Physica A 492 (2018) 1206–1215

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Characterizing core–periphery structure of complex network by *h*-core and fingerprint curve

Simon S. Li [a,b,1], Adam Y. Ye [c,d,1], Eric P. Qi [a,b], H. Eugene Stanley [e,*], Fred Y. Ye [a,b,**]

[a] School of Information Management, Nanjing University, Nanjing 210023, China
[b] Jiangsu Key Laboratory of Data Engineering and Knowledge Service, Nanjing 210023, China
[c] Center for Bioinformatics, School of Life Sciences, Peking University, Beijing 100871, China
[d] Peking-Tsinghua Center for Life Sciences , Beijing 100871, China
[e] Department of Physics and Center for Polymer Studies, Boston University, Boston, MA 02215, USA

HIGHLIGHTS

- We suggest to characterize the core–periphery structure of complex network with using network *h*-core and fingerprint curve.
- The feature of core structure is described by network *h*-core and the feature of periphery structure is represented by fingerprint curve.
- We also propose Fourier-like analysis as a potential methodology for network analysis.

ARTICLE INFO

Article history:
Received 3 February 2017
Received in revised form 17 September 2017
Available online 2 December 2017

Keywords:
Core–periphery structure
h-core
Fingerprint curve
Rose curve
Fourier-like analysis
Complex network

ABSTRACT

It is proposed that the core–periphery structure of complex networks can be simulated by *h*-cores and fingerprint curves. While the features of core structure are characterized by *h*-core, the features of periphery structure are visualized by rose or spiral curve as the fingerprint curve linking to entire-network parameters. It is suggested that a complex network can be approached by *h*-core and rose curves as the first-order Fourier-approach, where the core–periphery structure is characterized by five parameters: network h-index, network radius, degree power, network density and average clustering coefficient. The simulation looks Fourier-like analysis.

1. Introduction

After complex network study was recalled [1,2], it has developed rapidly[3–6] and produced important theoretical results [7,8] and practical applications, concerning weighted and unweighted [9,10] as well as single-layer and multi-layer [11,12] networks. The studies of complex networks have used both global and local measures, and have been carried out on both homogeneous and heterogeneous structures [13,14]. Although complex network study has become increasingly complicated, focusing on the core structure enables a simplification [15].

* Corresponding author.
** Corresponding author at: Jiangsu Key Laboratory of Data Engineering and Knowledge Service, Nanjing 210023, China.
E-mail addresses: hes@bu.edu (H.E. Stanley), yye@nju.edu.cn (F.Y. Ye).
1 Co-first authors: the authors contribute equally and are listed alphabetically.

https://doi.org/10.1016/j.physa.2017.11.048

To study the core structure of networks, the small core or bone structure can be extracted [16]. The h-index [17] has also been applied to networks [18], and this has produced such measures and structures as the h-degree [19] and the h-subnet [20] in single-layer networks, and the h-crystal [21] in multilayer networks.

Since 2000 the core–periphery structure has introduced [22,23], a small number of studies have been carried out on periphery structure. The algorithm of core–periphery is a standard method in network analysis, and it has yielded a series of improvements and developments [24–28].

Combined with the idea of network fingerprint [29,30], it is feasible to choose a curve as fingerprint curve for simulating the periphery structure. Supposing that the fingerprint curve should be simple, with keeping two entire-network parameters in the curve, we choose the spiral or rose curves as fingerprint curves.

The core structure can be described by network h-core, and then the core–periphery structure of a complex network will be similar with h-core plus fingerprint curve linking to entire-network parameters. We apply following definitions.

The h-core of a network consists of nodes and their links measured using the h-index. In an unweighted network, the h-core is a subgraph of nodes and their links ranked by node degree produced by the h-index. In a weighted network, the h-core is a set of n nodes in which all nodes have an h-degree [19] (d_h) equal to $d_h(n)$. Note that the h-degree (d_h) or the degree h-index is a node-based measure, but that the h-core is a structure that consists of the core nodes and their links. Thus the network h-core will be unique. Other research has shown that the h-indices of unweighted network nodes converge to coreness [31].

The entire network parameters of a network concern the total node (vertices) and total link (edges) measurements and describe the entire network. These include the average node degree, the average link strength, the average clustering coefficient (c), the network density (the average degree centrality A), the network diameter or radius (R), and the power index (k) of the degree distribution. Among these parameters, the network density (A) and average clustering coefficient (c) are suitable to use for plotting the (logarithmic) spiral curve ($r = A\exp(c\varphi)$), while the network radius (R) and power index (k) are suitable to use for plotting the rose curve ($r = R\cos(k\varphi)$), as rose keeps a relatively fixed radius and spiral scatters its density and cluster.

In more general, for characterizing the core–periphery structure of networks, including unweighted and weighted and single-layer and multilayer networks, we can introduce a Fourier-like analysis for deeper exploration, in methodology.

2. Methodology

There are two steps in characterizing the core–periphery structure of networks. (i) Using the h-core algorithm to find the unique core structure of the network. (ii) Applying the network diameter and degree power and the network density and average cluster coefficient (based on entire network) to find the characteristic parameters of the network. We then merge the core structure with the characteristic parameters and use the rose curve $r = (D/2)\ \cos(k\varphi)$ to produce the periphery structure while core is h-core.

Both indicators and models are important in any possible applications.

(1) Indicators

When describing the entire network, network diameter D, average cluster coefficient c, node degree d, and degree distribution $p\,(d, k)$ are the common indicators. The general random network formula is

$$D = 2R = \frac{\ln N}{\ln pN} = \frac{\ln N}{\ln\langle d\rangle} \tag{1}$$

$$c = \frac{\langle d\rangle}{N} \times \frac{N}{N} = \frac{\langle d\rangle}{N} \tag{2}$$

$$p(d, k) = \frac{1}{\varsigma(k)} d^{-k};\ d \geq 1, \tag{3}$$

where R is the radius, N the total nodes in the network, $\langle d\rangle$ the average degree of nodes, p the connection probability, d the degree, k the power index (note that this differs from common usage in which k is the degree and α or γ the power index), and $\zeta(k)$ is the Riemann ζ-function. Another important indicator is the degree centrality. The average degree centrality A is equal to the network density

$$A = \frac{1}{N}\sum_{i=1}^{N}\frac{d(i)}{N-1} \tag{4}$$

in which $d(i)/(N-1)$ is the degree centrality of node i.

The parameters R, A, c, and k describe the entire network, then are transformed into spiral or rose curve parameters. As rose curve has its radius and power index gives key factor, we choose R and k for charactering rose curve, and then remains A and c for spiral one. The selection looks a natural choice.

We use the h-core to reveal the core. In an unweighted network the h-core is a subgraph that consists of nodes with links ranked by node degree according to the principle of h-index. In a weighted network the h-core is set of nodes all have links of at least h-degree [19], where the h-degree (d_h) of node n is equal to $d_h(n)$ if $d_h(n)$ is largest natural number such that node n has at least $d_h(n)$ links each with a strength of at least $d_h(n)$. In total, there are five indicators $\{h, R, k, A, c\}$ that are the main

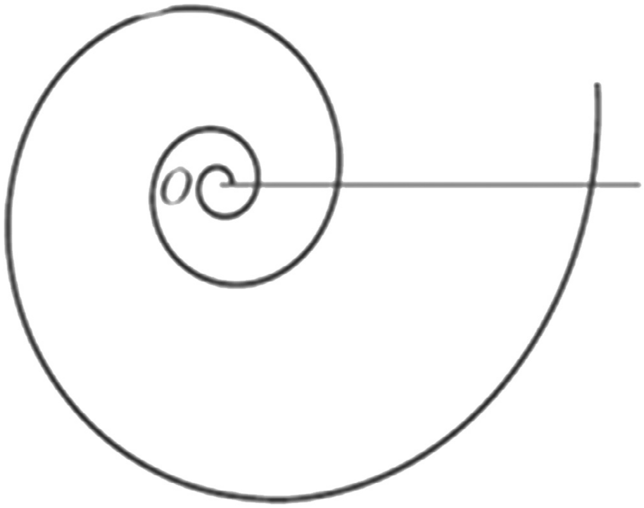

Fig. 1. The basic shape of the spiral.

parameters for describing the core–periphery structure of a network, where h is the core and R, k, A, and c are entire-network parameters for charactering periphery.

(2) Models

A spiral curve is described by

$$r(A, c) = A\exp(c\varphi). \tag{5}$$

Here A is a size factor that does not affect the shape of the spiral curve. Thus c is the key factor. Fig. 1 shows the basic shape of the spiral.

The simple equation for the rose curve in a polar coordinate system (r, φ) is

$$r(R, k) = R\cos(k\varphi). \tag{6}$$

The equivalent equation in a Cartesian coordinate system (x, y) is

$$\left.\begin{array}{l} x = R\cos(k\varphi)\cos\varphi \\ y = R\cos(k\varphi)\sin\varphi \end{array}\right\}. \tag{7}$$

Although a rose curve can be also expressed using a sine function, we apply a cosine function for charting, because $\cos(z)$ is the real part of $\exp(iz)$. The parameters R and k determine the shape of the rose curve. Because R is only a size factor, k is the key factor that affects the shape, as shown in Fig. 2.

Fig. 2 shows that k is the sensitive parameter for rose shape, and that a small difference in k strongly affects the rose curves.

Spiral or rose, which is better to be fingerprint curve? We will solve the problem via empirical studies.

When we use rose curves, one important finding concerns the merging and overlying of networks. When two networks are merged, the process can be modeled by merging their rose curves in special conditions.

Suppose we have two rose curves,

$$r_1(R_1, k_1) = R_1\cos(k_1\varphi) \tag{8}$$

and

$$r_2(R_2, k_2) = R_2\cos(k_2\varphi). \tag{9}$$

When we overlay the two curves, the synthesized curve is

$$r(R, k) = r_1 + r_2 = R_1\cos(k_1\varphi) + R_2\cos(k_2\varphi). \tag{10}$$

When $R_1 = R_2 = R$, $k_1 = k_2 = k$, Eq. (7) can be simplified to be

$$r(R, k) = 2R\cos\frac{k\varphi + k\varphi}{2}\cos\frac{k\varphi - k\varphi}{2} = 2R\cos k\varphi. \tag{11}$$

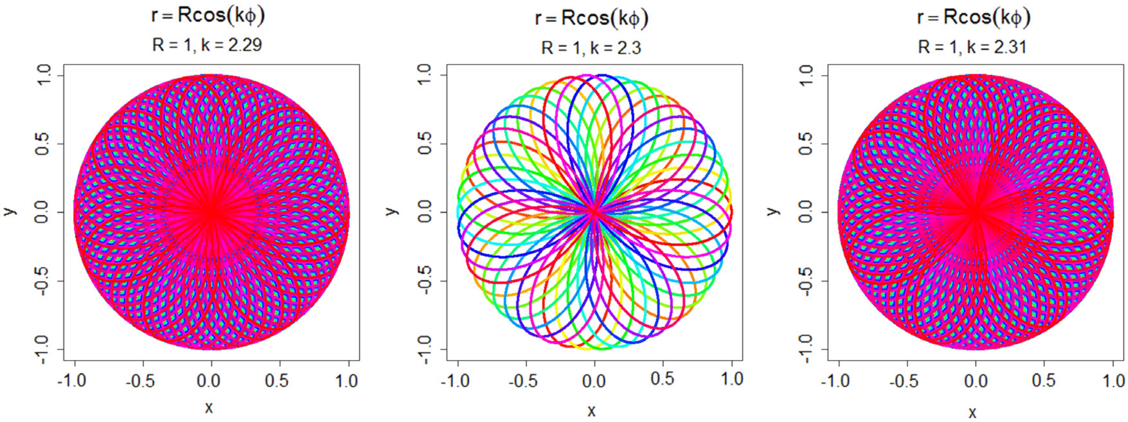

Fig. 2. The different rose curves with $k = 2.29$, 2.30, and 2.31, and $R = 1$.

When the situation is $R_1 \approx R_2 \approx R$, $k_1 \approx k_2 \approx k$, the fusion of two similar curves will generate a larger network on nodes with a similar degree distribution.

A related finding concerns the core stability. When a network is complex (particularly when it is a multilayer or multiplex network), its core is approximately stable. This property is shown in Eq. (11), where the k value is approximately the same as k_1 or k_2 when $k_1 \approx k_2$. This indicates that the core remains relatively stable when the network is extended [32]. When the two layers are merged, the multiplex network effects are the same.

Solving the partial derivatives, we find that

$$\frac{\partial^2 r(R,k)}{\partial R^2} = 0 \tag{12}$$

and

$$\frac{\partial^2 r(R,k)}{\partial k^2} = -2R\varphi^2 \cos k\varphi. \tag{13}$$

Analytically, the spiral curve displays clearer properties. The change rate of its radius is

$$\frac{\partial r(A,c)}{\partial \varphi} = Ac\exp(c\varphi) = cr. \tag{14}$$

Using the Euler formula

$$\exp(iz) = \cos z + i\sin z \tag{15}$$

we can introduce the relation between the spiral or rose curves when $A = R$ and $c = k$,

$$r(A, ic) = A\exp(ic\varphi) = A\cos c\varphi + iA\sin c\varphi = r_{\cos}(R,k) + ir_{\sin}(R,k) \tag{16}$$

which means that overlaying a real cosine rose curve and an imaginary sine rose curve generates a complex spiral curve.

(3) Possible extension: Fourier-like analysis

Eq. (16) looks Fourier-like elements. In Fourier analysis, any periodic function $f(x)$ can be expanded as a series at $[-\pi, \pi]$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx) = \sum_{n=-\infty}^{\infty} c_n \exp(inx) \tag{17}$$

where

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt dt \; (n = 0, 1, 2, \ldots) \tag{18}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt dt \; (n = 1, 2, \ldots) \tag{19}$$

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t) exp(-\text{int}) dt \; (n = 0, \pm 1, \pm 2, \ldots). \tag{20}$$

Similarly, in network analysis any network $G(V, E)$ can be expanded by approaching spiral–rose curves

$$G(V,E) = G(X,Y) \sim \sum_j X_j \exp(iY_j\varphi) = h + \sum_j (X_j conY_j\varphi + iX_j \sin Y_j\varphi), \tag{21}$$

where V and E are the vertices and edges of a network (graph), and $(X_j, Y_j) = f(V, E)$ are the characteristic parameters of the series of curves. When $j = 1$ (the first-order Fourier-approach), there is one spiral or one rose and thus five parameters $\{h, R, k, A, c\}$ can be used to describe the core–periphery network structure, where h describes the core and R, k, A, and c do the whole structure. We suggest that a Fourier-like analysis be used in future network studies, by means of which the simplest core–periphery structure of a complex network can be characterized by the first-order Fourier-approach described above in the single spiral–rose curve model.

Decomposing X_j

$$\left.\begin{aligned} X_{-j} &= \frac{1}{2}(a_j + ib_j) \\ X_{+j} &= \frac{1}{2}(a_j - ib_j) \end{aligned}\right\}, \tag{22}$$

where a_j and b_j are expected to link with characteristic parameters and spectra of complex network. We insert (22) into (21) and let $X_0 = a_0$, then obtain

$$\begin{aligned} G(X,Y) &= a_0 + \sum_{j=1}^{+\infty}[\frac{1}{2}(a_j - ib_j)\exp(iY_j\varphi)] + \sum_{j=1}^{+\infty}[\frac{1}{2}(a_j + ib_j)\exp(-iY_j\varphi)] \\ &= a_0 + \sum_{j=1}^{+\infty}\{\frac{1}{2}a_j[\exp(-iY\varphi) + \exp(iY_J\varphi)] + \frac{i}{2}b_j[\exp(-iY_j\varphi) - \exp(iY_j\varphi)]\}. \end{aligned} \tag{23}$$

Using the Euler formula, we have

$$\left.\begin{aligned} \frac{1}{2}[\exp(-iY_j\varphi) + \exp(iY_j\varphi)] &= \cos(Y_j\varphi) \\ \frac{i}{2}[\exp(-iY_j\varphi) - \exp(iY_j\varphi)] &= \sin(Y_j\varphi) \end{aligned}\right\}, \tag{24}$$

which allows us to rewrite Eq. (23) to be at a_0=h.

$$G(X,Y) = h + \sum_{j=1}^{+\infty}[a_j \cos(Y_j\varphi) + b_j \sin(Y_j\varphi)]. \tag{25}$$

Eq. (25) transforms the field from a complex to a real field in which h describes h-core, and the rose curves are determined by the characteristic parameters Y_j. The complete Fourier-like transformation is thus

$$F(U, \mathrm{V}) = \frac{1}{2\pi}\int_{-\pi}^{+\pi}\int_{-\pi}^{+\pi} G(X, \mathrm{Y})\exp(iw)\exp(iz)dwdz. \tag{26}$$

This approach promises to be a potentially useful methodology for future network research.

In our empirical study we searched the Web of Science (WoS) database for articles, letters, and reviews published in *Nature* and *Science*. We divided them into three time periods (1981–1990, 1991–2000, and 2001–2010). We treated each article, letter, and review in these datasets as a node and charted their bibliographic coupling networks.

3. Results

The bibliographic coupling networks of *Nature* and *Science* are separately set up and then merged them together. All the network parameters are shown in Table 1.

Each time period contains more than 11,000 papers. These constitute the nodes of the bibliographic coupling network. Figs. 3–5 show their core–periphery structures with using both spiral and rose approaches, respectively, for comparison, in which network parameters are drawn using Python programming (NetworkX from https://networkx.github.io/ and power law package [33]) and spiral–rose curves using *R* programming.

Table 2 shows the empirical and theoretical network parameters when the *Nature* and *Science* networks are merged into one (*Nature* + *Science*).

The theoretical estimated parameters are computed using separate *Nature* and *Science* data with an average estimation of $k = (k_1 + k_2)/2$ when $k_1 \approx k_2$ and $c = (c_1 + c_2)/2$ when $c_1 \approx c_2$, while the empirical computation is based on real data of *Nature* + *Science*. The errors occur when $k_1 \neq k_2$ and $c_1 \neq c_2$. When $k_1 = k_2$ and $c_1 = c_2$, the theoretical estimation will be $k = (k_1 + k_2)/2$ and $c = (c_1 + c_2)/2$ strictly.

Table 1
Network parameters of sample datasets.

Sample	Parameter	Time Periods		
		1981–1990	1991–2000	2001–2010
Nature	Total nodes	16 097	14 127	11 388
	Total links	660 213	234 022	221 975
	Average node degree	82.0293	33.1312	38.9840
	Average link strength	1.8042	2.6570	2.9746
	Average clustering coefficient	0.4120	0.3733	0.3588
	Network density	0.0051	0.0023	0.0034
	Network diameter	15	15	12
	Network power	1.3343	1.3763	1.3502
	h-degree	12	9	10
	Nodes of *h*-core	21	18	10
Science	Total nodes	10 755	12 032	10 912
	Total links	247 937	206 714	101 893
	Average node degree	46.1064	34.3607	18.6754
	Average link strength	2.6194	3.1827	3.9463
	Average clustering coefficient	0.3483	0.3549	0.3338
	Network density	0.0043	0.0029	0.0017
	Network diameter	15	12	19
	Network power	1.3861	1.3583	1.4054
	h-degree	11	9	7
	Nodes of *h*-core	20	12	23
Nature + Science	Total nodes	26 852	26 159	22 300
	Total links	1 607 079	813 134	539 486
	Average node degree	119.6990	62.1686	48.3844
	Average link strength	1.7339	2.2598	2.5850
	Average clustering coefficient	0.3978	0.3743	0.3566
	Network density	0.0045	0.0024	0.0022
	Network diameter	13	13	17
	Network power	1.3104	1.3150	1.3177
	h-degree	13	11	10
	Nodes of *h*-core	30	21	22

Table 2
Empirical and theoretical parameters: A comparison.

Type	Parameters	*Nature* + *Science*
Empirical computation	power index (k)	1.3177
	average clustering coefficient (c)	0.3566
Theoretical estimation	power index (k)	1.3778
	average clustering coefficient (c)	0.3463

In above empirical examples, we saw that *Nature* and *Science* had different *h*-core and fingerprint curves, as well as different ones in different period for same journal. So the *h*-core plus fingerprint curves revealed core–periphery structure visually. As to the differences of *Nature* and *Science*, the reason came from their different contents of bibliographic coupling networks and then expressed as different core–periphery structure.

4. Discussion

Both rose curves and spirals are simple curves mastered by two parameters R and k, or by A and c. The symmetrically beautiful shape of the rose curve is sensitive to k and insensitive to R, but the curved spiral looks insensitive to both parameters A and c visually. Based on above empirical results, we suggest to use real rose curve as visual fingerprint curve, and ignore spiral curve. This simplified approach reveals the core–periphery characteristics of network, where the complexity of the entire network would be produced by merging and overlying the simple subgraphs, with keeping the network *h*-core stable relatively.

The reasons that we did not choose one-parameter curves such as the archimedean spiral $r = a\varphi$ or the hyperbolic spiral $r = a/\varphi$ as fingerprint curves focus on two points: (1) the mathematical terms of con and sin in Eqs. (21) and (25) contain two parameters, which are important for analytical methodology; (2) two parameters could keep both more entire-network parameters and necessary simplicity.

Using rose curves also provide advantages when focusing on analytical properties. Because trigonometric functions link with exponential functions (via the Euler formula) as well as Fourier analysis, they may prove beneficial in extending future theoretical developments. The methodology may be used on either weighted or unweighted single-layer networks, and may be also suitable for multilayer networks when we divide the multilayers into a series of single layers.

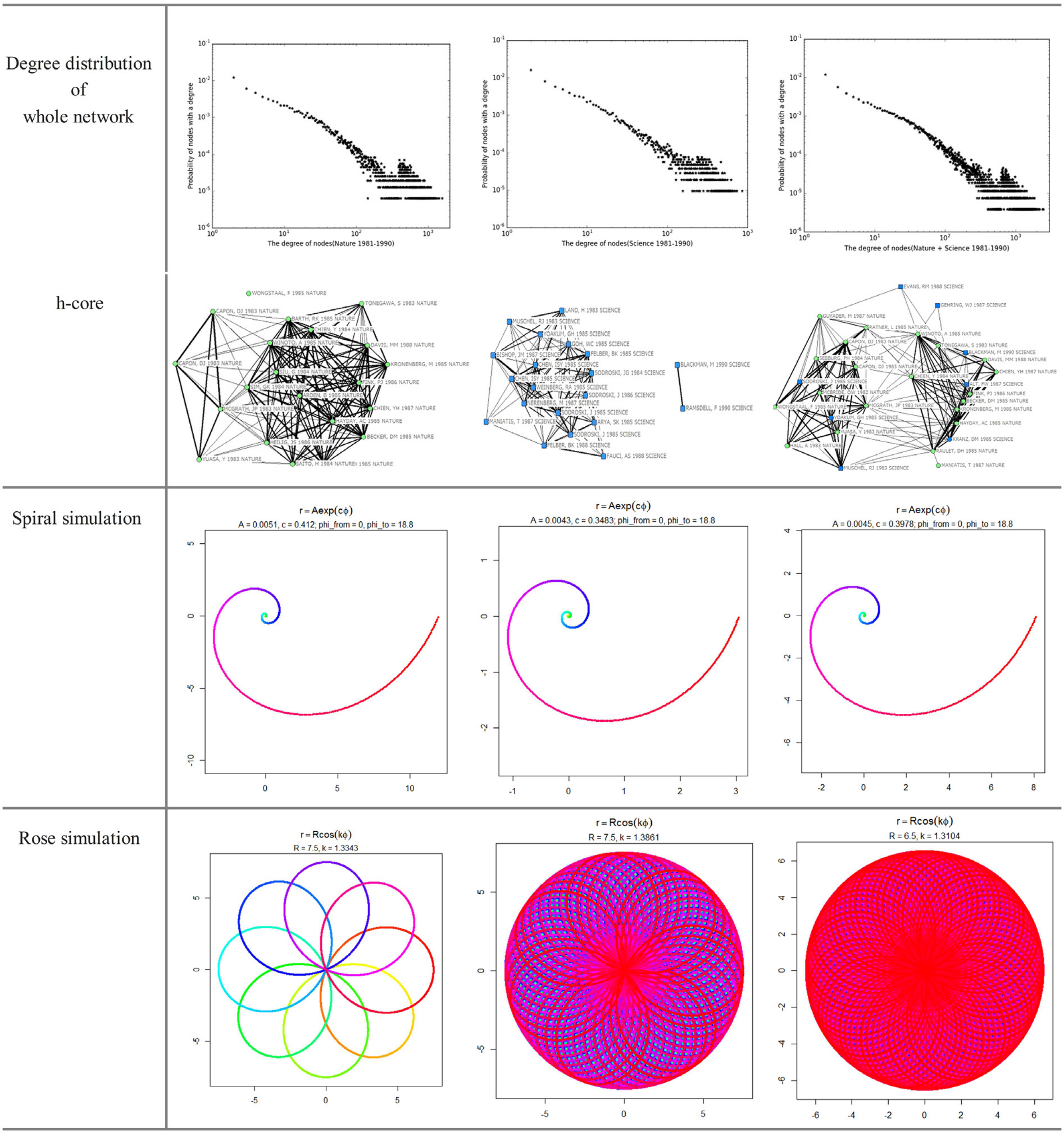

Fig. 3. The core–periphery structure using *h*-core and fingerprint curve at time period 1981–1990.

The limitations of the method are seen when it is applied to connected networks without branches. If a network is hierarchical it has many branches, the method is less effective, and a branch coefficient needs to be considered.

5. Conclusion

The core–periphery structure can be characterized by *h*-core and fingerprint curve in complex networks, in which the fingerprint curve can be rose curve linking to entire-network parameters.

When we describe the core–periphery structure with using the *h*-core and fingerprint curves, a complex network looks like a "flower" (such as rose) configuration. While the *h*-core characterizes the core structure, the fingerprint curves use the entire-network parameters (R, k) to simulate the periphery distributions, so that the "flower" reveals the core–periphery

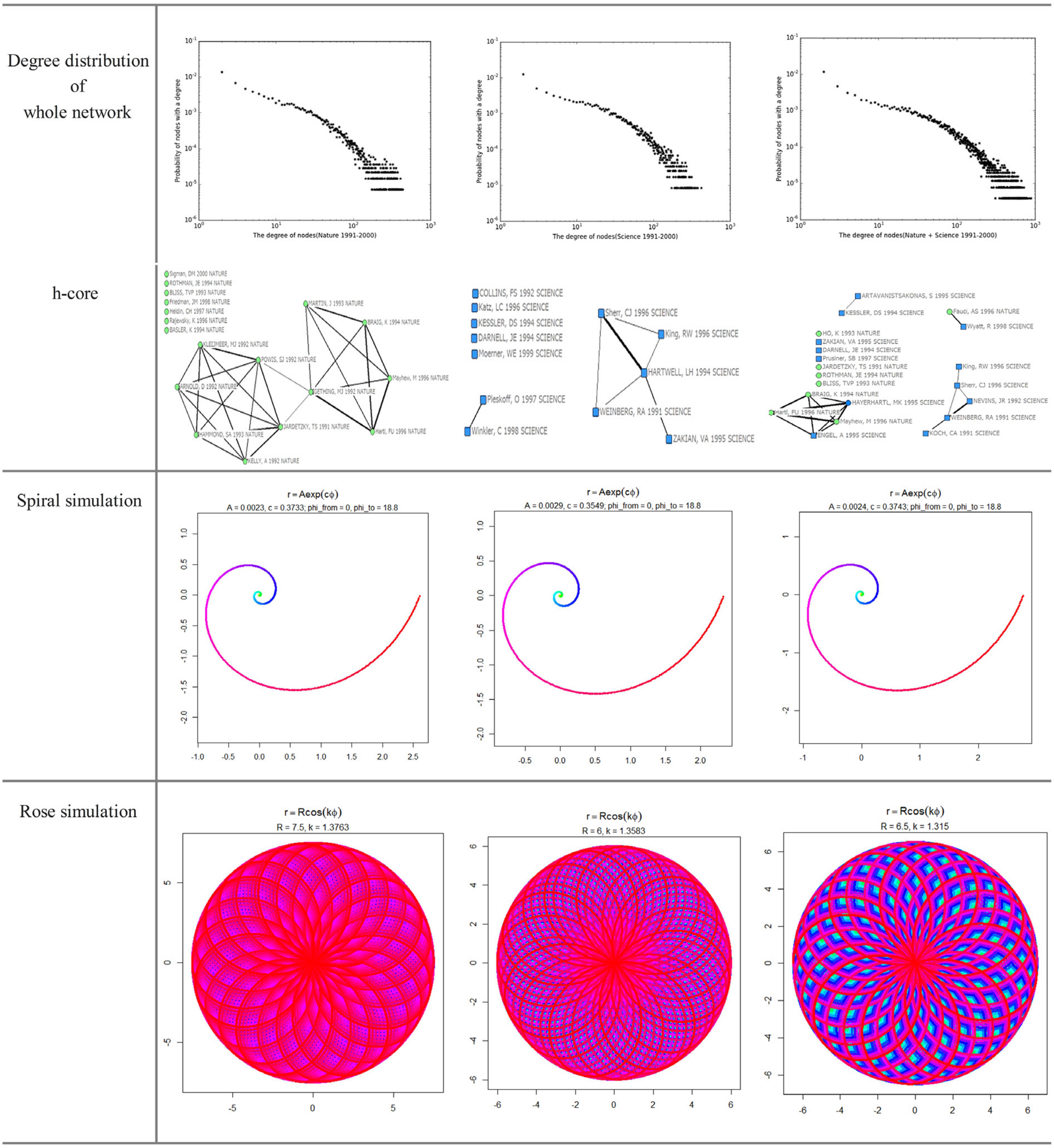

Fig. 4. The core–periphery structure using *h*-core and fingerprint curve at time period 1991–2000.

structure. The "flower" metaphor is useful because it enables us to characterize the network configuration, and different networks display differing "flower" structures. A Fourier-like analysis of networks may enable us to simplify complex networks by means of a simple rose expansion plus *h*.

Acknowledgments

The authors acknowledge National Natural Science Foundation of China Grant No. 71673131 and Jiangsu Key Laboratory Fund for partially financial supports. The Boston University Center for Polymer Studies is supported by NSF Grants PHY-1505000, CMMI-1125290, and CHE-1213217, by DTRA Grant HDTRA1-14-1-0017, and by DOE Contract DE-AC07-05Id14517.

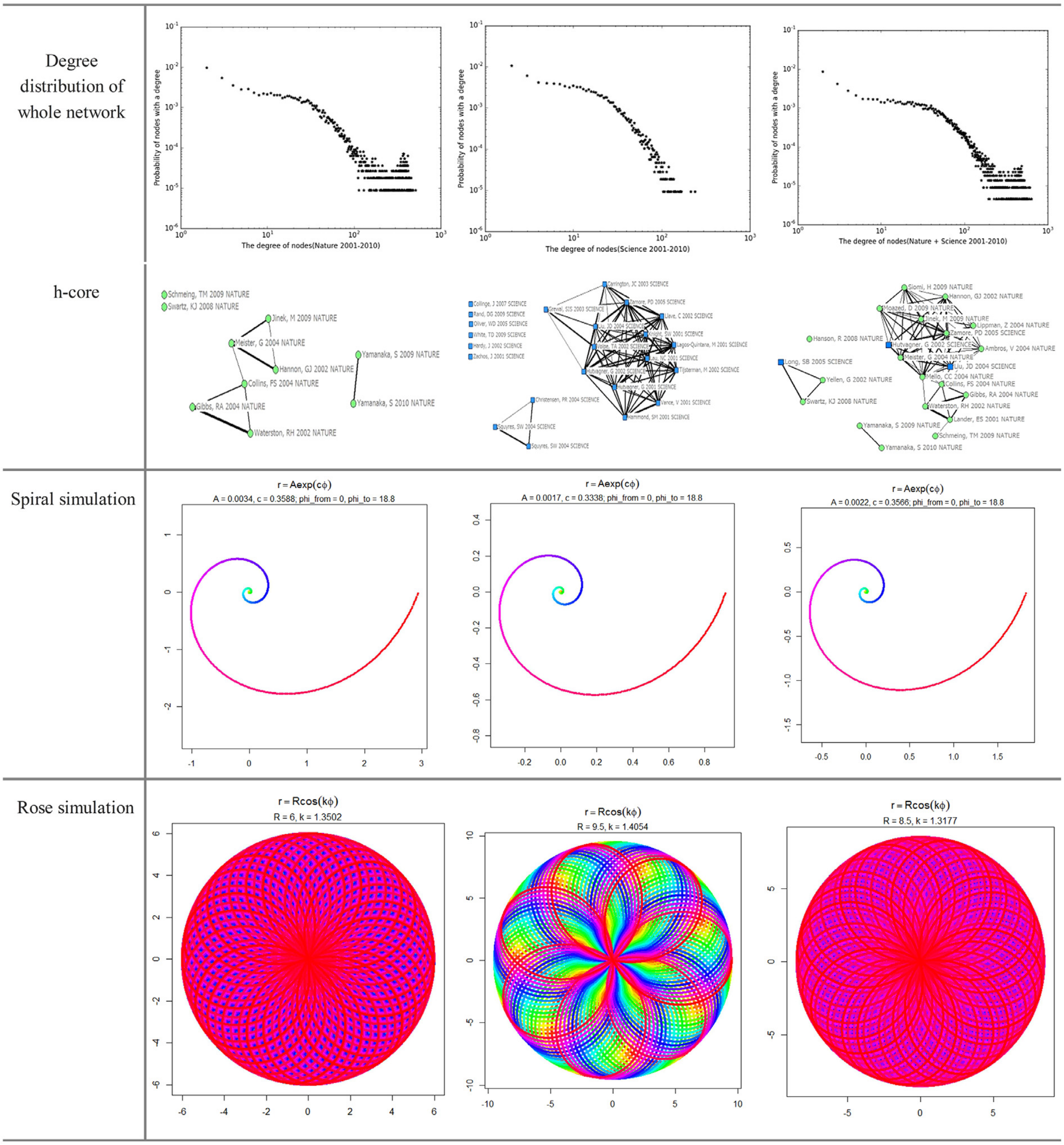

Fig. 5. The core–periphery structure using *h*-core and fingerprint curve at time period 2001–2010.

Author contributions

S.S.L. collected and processed data, A.Y.Y. coded program and optimized model, E.P.Q. assisted and improved analysis, H.E.S. checked the research and wrote the paper, and F.Y.Y. initiated and designed the research and wrote the paper.

References

[1] D.J. Watts, S.H. Strogatz, Collective dynamics of 'small-world' networks, Nature 393 (1998) 440–442.
[2] A.-L. Barabási, R. Albert, Emergence of scaling in random networks, Science 286 (1999) 509–512.
[3] M.E.J. Newman, The structure and function of complex networks, SIAM Rev. 45 (2003) 167–256.
[4] M.E.J. Newman, Networks: An Introduction, Oxford University Press, Oxford, 2003.

[5] S. Boccaletti, V. Latora, Y. Morenod, M. Chavez, D.-U. Hwang, Complex networks: Structure and dynamics, Phys. Rep. 424 (2006) 175–308.
[6] S. Boccaletti, G. Bianconi, R. Criado, et al., The structure and dynamics of multilayer networks, Phys. Rep. 544 (2014) 1–122.
[7] R. Albert, A.-L. Barabási, Statistical mechanics of complex networks, Rev. Modern Phys. 74 (2002) 47–97.
[8] M. De Domenico, A. Sole-Ribalta, E. Cozzo, et al., Mathematical formulation of multilayer networks, Phys. Rev. X3 (2013) 041022.
[9] G. Menichetti, D. Remondini, P. Panzarasa, R.J. Mondragon, G. Bianconi, Weighted multiplex networks, Plos One 9 (6) (2014) e97857.
[10] A. Barrat, M. Barthélemy, R. Pastor-Satorras, A. Vespignani, The architecture of complex weighted networks, Proc. Natl. Acad. Sci. USA 101 (2004) 2747–3752.
[11] F. Battiston, V. Nicosia, V. Latora, Structural measures for multiplex networks, Phys. Rev. E 89 (2014) 032804.
[12] G. Bianconi, Statistical mechanics of multiplex networks: entropy and overlap, Phys. Rev. E 87 (2013) 062806.
[13] A. Clauset, C. Moore, M.E.J. Newman, Hierarchical structure and the prediction of missing links in networks, Nature 453 (2008) 98–101.
[14] Y.-Y. Ahn, J.P. Bagrow, S. Lehmann, Link communities reveal multiscale complexity in networks, Nature 466 (2010) 761–765.
[15] S.N. Dorogovtsev, A.V. Goltsev, J.F.F. Mendes, K-core organization of complex networks, Phys. Rev. Lett. 96 (2006) 040601.
[16] M.A. Serrano, M. Boguñáb, A. Vespignani, Extracting the multiscale backbone of complex weighted networks, Proc. Natl. Acad. Sci. USA 106 (2006) 6483–6488.
[17] J.E. Hirsch, An index to quantify an individual's scientific research output, Proc. Natl. Acad. Sci. USA 102 (2005) 6569–16572.
[18] A. Schubert, A. Korn, A. Telcs, Hirsch-type indices for characterizing networks, Scientometrics 78 (2009) 375–382.
[19] S.X. Zhao, R. Rousseau, F.Y. Ye, h-degree as a basic measure in weighted networks, J. Inform. 5 (2011) 668–677.
[20] S.X. Zhao, P.L. Zhang, J. Li, A.M. Tan, F.Y. Ye, Abstracting core subnet of weighted networks based on link strengths, J. Assoc. Inf. Sci. Technol. 65 (2014) 984–994.
[21] S.S. Li, X. Lin, X. Liu, F.Y. Ye, H-crystal as a core structure in multilayer weighted networks, Am. J. Inf. Sci. Comput. Eng. 2 (2016) 29–44.
[22] S.P. Borgatti, M.G. Everett, Model of core/periphery structures, Social Networks 21 (2000) 375–395.
[23] M.P. Rombach, M.A. Porter, J.H. Fowler, P.J. Mucha, Core–periphery structure in networks, SIAM J. Appl. Math. 74 (2014) 167–190.
[24] P. Holme, Core–periphery organization of complex networks, Phys. Rev. E 72 (2005) 046111.
[25] M.R.D. Silva, H. Ma, A.P. Zeng, Centrality, network capacity, and modularity as parameters to analyze the core–periphery structure in metabolic networks, Proc. IEEE 96 (2008) 1411–1420.
[26] F.D. Rossa, F. Dercole, C. Piccardi, Profiling core–periphery network structure by random walkers, Sci. Rep. 3 (2013) 1467.
[27] T. Verma1, F. Russmann, N.A.M. Araujo, J. Nagler, H.J. Herrmann, Emergence of core–peripheries in networks, Nature Commun. 7 (2016) 10441.
[28] M.T. Gastner, M.E.J. Newman, Shape and efficiency in spatial distribution networks, J. Stat. Mech (2006) P01015.
[29] M. Jungsbluth, B. Burghardt, A.K. Hartmann, Fingerprinting networks: Correlations of local and global network properties, Physica A 38 (2007) 444–456.
[30] X. Cui, H. He, F. He, et al., Network fingerprint: a knowledge-based characterization of biomedical networks, Sci. Rep. 5 (2015) 13286.
[31] L. Lv, T. Zhou, Q.-M. Zhang, H.E. Stanley, The h-index of a network node and its relation to degree and coreness, Nat. Commun. 7 (2015) 10168.
[32] F.Y. Ye, P.L. Zhang, S.X. Zhao, R. Rousseau, An experimental study on revealing domain knowledge structure by co-keyword networks, J. China Soc. Sci. Tech. Inf. 31 (2012) 1245–1251 (in Chinese).
[33] J. Alstott, E. Bullmore, D. Plenz, Power law: A Python package for analysis of heavy-tailed distributions, PLoS One 9 (1) (2014) e85777.