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Opinion dynamics in activity-driven networks

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Abstract – Social interaction between individuals constantly affects the development of their personal opinions. Previous models such as the Deffuant model and the Hegselmann-Krause (HK) model have assumed that individuals only update their opinions after interacting with neighbors whose opinions are similar to their own. However, people are capable of communicating widely with all of their neighbors to gather their ideas and opinions, even if they encounter a number of opposing attitudes. We propose a model in which agents listen to the opinions of all their neighbors. Continuous opinion dynamics are investigated in activity-driven networks with a tolerance threshold. We study how the initial opinion distribution, tolerance threshold, opinion-updating speed, and activity rate affect the evolution of opinion. We find that when the initial fraction of positive opinion is small, all opinions become negative by the end of the simulation. As the initial fraction of positive opinions rises above a certain value —about 0.45— the final fraction of positive opinions sharply increases and eventually equals 1. Increased tolerance threshold δ is found to lead to a more varied final opinion distribution. We also find that if the negative opinion has an initial advantage, the final fraction of negative opinion increases and reaches its peak as the updating speed λ approaches 0.5. Finally we show that the lower the activity rate of individuals, the greater the fluctuation range of their opinions.

Introduction. – When people face a social issue, such as which candidates to support, which research fields to pursue, or what kind of drink they would like, they will express their attitudes or opinions either consciously or unconsciously [1,2]. Individuals' opinions are shown in books, newspapers, TV, online social networks, or any other social media [3,4]. Generally, people spread opinions in their social networks by interacting with others [5,6]. The collective social behavior of large communities of individuals, such as culture dissemination, rumor spreading and the dynamics of opinion formation are widely studied using the concepts and methods of statistical physics [7–10]. Network science provides a very effective method to explore the collective social behavior [11–15].

Opinion dynamics is one of the social-dynamics problems that can be closely related to physical problems. In everyday life, almost all social interactions are affected and shaped by attitudes or opinions. Opinion dynamics models can be classified into discrete and continuous opinion dynamics models. In some models, opinions are represented by a discrete variable which can take two values: 0 or 1. Examples include the Sznajd model [16], the voter model [17], and the Galam majority-rule model [18] which are similar to the Ising spin model in statistical physics. Two famous continuous opinion models are the Deffuant model [19] and the Hegselmann-Krause model [20] in which individuals' opinions distribute in a real space from 0 to 1 and agents only interact with people whose opinion is close to their own under a given confidence level. All of these traditional opinion models evolved in static networks. However, not only do the opinion dynamics taking place on a network control the network structure, but the network structure also influences the dynamics [21,22]. Kozma and Barrat studied how an adaptive network of

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interacting agents and the agents opinions influence each other [23]. Grauwin and Jensen investigated the opinion dynamics on a dynamic network where the agents are able to break their links and rewire them at random [24]. Li *et al.* researched the adaptive network models of collective decision-making in swarming systems [25]. Fu and Wang studied co-evolutionary dynamics of opinions and networks; the results showed that the diverse opinions disappear in a population in which all individuals share a uniform opinion when the model parameter exceeds a critical value [26].

Agents typically have ongoing relationships with others and interact with peers selected from the related agents [27]. The structure of these relationships plays an important role in social dynamics, and networks can be used to describe these relations (social networks) [28,29]. The social influence and interaction behavior between individuals constantly affect their opinions [30,31]. As we know, the interaction between individuals cannot be a continuous process, for instance, one person attends a party or meets friends once a week or even once a month; that is to say, an individual's activity is an intermittent process in which the individual communicates with others at some time and does not communicate with others at any other time. Here, we derive an analytical network for the study of opinion dynamics specifically devised for a class of time-varying networks, namely activity-driven networks. In recent years, there have been extensive research efforts in activity-driven networks that evolve on a time scale comparable to the time scale of the diffusion process taking place on the network [32]. The activity-driven structure of link activations affects the network dynamics from disease contagion to information diffusion. Therefore, the temporal variation in network connectivity patterns and the ongoing dynamic processes are usually coupled in ways that still challenge our mathematical or computational modeling [33,34].

Considering the fact that many networks are highly dynamical and evolve in time with opinion dynamics, we propose a continuous opinion model and study the opinion dynamics in activity driven networks. Some classic models like the Deffuant model and the HK model assume an individual interacts with another agent only if their opinions are close enough. However, people sometimes communicate widely with all of their neighbors to gather their ideas and opinions, even if they encounter a number of opposing attitudes. When someone posts a negative comment about a stock on Facebook or Twitter, one might become slightly more pessimistic after reading the comment even if he previously favored the stock. Similarly, a pessimist may become an optimist after she or he comes across a positive comment [35]. More broadly, human cooperation commonly results from our evolutionary struggle for survival [36]. The conformist mentality and the convergence of thought and action can be thought of as "cooperative behavior" to some extent. The conformist mentality is a common social psychological phenomena [37,38], and this phenomenon also occurs widely in opinion dynamics. Indeed, people often follow their neighbors' thought and behavior. Here we propose a continuous opinion model with individuals' opinions ranging from -1 to 1. At each time step, an individual communicates with all of his neighbors and then calculates their average opinion. If the opinion difference between his current opinion and the average opinion of his neighbors is larger than his tolerance range or threshold, he will take his neighbors' opinion. Otherwise, he keeps his own opinion. Furthermore, we investigate the opinion dynamics in activity-driven networks, and study how the initial fraction of opinion, the tolerance threshold, and the opinion-updating speed affect the evolution of opinion.

Model of opinion dynamics in activity-driven networks. – To describe more exactly the interaction activities between individuals in real social networks, we consider a network with N individuals, and each agent i is characterized by an activity rate α_i. The activity rates are the ability to create contacts or interactions with other individuals per unit time, and are assigned according to a given probability distribution $F(\alpha)$. Here we adopt heavy-tailed distributions $F(\alpha) \sim \alpha^{-\gamma}$ with activities restricted in the region $\alpha \in [\epsilon, 1]$ to avoid divergences for $\alpha \to 0$, and $\gamma \in (2,3)$ is the exponent [39]. At each time step, the network starts with N disconnected nodes, and then each individual i becomes active with probability α_i and generates m links that are connected to m other randomly selected nodes. At the next time step, all the edges in the network are deleted. This implies that the connections between nodes in activity-driven networks are non-persistent and memoryless, and the degree distribution of the integrated network at time t takes the form $P_t(k) \sim \frac{1}{tm} F[\frac{k}{tm}]$ [39,40].

Each individual i has a continuous opinion g_i represented by a real number in the interval $[-1, 1]$. When $g_i \in (0, 1]$, this means that individual i has a positive opinion. Accordingly, $g_i \in [-1, 0)$ implies individual i has a negative opinion. At the initial time, there are a fraction of g_0 and $1 - g_0$ individuals with positive and negative opinion respectively. The initial opinion is uniformly distributed in opinion space; the positive opinion is located in $(0, 1]$ with uniform distribution $g : U(0, 1)$, and the negative opinion is within $[-1, 0)$ with uniform distribution $g : U(-1, 0)$. As we know, the opinion of an individual is usually affected by his or her neighbors' opinion. In activity-driven networks, at time t, we represent the neighbor set which connects with node i as $N_i(t)$ while the number of node i's neighbors is written as $|N_i(t)| > 0$. Thus we can get the average opinion of the neighbors of node i at time t by $\bar{g}_i(t) = \frac{1}{|N_i(t)|} \sum_{j \in N_i(t)} g_j(t)$. Note that $|N_i(t)| = 0$ means individual i does not have any neighbors yet, so he also does not change his opinion at time t. If the opinion difference between an individual and his neighbors' average opinion are large, due to social pressure, this agent will change his opinion. Here we set an opinion tolerance threshold δ: if the opinion difference between an individual and his neighbors' average opinion is less than the tolerance threshold δ, that means the opinion difference is in his considerable range, so he keeps his current opinion. If the difference of opinion is larger than the tolerance threshold δ, then he adjusts his opinion during the next time step according to the opinion updating rules as follows:

$$g_i(t+1) = \begin{cases} g_i(t), & |\bar{g}_i(t) - g_i(t)| \leq \delta, \\ g_i(t) + \lambda(\bar{g}_i(t) - g_i(t)), & |\bar{g}_i(t) - g_i(t)| > \delta, \end{cases} \quad (1)$$

where λ is the convergence parameter and refers to the speed of updating the opinion of node i. In particular, when $\lambda = 0$, node i will never change his opinion no matter what his neighbors' opinions are. When $\lambda = 1$, once the opinion difference is larger than the tolerance threshold, he will immediately adopt the average opinion of his neighbors $g_i(t+1) = \bar{g}_i(t)$.

Numerical simulation and results. – Let us consider an activity-driven network with $N = 10000$ agents. The activity rates obey the distribution $F(\alpha) \sim \alpha^{-\gamma}$ with $\gamma = 2.1$ and $\epsilon = 0.1$. Based on the Monte Carlo method, we simulate the opinion evolution dynamics in activity-driven networks. Since the results differ for each Monte Carlo trial, here we present the results averaged over 100 independent runs.

Individuals form, reconsider, and possibly change their opinions by constantly interacting with others. Someone's internal psychological factors and external social influences, especially the external pressure from social interaction, strongly affect opinion formation. Individuals adaptively adjust their opinions to decrease the difference with their peers. When the opinion difference between individual i and the average opinion of his neighbors is more than his tolerance threshold $|\bar{g}_i(t) - g_i(t)| > \delta$, individual i will converge to his neighbors' average opinion with speed λ. We may intuitively believe that all of the opinions evolve towards consensus at steady state. Actually the tolerance threshold influences the evolution process; thus the opinions vary within a certain range. Figure 1 shows the opinion evolution process at different values of λ and δ. As we can see, the larger the value of tolerance threshold δ, the more types of opinion are preserved, but opinions reach consensus when the tolerance threshold is small (see $\delta = 0.1$). Interestingly, in the case of $g_0 = 0.5$, the convergence parameter almost has no effect on the evolution process that can be seen by comparing the three columns.

The pace of updating an individual's opinion also has a significant impact on the evolution of opinions. Let us take a look at how the updating speed λ affects the final fraction of opinions. Here N_p and N_n are the number of people with positive opinions and negative opinions, respectively. $f_p = N_p/N$ and $f_n = N_n/N$ are the fraction of positive opinions and negative opinions in the network, respectively. Although the final state of individuals opinion could converge to a certain range, the fraction of positive

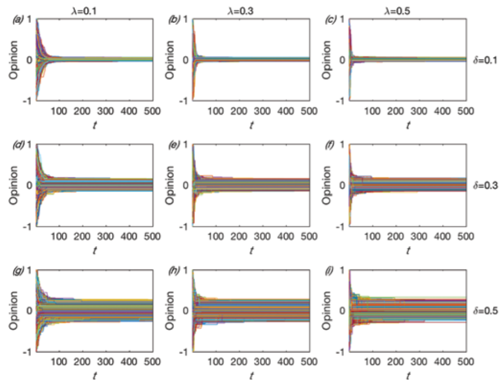

Fig. 1: (Colour online) The evolution of opinions varies with time in activity-driven networks for different convergence parameters λ and tolerance thresholds δ. The initial opinion uniformly distributed in the interval $[-1, 1]$. From the left column to the right column, the values of λ are 0.1, 0.3 and 0.5, respectively. The rows correspond to three different values 0.1, 0.3 and 0.5 of the tolerance threshold δ. The other parameters are set as $g_0 = 0.5$, and $m = 3$.

opinion $f_p = N_p/N$ and negative opinion $f_n = N_n/N$ may have a larger fluctuation.

The updating speed λ reflects how fast people adjust their opinion. If the adjusting speed is too low, *i.e.*, $\lambda \to 0$, then according to the opinion updating rules $g_i(t+1) = g_i(t) + \lambda(\bar{g}_i(t) - g_i(t))$, we derive $g_i(t+1) \approx g_i(t)$. This implies almost all of the neighbors have no impact on the individuals opinion at the next time step. However, they will gradually change their opinion with increasing λ. If the updating speed is fast enough ($\lambda \to 1$), then the individuals opinion rapidly tends to the average opinion of his neighbors: $g_i(t+1) \approx \bar{g}_i(t)$. Thus, λ can significantly affect the $f_p(\infty)$ and $f_n(\infty)$. In figs. 2(a1), (b1), we set the initial fraction of positive opinion as $g_0 = 0.45 < 0.5$, which means the number of negative opinion is larger than positive opinion at initial time. As we can see from figs. 2(a1), (b1), the fraction of negative opinion increases and reaches its peak when the updating speed is around 0.5. In figs. 2(a2), (b2), the initial fraction of positive opinion is $g_0 = 0.55 > 0.5$, which means the number of positive opinion is larger than negative opinion at initial time. Interestingly, the fraction of positive opinion increases and reaches its peak when the updating speed is around 0.5. When the initial fraction of positive (negative) opinion is less than 0.5 and the updating speed is larger than 0.5, the fraction of negative (positive) opinion gradually decreases. This result shows that the evolution of opinion significantly relates to the internal factors (*e.g.*, the speed or extent to which people are willing to change their opinions) and the external environment (*e.g.*, the neighbors opinion). It also illustrates that the initial fraction of opinion has an important impact on the final fraction of opinion. If the negative (positive) opinion has a numerical advantage at the initial time, the final fraction of negative (positive) opinion increases and reaches

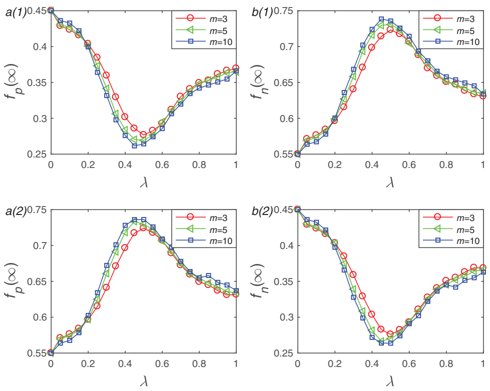

Fig. 2: (Colour online) The fraction of positive and negative opinion as a function of the updating speed λ for different numbers of active connections m. $f_p(\infty)$ and $f_n(\infty)$ are the fractions of positive and negative opinions (respectively) when $t \to \infty$. (a1), (b1): parameters are $g_0 = 0.45$ and $\delta = 0.4$; (a2), (b2): parameters are $g_0 = 0.55$ and $\delta = 0.4$.

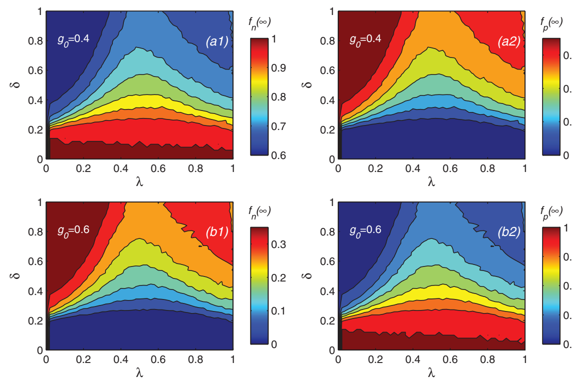

Fig. 3: (Colour online) The final fraction of positive and negative opinion as a function of the tolerance threshold δ and the rate of opinion updating λ. $f_p(\infty)$ and $f_n(\infty)$ are the fraction of positive and negative opinion when $t \to \infty$. (a1) and (a2) are the final fraction of negative and positive opinion at $g_0 = 0.6$, respectively. (b1) and (b2) are the final fraction of negative and positive opinion at , respectively. The parameter is $m = 3$.

its peak when the opinion updating speed λ is about 0.5. At the same time, fig. 2 shows that the number of connections m generated by people in an active state has less effect on the final fraction of opinion.

The tolerance threshold δ and the opinion updating pace λ have significant influence on the final distribution of opinion. The tolerance threshold δ reflects how willing an individual is to put up with difference of opinion from his surrounding environment. When the tolerance threshold is large, an individual tolerates or accepts the opinion difference of his neighbors and does not change his own opinion, even though he finds the difference unpleasant or unsatisfactory. However, when the tolerance threshold is small, once an agent recognizes the opinion difference, he will update his opinion to be the average opinion of his neighbors, which implies these people with low tolerance have strong social identity. Figure 3 shows that the final fraction of opinion varies with the tolerance threshold and the opinion updating speed for different initial fractions of positive opinion. Suppose we have an initial configuration with more positive opinions than negative opinions. Then with the increase of tolerance threshold δ, the final opinion configuration could become more negative than positive. Let us look more carefully at fig. 3(a1) and fig. 3(b2). In fig. 3(a1), the initial positive opinion is $g_0 = 0.4$; that is, the negative opinion has an advantage at the initial time, but we can see that the final fraction of negative opinion $f_n(\infty)$ decreases as the tolerance threshold δ increase. Figure 3(b2) shows the same phenomenon, the positive opinion is the dominant one in the initial number, the final fraction of positive opinion $f_p(\infty)$ also decreases with the increase of tolerance threshold δ. The reason behind this interesting phenomenon is that the increase of tolerance threshold δ may lead to a large level of opinion difference. At the same time, as the tolerance threshold is large, individuals do not want to change their

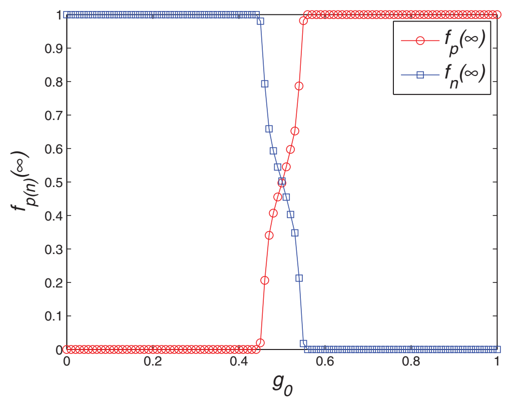

Fig. 4: (Colour online) The fraction of positive and negative opinion varies with the initial fraction of positive opinions g_0. Parameters are set as: $\delta = 0.1$, $\lambda = 0.5$ and $m = 3$.

own opinion. Therefore, even though the number of negative/positive opinion has an advantage at initial time, the final fraction of positive/negative opinion is not going to decrease sharply.

The initial fraction of opinion g_0 has an important effect on the evolution of opinion in the system. Without any interference from external environment, once when positive (negative) opinions overwhelm the other side, then the "herd effect" could be formed. Figure 4 studies how the final fractions of positive and negative opinion evolve for various initial fraction of positive g_0. As we can see from fig. 4, when g_0 is small, there is no positive opinion at the end of evolution time. Once g_0 is larger than a certain value (about 0.45), the final fraction of positive opinions sharply increases and then equals 1. When the negative opinion has an advantage at initial time, that is $g_0 < 0.5$, individuals will easily be influenced by surrounding neighbors with negative opinions, which leads

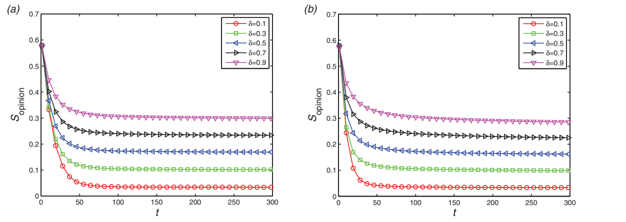

Fig. 5: (Colour online) The difference of opinion in the whole network $S_{opinion}$ varies with time t for different tolerance threshold. Parameters are set as $g_0 = 0.5$, $\lambda = 0.1$. Panels (a) and (b) show different situations as the value of parameter m are $m = 3$ and $m = 10$, respectively.

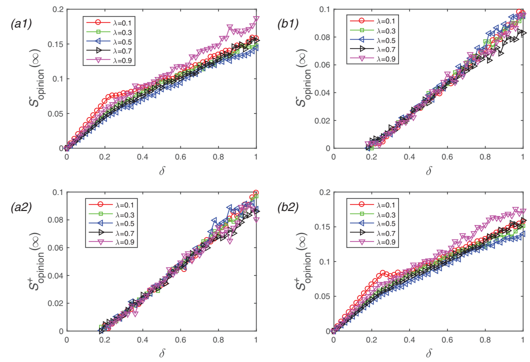

Fig. 6: (Colour online) The difference between the average positive and negative opinions varies with tolerance threshold for different opinion updating speed λ. $S^+_{opinion}(\infty)$ and $S^-_{opinion}(\infty)$ are the difference between the average positive and negative opinions when $t \to \infty$. (a1) and (a2) show the variation tendency of $S^+_{opinion}(\infty)$ and $S^-_{opinion}(\infty)$ at $g_0 = 0.4$, respectively; (b1) and (b2) show the variation trend of $S^+_{opinion}(\infty)$ and $S^-_{opinion}(\infty)$ at $g_0 = 0.6$, respectively. The other parameter is $m = 3$.

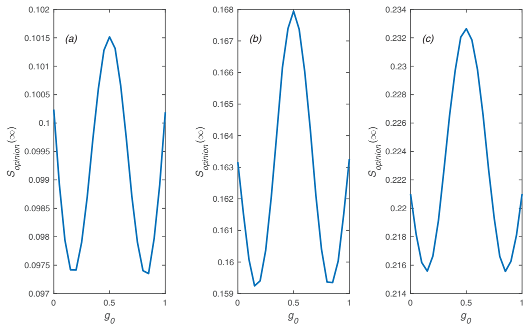

Fig. 7: (Colour online) The difference of opinion $S_{opinion}(\infty)$ as a function the initial fraction of positive opinion g_0. $S_{opinion}(\infty)$ is the difference of opinion when $t \to \infty$. Parameters are set as $\lambda = 0.1$ and $m = 3$. (a) The tolerance threshold $\delta = 0.3$; (b) the tolerance threshold $\delta = 0.5$; (c) the tolerance threshold $\delta = 0.7$.

the agents with positive opinion to gradually change their positive opinions and tend toward negative. This gradual process results in negative opinion dominating in the end. Thus the initial fraction of opinion g_0 is a crucial index to measure the final tendency of opinion evolution. Without any other external factors, on the basis of the mean-field method, we can calculate the average opinion of all nodes at initial time as $\bar{g}_0 = g_0 - 0.5$. When $g_0 < 0.5$, $\bar{g}_0 = g_0 - 0.5 < 0$, so the average opinion of all nodes in the whole network continually tends to negative opinion, and when $g_0 > 0.5$, the average opinion of all nodes similarly approaches positive opinion.

The difference of individuals' opinion in the whole network reflects the overall degree of bias of opinion in the whole network. To measure how individual opinions differ at each step, we here use the standard deviation of the individuals' opinions:

$$S_{opinion}(t) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(g_i(t) - \bar{g}(t))^2}. \quad (2)$$

Where $g(\bar{t})$ is the mean value of all indifiduals' opinion at time t.

Figure 5(a) and (b) show that the differences of opinion in the whole network decrease with time for different tolerance threshold. Furthermore, with the increase of tolerance threshold δ, the difference of opinion obviously increases. Since the threshold δ expresses the tolerance of individuals to their surrounding environment, the more the individuals' tolerance, the more the difference of opinion in the whole network. However, when the difference of opinion reaches a certain value, individuals' opinions reach a steady state. We can see that when $t > 200$, the difference of opinion $S_{opinion}$ is almost unchanged.

In order to further study the effect of tolerance threshold δ on the difference of opinion, we calculate the difference of positive and negative opinion respectively. Let $S^+_{opinion}(\infty)$ and $S^-_{opinion}(\infty)$ be the difference of positive and negative opinion, respectively. From fig. 6, we see that the difference of opinion $S_{opinion}(\infty)$ increases with the tolerance threshold δ regardless of whether the opinion is positive or negative. From fig. 6(a1) and fig. 6(b1), one sees that, for small g_0 values, $S^-_{opinion}(\infty) = 0$ only at $\delta = 0$ while $S^-_{opinion}(\infty)$ is zero at δ is not equal to 0 for large g_0 values ($g_0 = 0.6$ in fig. 6(b1)). For the large initial fraction of positive opinion $g_0 = 0.6$, as the tolerance threshold is less than 0.2, the difference of negative opinion is almost to 0. Since there are many positive opinions at the initial time, and the tolerance threshold is small, almost all of the individuals adopt a positive opinion. This leads to few (if any) negative opinions in the network. Comparing fig. 6(a2) and fig. 6(b2), we find the same phenomenon. Interestingly, the difference of opinion is relatively small when the updating speed is $\lambda = 0.5$ (see fig. 6(a1) and fig. 6(b2)). However, when the opinion updating speed is fast, for example, as $\lambda = 0.9$, the difference of opinion is large (see fig. 6(a1) and fig. 6(b2)).

As mentioned above, the initial fraction of opinion has a significant impact on the final fraction of opinion. Now let

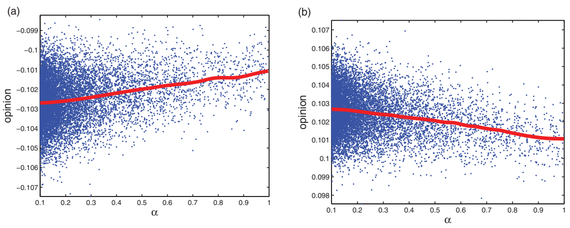

Fig. 8: (Colour online) The relationship between the final opinion distribution and activity rate. Parameters are set as $\lambda = 0.05$, $m = 3$ and $\delta = 0.1$. Panels (a) and (b) are for $g_0 = 0.4$ and $g_0 = 0.6$, respectively. The red line is the result of nonparametric fitting. (For more information about the nonparametric fitting method see the appendix.)

us take a careful look at how the initial fraction of positive opinion g_0 affects the difference of opinion. In fig. 7, we use the standard deviation to indicate the difference of individuals' opinions. On the one hand, the range of the g_i and $\bar{g}_i$ are $[-1, 1]$, $i = 1, 2, \ldots, N$; on the other hand, individuals' opinions could converge to within a certain interval, thus the fluctuations of $S_{opinion}(\infty)$ are relative small. As we can see from fig. 7(a), (b) and (c), with the increase of initial fraction of positive opinion g_0, there are a peak and two valleys in the difference of opinion $S_{opinion}$. Most importantly, even for different tolerance thresholds, the difference of opinion $S_{opinion}(\infty)$ reaches the peak and the valley at the same initial fraction of positive opinion g_0. The higher the tolerance threshold, the higher the peak and the greater the difference between the peak value and the valley value becomes. In addition, we find that $S_{opinion}(\infty)$ is maximum when $g_0 \approx 0.5$, which means that the difference of opinion $S_{opinion}(\infty)$ is large if the initial fractions of positive opinion and negative opinion are the same.

The activity-driven network described in the second section is applied to build the underlying connections between individuals. In general, the individuals with high activity rates could interact with more neighbors at each time. Figure 8(a) shows that when the negative opinion dominates initially (*i.e.*, the initial fraction of positive opinion is $g_0 = 0.4$), there is positive correlation between individuals' activity rates and individuals' opinions. However, when the positive opinion has an advantage at the initial time (*i.e.*, the initial fraction of positive opinion is $g_0 = 0.6$), fig. 8(b) shows that the relationship between individuals' activity rates and individuals' opinions is negatively correlated. The above results demonstrate that there is a strong relationship between the degree of activity rate and individual' opinion, and the higher the activity rate of the individual, the more likely it is for the individuals' opinions to tend to 0. We also find that the lower the activity rate of the individuals, the greater the fluctuation range of their opinions; adversely, the higher the activity rate of the individuals, the smaller the fluctuation range of their opinions. The reason is that the individuals with low activity rates only connect with a few other agents at each time step. The frequency of opinion updating for people with low activity rates is therefore less than the opinion-updating frequency for those with high activity rate, and this finally leads to large fluctuations in opinion for those with a low activity rate.

Conclusions. – Quantitative analysis of human behavior is an interesting and important project. The interaction topology connecting individuals often varies with time, especially considering modern online social networks which provide a convenient way for people to interact with different users at different times. In the previous studies, researchers proposed many opinion spreading models, such as the Deffuant model and the Hegselmann-Krause model. On the one hand, almost all of those traditional opinion models evolved in static networks, on the other hand, they assume that an individual interacts with another agent only if their opinions are close enough. Therefore, we have built a realistic continuous opinion model to study the opinion dynamics of activity-driven networks. By studying the behavior of this opinion dynamics system, we could improve our understanding of the evolution of opinion.

After a large number of numerical simulations, we find that there exists a critical value of λ (the updating speed) at about 0.5. Interestingly, the difference of opinion is relatively small at this critical value. We find that a higher tolerance threshold δ leads to less agreement among agents. We also observe that when the initial fraction of positive opinion g_0 is small, there are no positive opinions once the system reaches a steady state, and $S_{opinion}(\infty)$ begins to change abruptly at $g_0 = 0.5 \pm 0.05$. In addition, we find that $S_{opinion}(\infty)$ is maximized when $g_0 \approx 0.5$. This value must in fact equal 0.5 due to the system's symmetry with respect to positive and negative opinions. Our final observation is that the lower the activity rate of the individuals, the greater the range of fluctuation of their opinions.

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Appendix: nonparametric regression. – We use nonparametric regression procedures to obtain a smooth set of points from each set of scattered data (k_i, n_i), $i = 1, \ldots, N$ [41]. Nadaraya-Watson: we construct the kernel smoother function

$$m_h(k) = \frac{\sum_i^N K_h(k - k_i) n_i}{\sum_i^n K_h(k - k_i)}. \tag{A.1}$$

Here $K_h(k - k_i)$ is a Gaussian kernel of the form

$$K_h(k - k_i) = \frac{1}{\sqrt{2N}} \exp\left[-\frac{(k - k_i)^2}{2h^2}\right]. \tag{A.2}$$

The optimal bandwidth h suggested by Bowman and Azzalini is

$$h = \sqrt{h_k h_n}, \tag{A.3}$$

where $h_{k(n)} = (\frac{4N}{3})^{\frac{1}{5}}\delta_{k(n)}$, $\delta_{k(n)} = \frac{\text{median}\{|k_i(n_i)-\text{median}\{k_i(n_i)\}|\}}{0.6754}$, $i = 1, 2, \ldots, N$.

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