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Uncertainty in oil production predicted by percolation theory

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Abstract

In this paper, we apply scaling laws from percolation theory to the problem of estimating the time for a fluid injected into an oilfield to breakthrough into a production well. The main contribution is to show that when these previously published results are used on realistic data they are in good agreement with results calculated in a more conventional way, but they can be obtained significantly and more quickly. As a result, they may be used in practical engineering circumstances and aid decision-making for real field problems. © 2002 Published by Elsevier Science B.V.

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1. Introduction

Oil reservoirs are extremely complex containing geological structures on all length scales. These heterogeneities have a significant impact on hydrocarbon recovery. The conventional approach to estimating recovery is to build a detailed geological model (of around 10 million numerical grid cells), populate it with flow properties, coarse grain it and then perform a flow simulation. In order to estimate the uncertainty in production a number of possible geological realisations are constructed (with associated probabilities) and this procedure repeated many times. A simple order of magnitude estimate of

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computing times (given today's model sizes and computing speeds) indicates that this could take many hundreds of days. Clearly, this is completely impractical for many purposes.

Given this practical limitation a number of approaches have been taken, for example improved coarse graining methods [1,2], fast simulation [3,4], and so on. In this paper, we adopt a different perspective. We simplify the geological model and flow physics such that quasi-analytical predictions of uncertainty can be made extremely quickly. The advantage is that the effects of the complex geometry which influence the flow can be readily estimated. Clearly, the disadvantage is that much of the flow physics and subtleties of the heterogeneity distribution are missed. Whilst, it is the aim of future research to address those issues we show, in this paper, that this simple model can already give reasonable estimates of the production performance when applied to a real data set.

We start by simplifying the rock heterogeneity by assuming that the permeability can be split into "good" rock (i.e., finite, non-zero permeability) and "poor" rock (low or zero permeability). For all practical purposes the flow takes place just in the good rock. It is the interconnectivity of the permeable rock that controls the flow. The spatial distribution of the sand is also governed by the geological process but can frequently be considered as independent or of a short range correlation. Hence, the problem of the connectivity of the sandbodies is precisely a continuum percolation problem. The place of the occupancy probability p of percolation theory is taken by the volume fraction of good sand (the net to gross ratio). This percolation view of sandbody connectivity has been used before [5], but here we look not just at the static connectivity but also at the dynamic displacement on this percolating system.

The second simplification is of the flow physics. Here we shall assume that the displacement is like passive tracer transport. In other words, we have single-phase flow from injector to producer (we only consider a single well pair) and we assume that the injected fluid is passively convected along these streamlines. To be specific, we shall consider the time to breakthrough (or the first passage time for a passive tracer) as the measure of performance. These are gross simplifications which enable us to use the scaling laws of percolation theory [6] to determine production performance and its associated uncertainty.

2. Flow model

To simplify the model we shall assume that the permeability is either zero (shale) or one (sand). The sandbodies are cuboidal. They are distributed independently and randomly (i.e., as a Poisson process) in space to a volume fraction of p. Further, we shall assume that the displacing fluid has the same viscosity and density as the displaced fluid. This has the advantage that as the injected fluid displaces the oil the pressure field is unchanged. This pressure field is determined by the solution of the single-phase flow equations ($\nabla \cdot K\nabla P = 0$). The injected flow then just follows the streamlines (normals to the isobars, pressure is P) of this flow. In dimensionless units the permeability (K) is either zero or one as described before. The boundary conditions are fixed pressure of +1 at the injection well and 0 at the production well. In this work, we shall only consider a single well pair separated by a Euclidean distance r. The breakthrough time then corresponds to the first passage time for transport between the injector and the producer.

For a given model of the reservoir we can then sample for different realisations of the locations of the wells (or equivalently for the same well locations for different models of the reservoir with the same underlying statistics) and plot the distribution of breakthrough times. This is the conditional probability that the breakthrough time is t_{br} given that the reservoir size (measured in dimensionless units of sandbody length) is L and the net to gross is p, i.e., $P(t_{br}|r, L, p)$. In previous studies [7,8], we have shown that this distribution obeys the following scaling:

$$P(t_{br}|r,L,p) \sim \frac{1}{r^{d_t}}\left(\frac{t_{br}}{r^{d_t}}\right)^{-g_t} f_1\left(\frac{t_{br}}{r^{d_t}}\right) f_2\left(\frac{t_{br}}{L^{d_t}}\right) f_3\left(\frac{t_{br}}{\xi^{d_t}}\right) , \tag{1}$$

$$f_1(x) = \exp(-ax^{\phi}) ,$$

$$f_2(x) = \exp(-bx^{\psi}) ,$$

$$f_3(x) = \exp(-cx^{\pi}) .$$

Currently, the best estimates of the various coefficients and powers (as found from detailed numerical experiments on lattices and theory, see Ref. [9]) in this are

$$d_t = 1.33 \pm 0.05, \qquad g_t = 1.90 \pm 0.03, \qquad a = 1.1, \qquad b = 5.0,$$

$$c = 1.6(p < p_c)2.6(p > p_c), \qquad \phi = 3.0, \qquad \psi = 3.0, \qquad \pi = 1.0 \text{ and}$$

$$\xi = |p - p_c|^{-\nu} \quad \nu = \tfrac{4}{3}, \qquad p_c = 0.668 \pm 0.003$$

(for continuum percolation).

In this paper, we will not discuss the background to this scaling relationship, but concentrate on how well it succeeds in predicting the breakthrough time for a realistic permeability field. However, it is worth spending some time describing the motivation behind the form of the various functions. The first expression (f_1) is an extension to the expression developed by others (see Ref. [6] for a detailed discussion) for the shortest path length in a percolating cluster between two points. The breakthrough time is strongly correlated with the shortest path length (or chemical path).

To this there are some corrections for real systems. In a finite size system very large excursions of the streamlines are not permitted because of the boundaries so there is a maximum length permitted (and also a maximum to the minimum transit time). This cut-off is given by the expression f_2. Away from the percolation threshold the clusters of connected bodies have a "typical" size (given by the percolation correlation length, ξ) which also truncates the excursion of the streamlines. This leads to the cut-off given by the expression f_3. The multiplication together with these three expressions is an assumption that has been tested by Dokholyan et al. [7]. Also a more detailed derivation of this form is given there and the references therein. Here, we shall concentrate on using this scaling form to make predictions about the distribution of breakthrough times for a realistic data set.

3. Application to a real field

We took as an example a deep water turbidite reservoir. The field is approximately 10 km long by 1.5 km wide by 150 m thick. The turbidite channels, which make up most of the net pay (permeable sand) in the reservoir, are typically 8 km long by 200 m wide by 15 m thick. These channels have their long axes aligned with that of the reservoir. The net to gross ratio (percolation occupancy probability, p) is 50%. The typical well spacing was around 1.5 km either aligned or perpendicular to the long axis of the field. In order to account for the anisotropy in the shape of the sand bodies and the field we first make all length units dimensionless by scaling with the dimension of the sand body in the appropriate direction (so the field dimensions are then L_x, L_y and L_z in the appropriate directions). Then scaling law, Eq. (1), can be applied with the minimum of these three values ($L = \min(L_x, L_y, L_z)$). The validity of using just the minimum length has been previously tested [9].

The real field is rather more complex than this, and a more realistic reservoir description was made and put into a conventional flow simulator. We could then enter these dimensions into the scaling formula, Eq. (1). It should be noted that, first the dimensionless units were converted into real field units to compare with the conventional simulation results. Using these data we find breakthrough times of around one year. The full probability distribution of breakthrough times from the scaling law is given by the solid curve in Fig. 1.

In addition conventional numerical simulations were carried out for the field. We could then collect the statistics for breakthrough times for the various well pairs to compare with this theoretical prediction. Not all pairs exhibited breakthrough in the timescale over which the simulations were run and there were only three injectors so there were only 9 samples. The histogram of breakthrough times is also shown on Fig. 1. Clearly, with such a small sample these results cannot be taken as conclusive however, certainly they are indicative that the percolation prediction from the simple

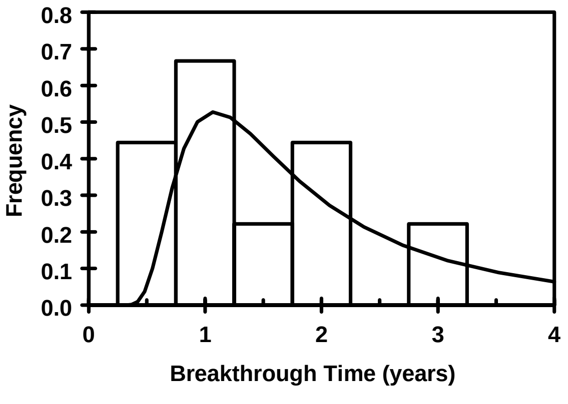

Fig. 1. Comparison of probability distribution of breakthrough times for example reservoir obtained from percolation theory (smooth curve) and from full field model (histogram).

model is consistent with the results of the numerical simulation of the more complex reservoir model. The agreement with the predictions is certainly good enough for engineering purposes. We would hope that if the simulation had been run for longer and more well pairs had broken through that better statistics could have been collected. The main point being that the scaling predictions took a fraction of a second of cpu time (and could be carried out on a simple spreadsheet) compared with the hours required for the conventional simulation approach. This makes this a practical tool to be used for making engineering and management decisions.

4. Conclusions

We have applied results obtained earlier for the scaling law for breakthrough time distributions for oilfield recovery to realistic field data. We have shown that by making a number of simplifying assumptions we can readily use previous results from percolation theory to make extremely rapid estimates of the uncertainty in breakthrough time. The agreement between the theory and the conventional simulation approach is accurate enough for engineering purposes and therefore makes it a practical tool for supporting decision making.

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