(\Delta t)$ of the experimental data. The two measured quantities differ in the full interval implying that the profile of the probability density function (PDF) must be non-gaussian. A power-law behaviour is observed for the entire time interval spanning three orders of magnitude. The slope of the best linear fit is -0.71 ± 0.025 . The difference between the two quantities is decreasing when Δt increases, implying a convergence to a gaussian process for high values of Δt . b, Probability of return to the origin $P(O)$ (circles) and $P_g(O)$ (filled squares) (defined in a) as functions of the time-sampling interval Δt for the velocity of the fully turbulent fluid. Again, the two measured quantities differ in the full interval, implying that the profile of the PDF must be non-gaussian. However, in this case, a single scaling power-law behaviour does not exist for the entire time interval spanning three orders of magnitude. The slope of the best linear fit (which is of quite poor quality) is -0.59 ± 0.11 .