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SELF-AVOIDING WALKS ON RANDOM NETWORKS OF RESISTORS AND DIODES

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We study the self-avoiding walks (SAW) on a square lattice whose various degrees of randomness encompasses many different random networks, including the incipient clusters of the directed, mixed and isotropic bond percolation. We apply the position-space renormalization group (PSRG) method and demonstrate that within the framework of this method one is bound to find that the critical exponent ν of the mean end-to-end distance of SAW on various two-dimensional random networks should be equal to the critical exponent of SAW on the ordinary square lattice. A detailed analysis of this finding, and similar findings of other authors, lead us to conclude that a debatable opposite finding, which has been predicted on the basis of different approaches, could be attained after a substantial refinement of the method applied.

1. Introduction

The self-avoiding walk (SAW) is a random walk that must not contain self-intersections. Statistical properties of the walk on a lattice are expected to expose criticality when the number of steps N approaches infinity. There has appeared an intriguing question that concerns the criticality of SAW when the translational symmetry of the underlying lattice is randomly perturbed, for instance by a quenched dilution of bonds. The most frequently studied quantity is the mean squared end-to-end distance critical exponent ν. In the last decade, almost twenty papers have been published[1–19]) offering different and, in some cases, conflicting answers. Roughly speaking there are two sides. On one side there are authors who claim that the critical exponent ν of SAW should be changed when a translationally invariant lattice is perturbed. On the other side there are those who claim that ν should not be changed. Various theoretical methods have been applied in these studies. In particular, the renormalization

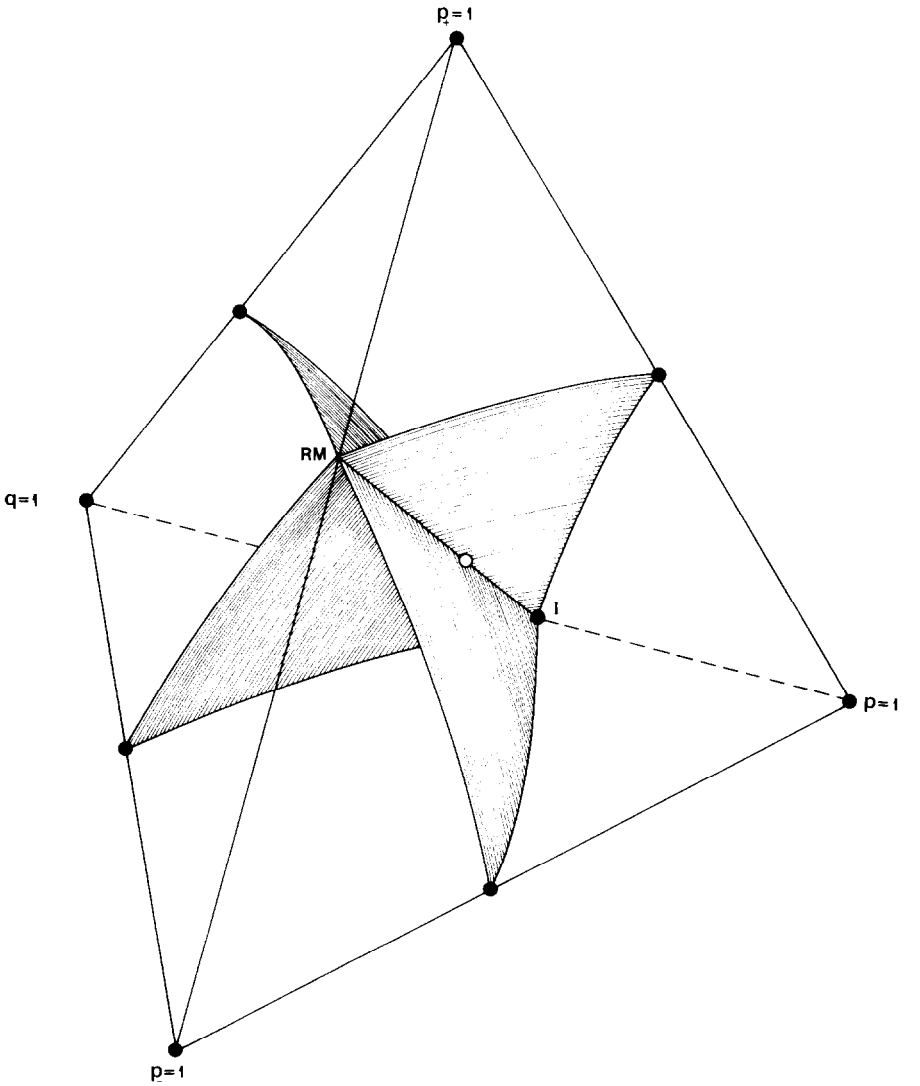

Fig. 1. The composition tetrahedron and the phase diagram of the four component random resistor–diode networks. For each point within the tetrahedron (1.1) is satisfied, that is to say the perpendicular distance to any face is equal to the probability of having the type of bond labelled at the opposite vertex. The points on the shaded surfaces correspond to the second-order percolation phase transition, and thus the line where they meet, i.e. the line that connects the point I and RM, is a multicritical line[21]). Fixed points of the RG transformations[21]) for the four probability parameters are depicted by the heavy dots, except for the mixed percolation point M which is indicated by the open circle. The point $p = 1$ corresponds to the ordinary square lattice, and, in the text, it will be designated as OSL.

group method has been used *on both sides*. This fact motivates our study of the problem.

We study the SAW problem on a square lattice whose randomness encompasses many different random networks. Namely, we assume that a bond of the square lattice can be present or removed with the probabilities p and q respectively. Furthermore, we assume that a bond can be eastward or northward oriented with the probability p_+, whereas it can be southward or westward oriented with probability p_-. The four probabilities satisfy the following relation:

$$p + p_+ + p_- + q = 1 . \tag{1.1}$$

The self-avoiding walker can pass along a non-oriented bond in any direction (of course, only for one time), while the oriented bonds can be passed only in the prescribed directions. From the physical point of view, the non-oriented bonds can be conceived as resistors, the oriented bonds can be conceived as diodes, and consequently the removed bonds should be viewed as vacancies. The corresponding network of bonds we shall call the random resistor–diode (RRD) network. The percolation problem[20]) of the RRD network was studied by Redner[21]). He applied the position-space renormalization group (PSRG) method and found that, depending on the set of values (p, p_+, p_-, q), there are various percolation thresholds (cf. fig. 1). Thus the point $I = (\frac{1}{2}, 0, 0, \frac{1}{2})$ corresponds to the standard (isotropic) bond percolation threshold[20]) and appears to be a fixed point[21]), whereas the so-called random Manhattan point $\mathrm{RM} = (0, \frac{1}{2}, \frac{1}{2}, 0)$ corresponds to a multiple percolation threshold[17,21]) whose critical properties are governed by the mixed percolation fixed point $M \cong (0.2563, 0.2437, 0.2437, 0.2563)$. In fact, every point on the line defined by $p = q \neq \frac{1}{2}$ and $p_+ = p_- \neq 0$ corresponds to a specific percolation threshold with the critical properties determined by the fixed point M [21]).

In this paper we demonstrate that the critical exponent ν of SAW on *all* RRD networks that are represented by points on the multicritical line (the line between RM and I) and on the line between I and the $p = 1$ point should be same, which means that ν of the translationally invariant lattice (the $p = 1$ case) should remain unaltered. This is proved within the framework of the PSRG method[22]) with an arbitrary rescaling factor. The details of our argument are presented in the next section, and later, in section 3, we present an overall discussion of our finding and findings of other authors.

2. Renormalization-group analysis

The relevant criticality of SAW is assumed[23]) to be captured by the two

power laws:

$$C_N \sim \mu^N N^{\gamma-1} \tag{2.1}$$

and

$$\langle R_N^2 \rangle \sim N^{2\nu}, \tag{2.2}$$

where C_N is total number of distinct N-step walks, $\langle R_N^2 \rangle$ is the mean squared end-to-end distance, μ is the connectivity constant (the ratio C_{N+1}/C_N approaches μ when $N \to \infty$), and γ and ν are the critical exponents. Further, it is customary to introduce the weight factor K (fugacity) for each step of the walk and to define two generating functions:

$$Z(K) = \sum_{N=0}^{\infty} C_N K^N \tag{2.3}$$

and

$$\xi^2(K) = \sum_{N=0}^{\infty} K^N C_N \langle R_N^2 \rangle / Z(K), \tag{2.4}$$

so that their leading singular terms are of the form

$$Z(K) \sim (1 - K\mu)^{-\gamma}, \tag{2.5}$$

$$\xi^2(K) \sim (1 - K\mu)^{-2\nu}, \tag{2.6}$$

when K approaches $1/\mu$ from below.

To study the criticality of SAW on a randomly perturbed lattice we look for an appropriate generalization of the generating functions (2.3) and (2.4). To this end let us rewrite (2.3) so that each individual walk is represented by a separate term in the sum

$$Z(K) = \sum_S K^{N_S}, \tag{2.7}$$

where S enumerates all possible walks. Now, when the lattice is randomly perturbed, each conceivable walk should be weighted by the probability that the underlying combination of bonds is properly preserved. For instance, if the square lattice is transformed into an RRD network, the four-step walk from point A to point B (cf. fig. 2) should be weighted by the probability

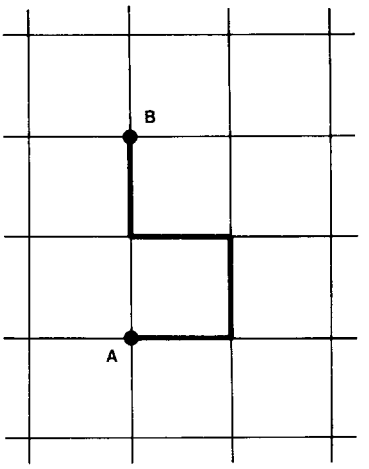

Fig. 2. The probability P_S of the feasibility of the four-step walk (SAW) from the point A to B, on a RRD network, is equal to $(p+p_+)^3(p+p_-)$ since the first, second and fourth step are feasible only if the underlying bond is either a resistor or a diode that is positively oriented, whereas the third step is feasible only if the corresponding bond is either a resistor or a negatively oriented diode.

$(p+p_+)^3(p+p_-)$. Thus the generating function for SAW on an RRD network should be of the form

$$Z(K, p, p_+, p_-, q) = \sum_S P_S K^{N_S}, \tag{2.8}$$

where P_S is the probability that the combination of bonds which corresponds to the Sth walk is preserved within the RRD network. It should be noticed that every P_S can be written as the product

$$P_S = \prod_{i=1}^{N_S} p_i, \tag{2.9}$$

where p_i is equal to either $p+p_+$ or $p+p_-$. Hence the generating function (2.8) is, in fact, a function of the products $K(p+p_+)$ and $K(p+p_-)$. Similar arguments are applicable to the generating function (2.4).

If we apply the PSRG method[22]) to study the SAW problem on an RRD network, our first step should consist in rescaling the RRD network under study into a new coarse-grained RRD network. For this purpose we shall use the cell-to-bond mapping with the scale factor b and the percolation rule[21]) illustrated in fig. 3. According to this mapping, a $b \times b$ cell of the underlying square lattice maps into the 1×1 cell, so that the vertical (horizontal) bond of the new cell becomes a positively oriented diode if the original cell carries such

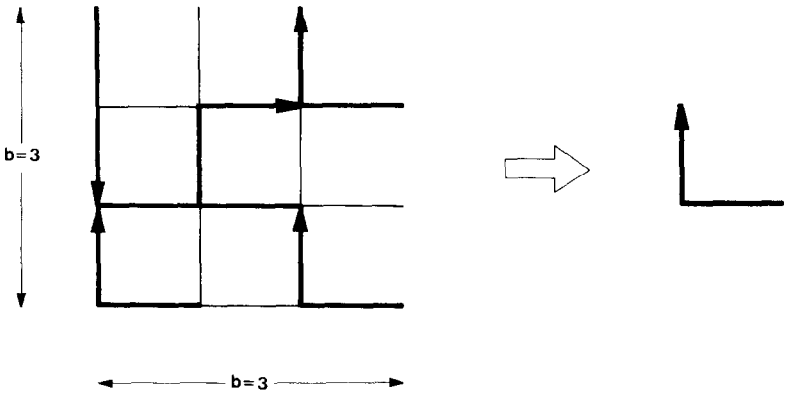

Fig. 3. Rescaling of a 3×3 cell of the square lattice into the 1×1 cell of a new coarse-grained lattice, and mapping of a part of the RRD network (heavy lines) into a part of the new coarse-grained RRD network. The vertical bond of the new cell is mapped into the northward oriented bond because the original configuration of bonds can be traversed vertically only in the northward direction, whereas the horizontal bond of the new cell is mapped into a nonoriented bond since the original configuration of bonds can be traversed in both horizontal directions.

a configuration of diodes and resistors which sustains only the northward (eastward) directed percolation. Similarly, if the original cell carries such a part of the RRD network which can be traversed in both vertical (horizontal) directions then the vertical (horizontal) bond of the new coarse-grained cell maps to a resistor. Finally, a nonpercolating configuration of bonds on the original cell maps to a vacancy in the new cell. Thus a regular array of new cells brings on a new RRD network. The renormalized probabilities p', p'_+, p'_- and q' that a bond of the coarse-grained RRD network is respectively a resistor, one of the two kinds of diodes, or a vacancy, are definite functions of the original probabilities

$$\begin{aligned} p' &= f_b(p, p_+, p_-, q), \quad & p'_+ &= \varphi_b(p, p_+, p_-, q), \\ p'_- &= \phi_b(p, p_+, p_-, q), \quad & q' &= \psi_b(p, p_+, p_-, q). \end{aligned} \tag{2.10}$$

These functions appear to be sums of probabilities of having appropriate configurations of bonds on the original $b \times b$ cell (Redner[21]) displayed their explicit form in the case $b = 2$). They are in fact the RG transformations, with certain well established[21]) symmetry properties, which elicited the phase diagram shown in fig. 1. One of the important features of the phase diagram is that the line which connects points RM and I remains invariant under the transformations (2.10).

The second step in applying the PSRG method to the SAW problem should

consist in establishing an RG transformation for the fugacity parameter K. If we want to study critical properties of the mean squared end-to-end distance only, it is sufficient to renormalize all the walks in one particular direction[22]). Therefore we consider only the walks from the south to the north, and, in accordance with the accepted form of the generating function (2.8), we require that the weight K' of one step on a bond of the coarse-grained RRD network, multiplied by the probability $p' + p'_+$, should be equal to the sum of weights of all northward oriented walks on all suitable configurations of bonds on a cell of the original lattice. To clarify this statement, we depict, as an example, all walks on a 2×2 cell (cf. fig. 4) whose respective weights are displayed on the right-hand side of the equation

$$\begin{aligned} K'(p' + p'_+) = \tfrac{1}{2}[&(p + p_+)^2 K^2 + (p + p_+)^3 K^3 + (p + p_+)^3 K^3 \\ &+ (p + p_+)^3 (p + p_-) K^4 + (p + p_+)^2 K^2 \\ &+ (p + p_+)^2 (p + p_-) K^3 + (p + p_+)^2 (p + p_-) K^3 \\ &+ (p + p_+)^3 (p + p_-) K^4], \end{aligned} \tag{2.11}$$

where the prefactor $\frac{1}{2}$ means that the two possible entries on the bottom of the 2×2 cell are equally weighted, which means that the latter equation is formed according to the so-called "equal-averaging"[24]) rule of the PSRG treatment of the SAW problem. However, it should be noticed that with any other possible rule one would similarly find the following RG transformation:

$$K'(p' + p'_+) = F_b[(p + p_+)K, (p + p_-)K], \tag{2.12}$$

where the function F_b turns out to be a polynomial with positive coefficients.

Eqs. (2.10) and (2.12) comprise the complete set of the PSRG transformations prepared for an analysis of the SAW problem on any RRD network, including, among other possible cases, the isotropic percolation network of bonds ($p + q = 1$, $p \geqq \frac{1}{2}$, $p_+ = p_- = 0$). Let us start with an arbitrary RRD network represented by a point on the line that connects points RM and I (cf.

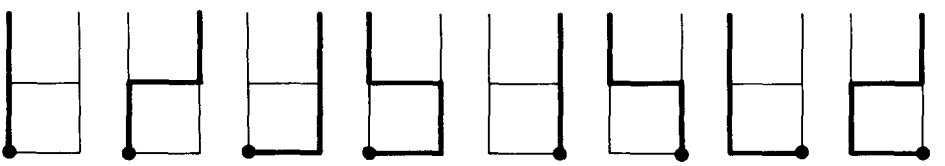

Fig. 4. Eight different northward oriented self-avoiding walks on a 2×2 cell. The respective weights of these walks are given on the right-hand side of eq. (2.11).

fig. 1). In such a case the following equalities are compatible[21]) with eqs. (2.10):

$$p + p_+ = \tfrac{1}{2}\,, \quad p + p_- = \tfrac{1}{2}\,,$$
$$p' + p'_+ = \tfrac{1}{2}\,, \quad p' + p'_- = \tfrac{1}{2}\,, \tag{2.13}$$

and, on this ground, eq. (2.12) reduces to

$$K' = 2F_b(\tfrac{1}{2}K, \tfrac{1}{2}K)\,, \tag{2.14}$$

or

$$K' = 2g_b(\tfrac{1}{2}K)\,, \tag{2.15}$$

where we have emphasized that K' appears to be a one-variable function. Therefore, the corresponding value of the critical exponent ν is independent of the values of p, p_+, p_- and q, that is to say it is the same for all RRD networks represented by the line between RM and I. Indeed, the critical exponent ν is determined by the formula[22])

$$\nu = \ln b / \ln \lambda\,, \tag{2.16}$$

where λ is the eigenvalue $\lambda = (\partial K'/\partial K)_{K=K^*}$ of the RG transformation (2.12), evaluated at the fixed point value K^*. In the case we are analysing the latter is given by

$$K_1^* = 2g_b(\tfrac{1}{2}K_1^*)\,, \tag{2.17}$$

where the subindex 1 has been introduced in order to distinguish it from the next case. One can see that K_1^* does not depend on the bond probabilities, and, thereby, the corresponding λ_1 given by

$$\lambda_1 = (\partial K'/\partial K)_{K=K_1^*} = g_b'(\tfrac{1}{2}K_1^*) \tag{2.18}$$

and the concomitant ν_1, evaluated according to (2.16), are independent of these probabilities.

The next case we are going to analyze is the ordinary square lattice (OSL) with no perturbations at all. Then $p = 1$ and $p_+ = p_- = q = 0$, so that the RG transformation (2.12) reduces to

$$K' = g_b(K)\,, \tag{2.19}$$

and the fixed point fugacity turns out to be given by

$$K_2^* = g_b(K_2^*)\,, \tag{2.20}$$

whereas the corresponding eigenvalue appears to be

$$\lambda_2 = g_b'(K_2^*)\,. \tag{2.21}$$

Comparing eq. (2.20) with (2.17), and (2.21) with (2.18), one can see that the equalities $2K_2^* = K_1$ and $\lambda_1 = \lambda_2$ hold, and consequently one finds that the critical exponent equality $\nu_1 = \nu_2$ holds as well. This result implies that SAW on any RRD network, represented by a point on the line between RM and I, has the same critical exponent ν it has on the ordinary square lattice.

The next question to be answered concerns the bond percolation networks that are represented on the phase diagram (cf. fig. 1) by points that lie on the tetrahedron's edge between $p = \frac{1}{2}$ and $p = 1$. The SAW critical exponent ν on these networks can be deduced by studying simultaneously the first equation of eq. (2.10) and eq. (2.12) written for $p_+ = p_- = 0$ and $q = 1 - p$. Thus, in the case $b = 2$, for instance, eq. (2.10) gains the form[21,22])

$$p' = p^5 + 5p^4q + 8p^3q^2 + 2p^2q^3\,, \tag{2.22}$$

whereas eq. (2.12) has the explicit form (2.11) which reduces to

$$K'p' = p^2K^2 + 2p^3K^3 + p^4K^4\,. \tag{2.23}$$

The flow diagram that stems from these two equations (cf. fig. 5) indicates that in the (p, K) plane there are two fixed points, one at $p = \frac{1}{2}$ and the other at $p = 1$. The fixed points are connected by a critical line, and the RG flow on the line is from the $p = \frac{1}{2}$ point to the $p = 1$ point. Hence one may conclude that the SAW critical exponent ν on all percolation networks with $\frac{1}{2} < p < 1$ is equal to the SAW critical exponent ν pertinent to the OSL ($p = 1$). In the particular case $b = 2$ one finds, from (2.16) and (2.23), $\nu = 0.7153$ for $p = 1$. However, for larger b, values for ν (at the $p = 1$ point) monotonically increase[21]) and it can be expected[22]) that in the infinite cell-size limit the PSRG method would attain the presumably exact value[25]) $\nu = \frac{3}{4}$. Similarly, it can be expected[22]) that for larger b the pattern of the flow diagram could not change, and, accordingly, the conclusion that the SAW critical exponent ν is same for all networks defined by $\frac{1}{2} < p < 1$ (and $p_+ = 0$ and $p_- = 0$) should stay valid for any b.

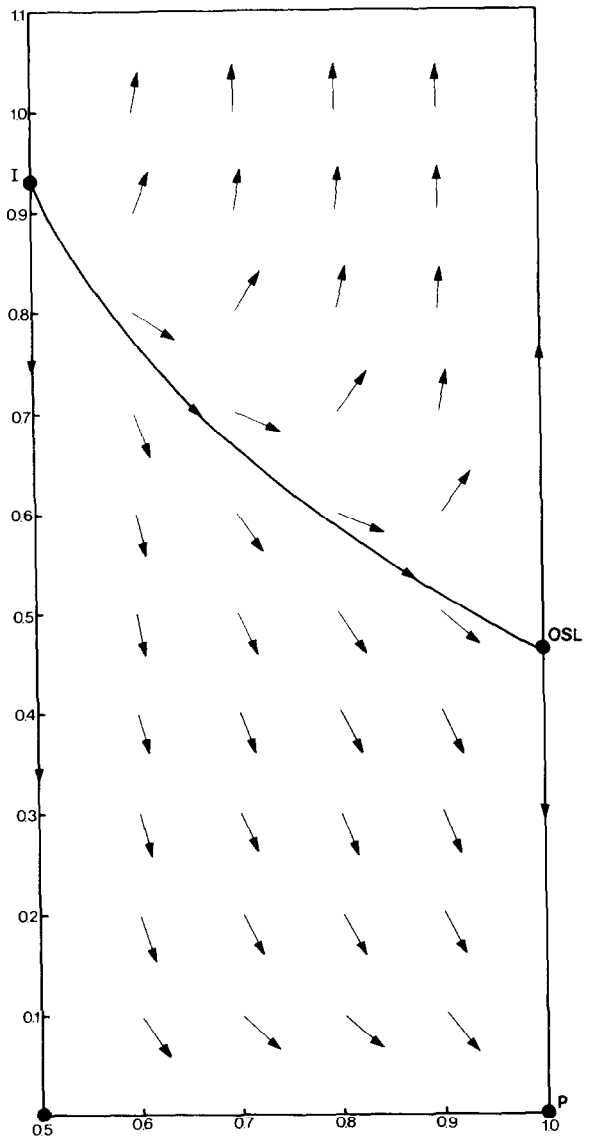

Fig. 5. Flow diagram generated by the RG transformations (2.22) and (2.23). The nontrivial fixed points $I = (0.5, 0.9311)$ and OSL $= (1, 0.4656)$ correspond, respectively, to SAW at the percolation threshold ($p_c = 0.5$) and to SAW on the ordinary square lattice ($p = 1$).

3. Discussion

In the preceding section we have demonstrated that within the framework of the PSRG method one should find that the critical exponent ν of the mean squared end-to-end distance of SAW on various two-dimensional random

networks should be equal to the critical exponent of SAW on the ordinary square lattice (OSL). These networks include percolating clusters, and, for this reason, they can be termed fractal networks[18]). As was mentioned in the introduction, the question whether the critical exponent ν of SAW on OSL, for instance, should change when the translational invariance of the underlying lattice is perturbed has been given different answers. The kind of answer that we have obtained in the preceding section was suggested previously[7]). The suggestion was corroborated by the field theoretical RG calculations[8]) and later it was disputed[10,12]) by using different qualitative and quantitative reasons. As the disputing question has not yet been irrevocably answered, our finding should be, before drawing any other conclusion, contrasted with those adverse findings which were obtained by the same method, that is to say by the PSRG method.

In fact, within the framework of the PSRG method there is no great diversity of results. The case of a random network of diodes (with no resistors and vacancies), or the so-called random Manhattan lattice (RM), was studied by the PSRG method[17]). The majority rule mapping, instead of the percolation rule mapping, was applied[17]) and by constructing a convenient flow diagram it was concluded that the critical exponent ν of SAW on RM (cf. fig. 1) should be equal to the critical exponent on OSL. This result appears to be a particular case of the results obtained in the preceding section, and the latter should be regarded as a vindication of the former, for the percolating rule engenders a more exact mapping than the majority rule. On the other hand, several papers[5,13,14]) have been published in which the SAW problem on the random networks that are represented by points on the line between I and OSL (cf. fig. 1) has been studied by means of the PSRG method. Their findings are in agreement with ours, except for the claim that the critical exponent ν for SAW at the percolation threshold (at the point I) should be found different from ν that is pertinent to OSL. However, we are going to argue that within the framework of the PSRG method this claim springs from artifacts.

The three papers[5,13,14]) are very similar, and we are going to analyze the paper by Sahimi[14]) which is the most elaborate. Sahimi studied[14]) the SAW problem on the site percolation clusters[20]) and, before applying the PSRG method, made a plausible suggestion that there should exist three different critical exponents ν, ν_1 and ν_2 that, respectively, describe SAW on an unperturbed lattice, SAW on the largest percolation cluster and SAW on all clusters. When applied the PSRG method, Sahimi claimed that he calculated ν_1 and found that it should be equal to ν for all $p_c < p < 1$ except for $p = p_c$, when the inequality $\nu_1 > \nu$ should hold. Yet, we shall demonstrate that the application of the PSRG method presupposes an evaluation of ν_2 rather than ν_1 (the first counter-fact), and the inequality $\nu_1 > \nu$ (or $\nu_2 > \nu$) at $p = p_c$ stems from an improper use of the method (the second counter-fact).

To demonstrate the first counter-fact we should recall that the first step in applying the PSRG consists in rescaling (coarse-graining) of the underlying lattice. Thus, in the case of the site-percolation problem, a 2×2 cell is rescaled into a site of the new lattice, and the latter is considered to be occupied, or vacant, depending on whether the occupation of the original cell is such that it can be, or it cannot be, traversed in both (vertical and horizontal) directions (cf. fig. 6). This rescaling procedure, when applied to the whole lattice, does not conserve the largest percolation cluster. On the contrary, after one rescaling step the largest percolation cluster can be torn and new smaller clusters can be attached to it (cf. fig. 7). Therefore, the PSRG method, in the way it is generally applied, implies dealing with all clusters rather than with the largest percolation cluster alone.

The second counter-fact concerns the way Sahimi[14]) constructed the RG transformation for the fugacity parameter K. In our notation, Sahimi's RG transformation, for a $b \times b$ cell, has the form

$$p'^2 K' = \frac{1}{b} \sum_n \left\{ P_n \sum_S{}' K^{N_S} \right\}, \tag{3.1}$$

where the first sum runs over all possible configurations which span the cell, P_n is the probability of having such a configuration, and the second sum runs over all possible SAW that traverse the given configuration (in the vertical direction, for instance) *and* end on the lower layer of sites of the next upper cell. The disputing point is in that on the right-hand side of eq. (3.1) it was taken[14]) for certain (with probability $p = 1$) that the lower sites of the upper cell were occupied, whereas, at the same time, on the left-hand side of (3.1) the occupation of the upper cell was weighted with the probability p' (cf. fig. 8). With such an inconsistency, one can find[14]) that, in the case $b = 2$, $\nu = 0.7153$ for $p_c < p \leqq 1$, and $\nu = 0.7531$ for $p = p_c$, and thereby one may claim that ν is larger at the percolation threshold than ν pertinent to OSL ($p = 1$). However,

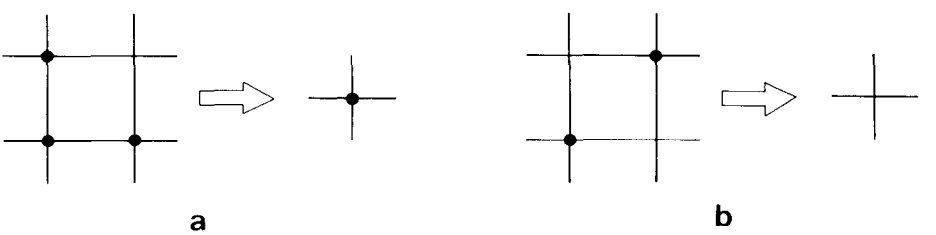

Fig. 6. A 2×2 cell, in the case of the site percolation, is mapped into an occupied site (a) when the original configuration of occupied sites spans the cell in both (vertical and horizontal) directions, whereas it is mapped into a vacancy (b) when the original configuration does not span the cell.

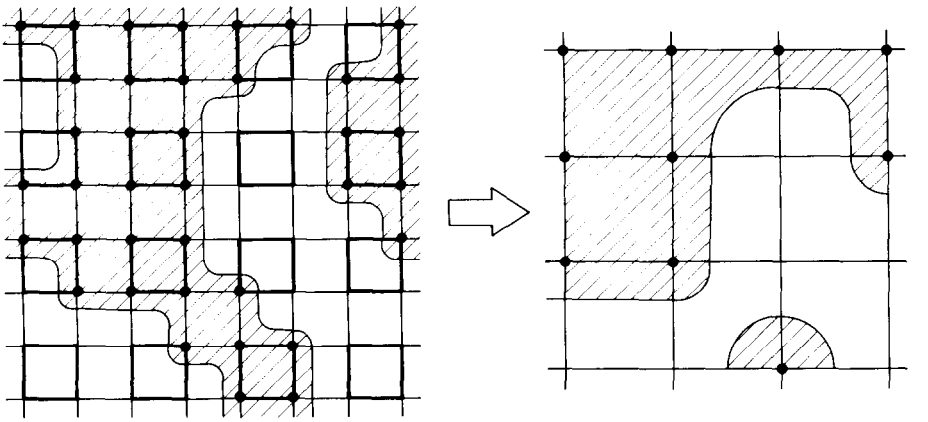

Fig. 7. Rescaling of a part of OSL, together with a part of the largest site percolation cluster (depicted on the left side of the original lattice) and a part of a smaller cluster (displayed on the right side of the original lattice). It should be noticed that, after the rescaling, the lower part of the largest cluster becomes separated, whereas the smaller cluster becomes attached to the largest one.

if the inconsistency is eliminated one retrieves the fact that ν is same for all $p_c \leqq p \leqq 1$. Indeed, there are two ways to treat eq. (3.1) consistently. First, one can take for sure that the upper cell is occupied and put on the left-hand side of (3.1) $p'K'$ instead of p'^2K'. Hence one obtains

$$p'K' = \frac{1}{b} \sum_n \left\{ P_n \sum_S{}' K^{N_S} \right\}, \tag{3.2}$$

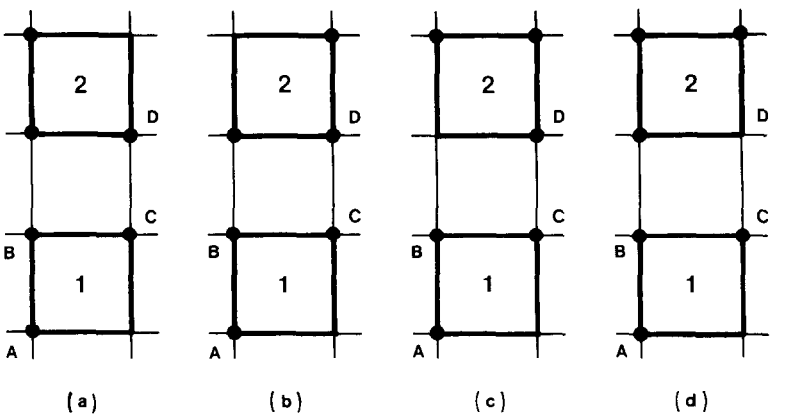

Fig. 8. Taking for granted that the lower edge of the cell 2 is occupied, the SAW from A to D was weighted[14]) with the simple term $p^3(1-p)K^3$. The fourth situation (d) shows that such an approach is not tenable.

which can be written in the form

$$p'K' = \frac{1}{b} \sum_{S} \left\{ \sum_{n}{}' P_n \right\} K^{N_S}, \tag{3.3}$$

where the first sum now runs over all SAW on a $b \times b$ cell, while the second sum runs over all possible occupations of the cell that make the given SAW feasible, and, for this reason, one can write the following equality:

$$\sum_{n}{}' P_n = p^{N_S}. \tag{3.4}$$

From (3.3) and (3.4) one finds

$$p'K' = \frac{1}{b} \sum_{S} (pK)^{N_S}, \tag{3.5}$$

or

$$p'K' = g_b(pK), \tag{3.6}$$

where $g_b(pK)$ is a polynomial, in pK, with positive coefficients. The last equation, written for $p = p_c$ and $p = 1$, is completely analogous to eqs. (2.15) and (2.19), respectively, and on this ground one can easily draw the previous conclusion that ν is same for $p = p_c$ and $p = 1$ (one should observe that the conclusion is not endangered by the fact that p_c is not equal to $\frac{1}{2}$ in the case of the site percolation).

Finally, we should comment on the second way to treat eq. (3.1) consistently. Now, one can keep the left-hand side of (3.1) unaltered, which is quite plausible, and weight each SAW, on the right-hand side of (3.1), with the probability p of having the last site of the walk occupied. Consequently, after the same rearrangements of (3.1) we have done in the preceding paragraph, one would obtain the new RG equation

$$p'^2K' = \frac{1}{b} \sum_{S} (pK)^{N_S} p. \tag{3.7}$$

However, since both values $p = p_c$ and $p = 1$ correspond to the fixed points of the RG equation for p, one can put $p' = p$ in (3.7), and by this means one will retrieve the same conclusion that was obtained from eq. (3.6). This completes our analysis of the work of Sahimi[14]). Before proceeding further, we have to mention that, in forming the RG equation for K, Roy and Chakrabarti[5]) made a calculational error, whereas Lam and Zhang[13]) overweighted those configu-

rations which have more exits than entrances (for the incoming self-avoiding walker), and thereby both works[5,13]) provided the inconsistent conclusion that ν is larger at $p = p_c$ than at $p = 1$.

In summary, we may repeat that we have demonstrated that within the framework of the PSRG method one is bound to find that the critical exponent ν of the mean squared end-to-end distance of SAW on various two-dimensional random networks, including the percolation clusters, should be equal to the critical exponent of SAW on OSL. It is important to emphasize that we do not claim that our finding completely unravels the confusion about the right values of ν (see the introduction). We merely assert that a proper application of the PSRG method to the SAW problem cannot lead to conflicting answers, as it seemed to be the case before this paper. The answer we have worked out may be a consequence of the approximate nature of the method applied, and may be eventually disproved. However, we claim that it is the single answer one can obtain, and to disprove it one would need a more sophisticated version of the PSRG method. In particular, although we are inclined to expect, on the basis of various qualitative arguments[12,14]), that ν of SAW on the largest percolation cluster can be larger than ν of SAW on OSL, we maintain that to vindicate this expectation one will have to invent a new version of the PSRG method which will conserve exactly the self-similar structure of the largest cluster.

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References

1) B.J. Hiley, J.L. Finney and T. Burke, J. Phys. A: Math. Gen. **10** (1977) 197.
2) B.K. Chakrabarti and J. Kertész, Z. Phys. B **44** (1981) 221.
3) K. Kremer, Z. Phys. B **45** (1981) 149.
4) B. Derrida, J. Phys. A: Math. Gen. **15** (1982) L119.
5) A.K. Roy and B.K. Chakrabarti, Phys. Lett. A **91** (1982) 393.
6) B.K. Chakrabarti, K. Bhadra, A.K. Roy and S.N. Karmakar, Phys. Lett. A **93** (1983) 434.
7) A.B. Harris, Z. Phys. B **49** (1983) 347.
8) Y. Kim, J. Phys. C: Solid State Phys. **16** (1983) 1345.
9) R. Rammal, T. Toulouse and J. Vannimenus, J. Physique **45** (1984) 389.

10) J.W. Lyklema and K. Kremer, Z. Phys. B **55** (1984) 41.
11) A.K. Roy and B.K. Chakrabarti, Z. Phys. B **55** (1984) 131.
12) B. Derrida, Phys. Rep. **103** (1984) 29.
13) P.M. Lam and Z.Q. Zhang, Z. Phys. B **56** (1984) 155.
14) M. Sahimi, J. Phys. A: Math. Gen. **17** (1984) L379.
15) Y. Liu and Z.Q. Zhang, J. Phys. A: Math. Gen. **18** (1985) 1027.
16) J.P. Nadal and J. Vannimenus, J. Physique **46** (1985) 17.
17) S. Milošević and A. Chernoutsan, in: Fractals in Physics, L. Pietronero and E. Tosatti, eds. (North Holland, Amsterdam, 1986), p. 115.
18) R. Blumenfeld, Y. Meir, A.B. Harris and A. Anharony, J. Phys. A: Math. Gen. **19** (1986) L791.
19) A.K. Roy and B.K. Chakrabarti, J. Phys. A: Math. Gen. **20** (1987) 215.
20) D. Stauffer, Introduction to Percolation Theory (Taylor and Francis, London, Philadelphia, 1985).
21) S. Redner, Phys. Rev. B **25** (1982) 3242.
22) H.E. Stanley, P.J. Reynolds, S. Redner and F. Family, in: Real-Space Renormalization, T.W. Burkhardt and J.M.J. van Leeuwen, eds. (Springer, Berlin, Heidelberg, 1982), pp. 171–208.
23) D.S. McKenzie, Phys. Rep. **27** (1976) 35.
24) S. Redner and P.J. Reynolds, J. Phys. A: Math. Gen. **14** (1981) 2679.
25) B. Nienhuis, Phys. Rev. Lett. **49** (1982) 1062.