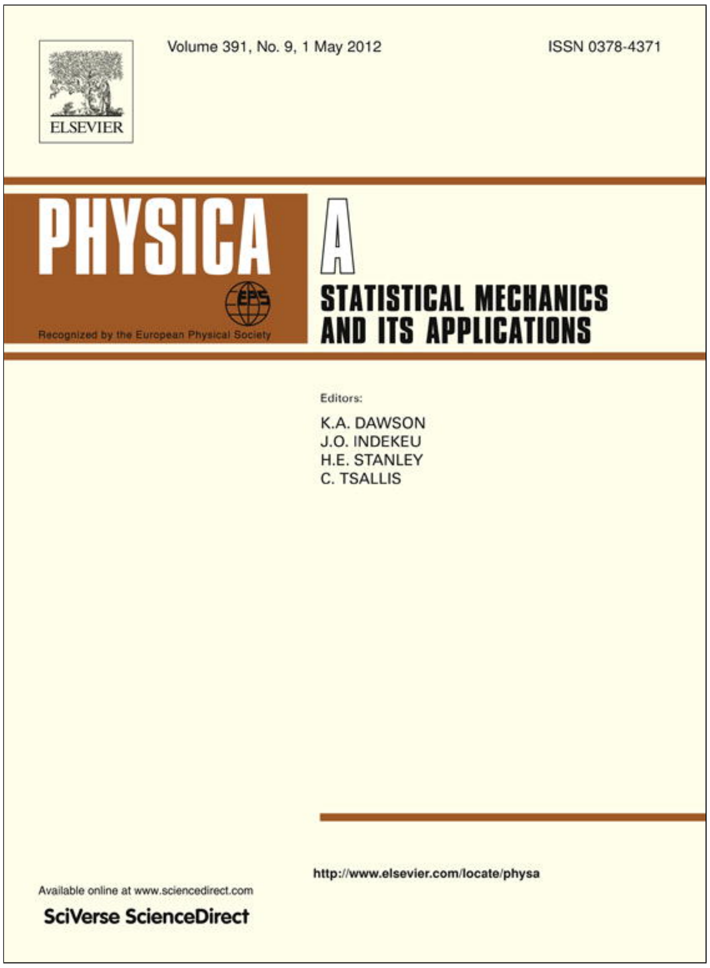
Volume 391, No. 9, 1 May 2012
ISSN 0378-4371
ELSEVIER
PHYSICA
A
STATISTICAL MECHANICS
AND ITS APPLICATIONS
Recognized by the European Physical Society
Editors:
K.A. DAWSON
J.O. INDEKEU
H.E. STANLEY
C. TSALLIS
http://www.elsevier.com/locate/physa
Available online at www.sciencedirect.com
SciVerse ScienceDirect

Physica A 391 (2012) 2860–2866

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

1/*f* behavior in cross-correlations between absolute returns in a US market

Igor Gvozdanovic [a], Boris Podobnik [b,c,d,a,*], Duan Wang [c], H. Eugene Stanley [c]

[a] *Zagreb School of Economics and Management, 10000 Zagreb, Croatia*
[b] *Faculty of Civil Engineering, University of Rijeka, 51000 Rijeka, Croatia*
[c] *Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA*
[d] *Faculty of Economics, University of Ljubljana, 1000 Ljubljana, Slovenia*

A R T I C L E I N F O

Article history:
Received 9 November 2011
Available online 29 December 2011

Keywords:
Finance
Cross-correlations
$1/f$ noise

A B S T R A C T

Employing detrended fluctuation analysis (DFA) and detrended cross-correlations analysis (DCCA), we analyze auto-correlations in the absolute returns for each of 30 Dow Jones Industrial Average (DJIA) constituents, S_i, and cross-correlations in the absolute returns between the DJIA and each S_i. We find that each DJIA member follows the DJIA in absolute returns, since the DCCA curve for each pair (S_i, DJIA_i) exhibits strong cross-correlations, with average DCCA exponent $\langle\lambda\rangle = 1.03 \pm 0.04$. This value for $\langle\lambda\rangle$ implies that the power-law cross-correlations are of the $1/f$ functional form. For the financial firms comprising the DJIA, we also find that the DFA and DCCA exponents controlling the duration of firm risk are somewhat larger than the corresponding values for the rest of the US financial industry.

1. Introduction

Investing in a variety of different assets (''portfolio diversification'') is a technique used in finance to reduce investment risk. Diversification lowers risk even if the returns of a portfolio's assets are positively – but not perfectly – correlated. When the returns of the assets are positively correlated, the diversified portfolio will be less risky than the weighted average risk of its constituent assets.

The theory of portfolio diversification assumes that there is no strong positive relationship among the returns of different assets. Here we test this assumption by calculating the cross-correlations between different assets, quantified by stock returns. Studying cross-correlations enables us to base our prediction of future outcomes on current information. In finance, we base our risk estimate on cross correlation matrices derived from asset and investment portfolios [1,2]. Many methods have been used to investigate cross-correlations between pairs of simultaneously recorded time series [3,4] or among a large number of simultaneously recorded time series [5,1,2,6].

To test how diversification can be helpful, we will estimate the cross-correlations among different assets [7,8]. Note that the greater the cross-correlations among a portfolio's assets, the smaller will be the benefit of cross-correlations. Cross-correlations exist throughout the entire market when either

(a) the cross-correlations are weak and exist only for zero lag (or for a small number of lags), or
(b) the cross-correlations are strong and long-range.

In case (a), once a single market becomes more volatile and the volatility is transmitted across different markets, the risk decays quickly. In case (b), cross-correlations are strong and the risk transferred from abroad decays slowly. Note that in case (b) diversification is less helpful because different assets follow each other for a longer time period.

* Corresponding author at: Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA.
E-mail address: bp@phy.hr (B. Podobnik).

0378-4371/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2011.12.020

Ref. [9] reports that, in the case of international stocks, cross-correlations between nine highly developed economies fluctuate strongly over time, and that fluctuations increased during periods of high market volatility. The finding that there is a link between zero-lag cross-correlations and market volatility is "bad news" for global money managers who attempt to reduce risk by diversifying stocks internationally. In order to determine whether the duration of a financial crisis will be short or long, Ref. [10] recently reported that, for six Latin American markets, the effects of a financial crisis were short-term, with each of the six markets returning to a low volatility regime after only two to four months following each crisis.

Ref. [11] report that the autocorrelation functions for Dow Jones and S&P500 absolute returns and their cross-correlation functions approximately overlap, are non-zero, and exhibit a long-range behavior. Two papers recently analyzed volatility cross-correlations for both the US market and the international market. Ref. [12] reports long-range cross-correlations among the 1340 members of the New York Stock Exchange (NYSE) Composite, analyzing 1340 time series with 2172 daily records during the 8.7-year period between the 2nd January 2001 to the 24th August 2009. Ref. [13] studies 48 world indices, one for each of 48 countries, finds long-range power-law cross correlations in the absolute values of returns that quantify risk, and finds that the correlations decay much more slowly than cross correlations between the returns. Both papers [12,13] report long-range magnitude cross-correlations in collective modes of financial data using time-lag random matrix theory (TLRMT). Because random matrix theory (RMT) is based on cross-correlation coefficients and thus is intended for stationary signals, the method is not reliable when measuring the power-law exponent precisely. Our goal is to determine the functional form of the long-range magnitude cross-correlations measuring cross-correlations not at a collective level, but between pairs of time series, and thus to find whether the exponents characterizing the cross-correlations exhibit universality or diversity.

2. Methods

Two methods have been proposed for studying long-range auto-correlations and cross-correlations in the presence of nonstationarity, (i) detrended fluctuation analysis (DFA) [14] and (ii) detrended cross-correlation analysis (DCCA) [3,15]. To understand the DFA method, we select a time series $x(i)$ where $i = 1, \ldots, L_{\max}$ ($L_{\max}$ being the length of the time series), we integrate the signal $x(i)$, and we obtain $y(k) = \sum x(i) - \langle x\rangle$, where $\langle x\rangle$ is the mean. We then divide the integrated signal $y(k)$ into boxes of equal size n. Into each box of size n, we fit $y(k)$, using a polynomial function of order l, which represents the trend in that particular box. The integrated signal $y(k)$ is then detrended by subtracting the local trend $y_n(k)$ in each box of size n,

$$F(n) = \sqrt{\frac{1}{L_{\max}} \sum [y(k) - y_n(k)]^2}. \tag{1}$$

When power-law auto-correlations are present, the DFA method, represented by $F(n) \propto n^{\alpha}$, and the power spectrum $S(f) \propto f^{-\beta}$ are related when the exponents are related as [16]

$$\alpha = \frac{1+\beta}{2}. \tag{2}$$

Thus $\alpha = 1$ corresponds to $1/f$ noise.

Ref. [3] analyzes the covariance between two time series in both the classical and detrended approaches. It shows that, for two stationary processes y_k and y'_k where the cross-correlation function scales as $C(n) \propto n^{-\gamma_X}$, the expected covariance between two random walks of n steps scales as $\langle (R_n - \langle R_n\rangle)(R'_n - \langle R'_n\rangle)\rangle \propto n^{\lambda}$, where $\lambda = 1 - 0.5\gamma_X$, and $R_n = \sum_{k=1}^{n} y_k$. Note that if each series is power-law auto-correlated, but there are no cross-correlations between the series $(\langle (y_k - y_k)(y'_l - y'_l)\rangle = 0)$ for any choice of l and k, then $\langle (R_n - \langle R_n\rangle)(R'_n - \langle R'_n\rangle)\rangle = 0$. In probability theory and statistics, covariance is a measure of how much two variables, e.g., X and Y, change together, Cov $(X, Y) = E[(X - \langle X\rangle)(Y - \langle Y\rangle)]$. Covariance is non-zero if errors $X - \langle X\rangle$ of X follow errors $Y - \langle Y\rangle$ calculated for Y. If the errors do not follow each other, there are no cross-correlations. In the Appendix we study a special case in which we define "covariance" between squared errors.

Utilizing the work presented in a seminal paper [3], we quantify the cross-correlations by applying detrended cross-correlation analysis (DCCA) [3]. We consider two long-range cross-correlated time series $\{y_i\}$ and $\{y'_i\}$ of equal length N and compute two integrated signals $R_k \equiv \sum_{i=1}^{k} y_i$ and $R'_k \equiv \sum_{i=1}^{k} y'_i$, where $k = 1, \ldots, N$. We divide the entire time series into $N - n$ overlapping boxes, each containing $n + 1$ values. For both time series, in each box (window) that begins at i and ends at $i + n$, we define the "local trend" to be the ordinate of a linear least-squares fit. We define the "detrended walk" to be the difference between the original walk and the local trend. We next calculate the covariance of the residuals in each box, $f^2_{\mathrm{DCCA}}(n, i) \equiv 1/(n-1) \sum_{k=i}^{i+n} (R_k - \widetilde{R_{k,i}})(R'_k - \widetilde{R'_{k,i}})$. We are then able to calculate the detrended covariance by summing over all overlapping $N - n$ boxes of size n,

$$F^2_{\mathrm{DCCA}}(n) \equiv 1/(N-n) \sum_{i=1}^{N-n} f^2_{\mathrm{DCCA}}(n, i). \tag{3}$$

If the detrended covariance vs. n is zero, there are no cross-correlations. However, for finite time series due to size effect, the absence of cross-correlations does not imply that the detrended covariance vs. n is zero, but only that it oscillates around

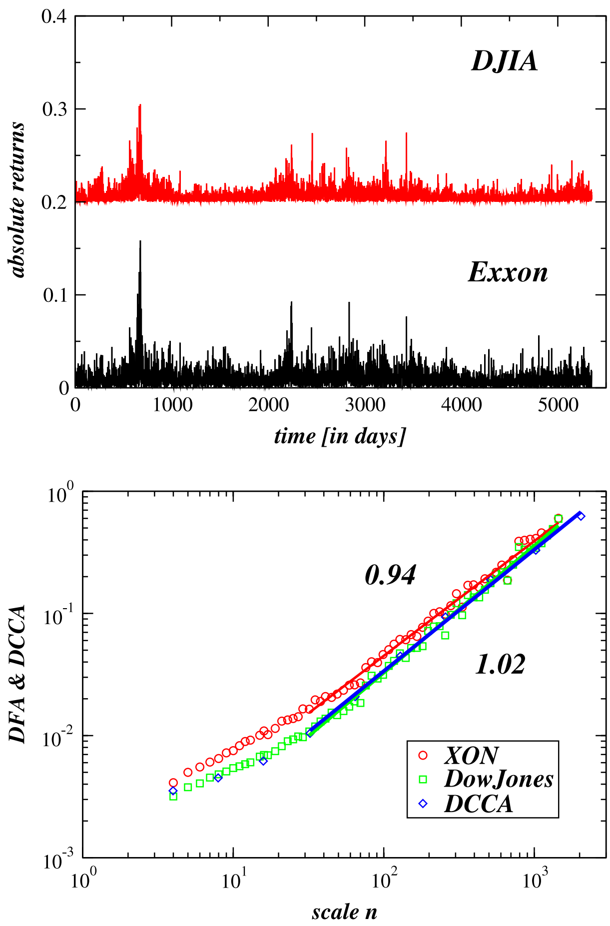

Fig. 1. $1/f$ behavior of cross-correlations between absolute returns of Exxon Mobil Corporation and Dow Jones Industrial Average (DJIA). (a) Exxon and DJIA follow each other in absolute returns implying the existence of cross-correlations. (b) Detrended fluctuation analysis DFA and detrended cross-correlations analysis DCCA calculated between the absolute values of returns of Exxon Mobil Corporation and Dow Jones Industrial Average. Each DFA curve and the DCCA curve follows a power-law regime. We show the fit in the asymptotic regime. The DCCA exponent $\lambda = 0.99 \pm 0.02$ implies power-law cross-correlations of the $1/f$ functional form.

zero. We then have either no cross-correlations or only short-range cross-correlations between $\{y_i\}$ and $\{y'_i\}$. In the special case when each series is power-law correlated and characterized by Hurst exponents H and H', generated by the ARFIMA process with a common error term, Ref. [3] numerically demonstrates and Ref. [17] derives that the root mean square (rms) of the detrended covariance vs. n follows a power law, $F_{DCCA}(n) \propto n^{\lambda}$, where the exponent λ is equal to the average of the Hurst exponents, $\lambda \approx (H + H')/2$.

Ref. [18] extends the DCCA by introducing the possibility of multifractality in time series [19]. Recently a new (detrended) cross-correlation coefficient has been proposed to quantify the presence of cross-correlations in a time series [20,21]. Extensions of DCCA based on a moving average [22] have also been proposed [23].

3. Analysis

Fig. 1(a) shows how absolute returns follow each other, especially during large price fluctuations, in the Exxon Mobil Corporation and the Dow Jones Industrial Average (DJIA) during the 21-year period from January 1989 to June 2011. Fig. 1(b) shows the DFA curves calculated for the absolute values of price returns. The DFA curves for Exxon Mobil and DJIA are known to exhibit strong power-law behavior [24], with DFA exponents 0.94 ± 0.01 and 1.02 ± 0.02, respectively. To estimate the level of co-movements in the volatility between these two time series, we estimate the cross-correlation level by employing DCCA. Fig. 1 shows the DCCA curve calculated between the absolute values of returns of the Exxon Mobil Corporation and the Dow Jones Industrial Average. The curve exhibits obvious power-law behavior with the DCCA exponent $\lambda = 0.99 \pm 0.02$, implying power-law cross-correlations, once again of the $1/f$ functional form.

Fig. 2 shows the auto-correlation properties of the absolute returns for each constituent of the DJIA. Fig. 2(a) shows that all DFA curves exhibit strong power-law behavior [24], in which the DFA exponent in the tails ranges between $\alpha = 0.89$ for Boeing and Merck & Company and $\alpha = 1.15$ for Bank of America (BoA). Fig. 2(b) shows the DFA exponent α for each DJIA constituent with the average $\langle\alpha\rangle = 0.97 \pm 0.03$. This result ($\alpha \approx 1$) supports the contention that the $1/f$ functional form is present in the auto-correlations of the absolute returns.

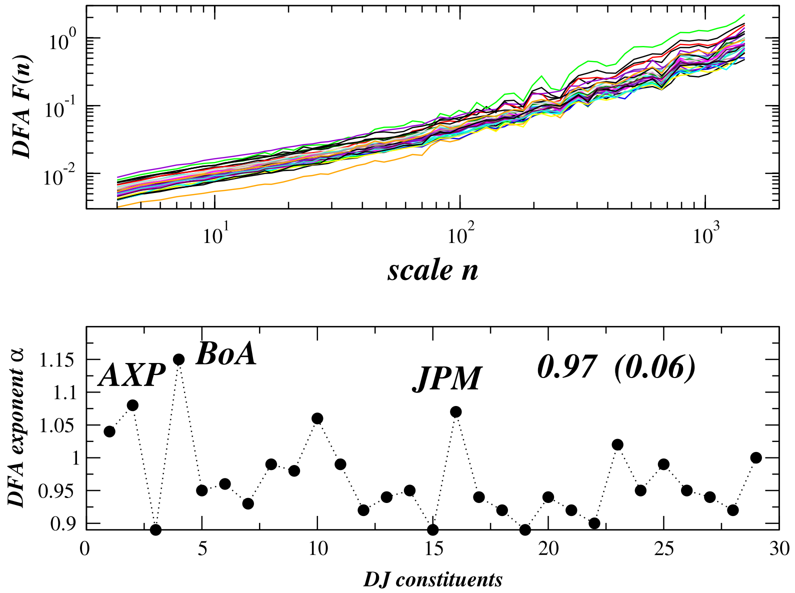

Fig. 2. Detrended fluctuation analysis DFA of absolute values of returns of 30 constituents of the Dow Jones Industrial Average. Each DFA curve follows a power law. The DFA exponent $\alpha = 0.97 \pm 0.06$ implies power-law auto-correlations of the $1/f$ functional form.

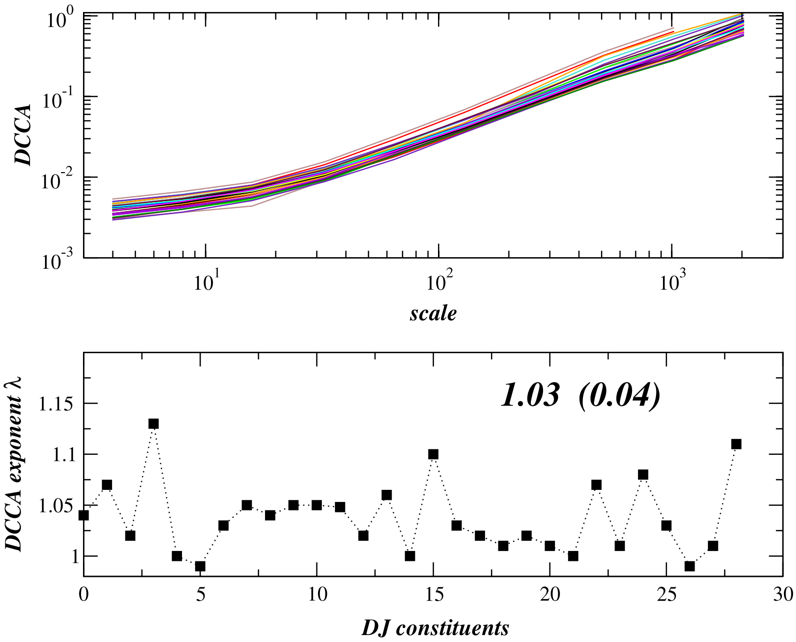

Fig. 3. Detrended cross-correlations analysis DCCA calculated between absolute values of returns calculated between Dow Jones Industrial Average (DJIA) and each of 30 constituents of the DJIA. Each DCCA curve follows a power law. The DCCA exponent $\lambda = 1.03 \pm 0.04$ implies power-law cross-correlations of the $1/f$ functional form.

Note that the largest DFA exponents α in Fig. 2(b) and in the table are frequently associated with financial institutions. The largest α corresponds to the Bank of America (BoA), American Express (AXP), and JP Morgan Chase. This result implies that, when auto-correlations are considered, financial institutions behave differently from all other firms. This result we confirm by analyzing several other financial institutions, including Goldman Sachs and Deutsche Bank, for which we find DFA exponents 1.07 ± 0.03 and 1.1 ± 0.02 and DCCA exponents 1.12 ± 0.02 and 1.13 ± 0.02, respectively. It is not uncommon that financial firms are treated differently from the rest of industry. When one evaluates bankruptcy risk, financial firms are assessed using models that differ from those used for manufacturing or service firms. Analyzing bankruptcy data, Ref. [25] shows the Zipf scaling for large debt-to-asset ratios and finds that the Zipf plot can be approximated by two power-law regimes, $\zeta = 0.57$ and $\zeta = 1.57$, in which the larger Zipf exponent corresponds primarily to financial firms. Note that the cumulative distribution function (cdf) is a simple transformation of the Zipf rank-frequency relation, where the cdf ζ' exponent and the Zipf exponent ζ are related as $\zeta = 1/\zeta$ [26]. Thus when financial firms go bankrupt, Ref. [25] shows that they strongly tend to be more indebted than other kinds of firms experiencing bankruptcy. We find that the DFA exponent controlling the duration of the firm risk also exhibits behavior for the financial firms that differs from that of the rest of

Table 1
The DFA and DCCA exponents for the DJIA members.

Firm	DFA α	DCCA λ	Firm	DFA α	DCCA λ
ALCOA	1.04 ± 0.02	1.04 ± 0.01	JP Morgan Chase	1.07 ± 0.02	1.10 ± 0.02
AXP	1.08 ± 0.02	1.07 ± 0.02			
Boeing	0.89 ± 0.02	1.02 ± 0.01	Coca-cola	0.94 ± 0.02	1.03 ± 0.01
BoA	1.15 ± 0.02	1.13 ± 0.04	McDonald's	0.92 ± 0.02	1.02 ± 0.01
Caterpillar	0.95 ± 0.02	1.0 ± 0.01	3M	0.92 ± 0.02	1.01 ± 0.01
Chevron	0.96 ± 0.02	0.99 ± 0.03	Merck	0.89 ± 0.01	1.01 ± 0.01
Cisco	0.93 ± 0.02	1.03 ± 0.02	Microsoft	0.94 ± 0.02	1.02 ± 0.01
duPont	0.99 ± 0.02	1.05 ± 0.01	Pfizer	0.92 ± 0.01	1.01 ± 0.01
Disney	0.98 ± 0.02	1.04 ± 0.01	Procter & Gamble	0.90 ± 0.01	1.00 ± 0.01
General electric	1.06 ± 0.02	1.05 ± 0.02	AT&T	1.00 ± 0.02	1.11 ± 0.01
Home depot	0.99 ± 0.02	1.05 ± 0.02	The travelers	1.02 ± 0.02	1.07 ± 0.03
Hewlett-Packard	0.92 ± 0.02	1.04 ± 0.02	United technologies	0.95 ± 0.02	1.01 ± 0.01
IBM	0.94 ± 0.02	1.02 ± 0.01	Verizon	0.99 ± 0.02	1.08 ± 0.01
Intel	0.95 ± 0.02	1.06 ± 0.01	Wal-mart stores	0.95 ± 0.02	1.03 ± 0.02
Johnson	0.89 ± 0.01	1.0 ± 0.01	Exxon mobil	0.94 ± 0.01	0.99 ± 0.02

the industry. Ref. [27] recently reported that the DFA exponent α obtained for price volatility increases with firm lifetime, suggesting that longer-lasting stocks tend to have a more persistent price movement (see Table 1).

Fig. 3 shows the cross-correlation levels in the absolute returns between the DJIA and each of 30 constituents, S_i, of the DJIA. Because of the high non-stationarity, we study the cross-correlations using DCCA [3] and find that each DJIA member follows the DJIA in absolute returns, i.e., the DCCA curve for each pair (S_i, DJAI_i) exhibits strong cross-correlations in which the DCCA exponent λ ranges between 0.99 ± 0.02 for Chevron Corporation and Exxon Mobil Corporation to 1.13 for the Bank of America. Fig. 3(b) shows the DCCA exponent λ for each pair (S_i, DJAI_i) with an average $\langle\lambda\rangle = 1.03 \pm 0.04$, implying that the power-law cross-correlations are of the $1/f$ functional form.

4. Conclusion

We have tested the limited effectiveness of diversification in reducing investment risk in the US market. We show that each of the 30 Dow Jones Industrial Average (DJIA) constituents, S_i, cross-correlates strongly with the DJIA when we analyze the cross-correlations in absolute returns. In particular, we find that the power-law cross-correlations are of the $1/f$ functional form, which is the strongest and longest-living correlation of all power-law functional forms. For the financial firms comprising the DJIA, we also find that the DFA and DCCA exponents controlling the duration of firm risk are somewhat larger than the corresponding values for non-financial US firms. Recent papers [28,29] have reported that interdependent networks, such as the financial systems of different countries, exhibit a lower failure-threshold value than isolated networks, i.e., system failure will occur at a lower damage level. Thus increasing the level of integration among the financial systems of different countries around the world may elicit power-law cross-correlations of the $1/f$ functional form at a world-wide level as well.

Acknowledgments

We thank the Ministry of Science of Croatia and the Keck Foundation for support.

Appendix

In contrast to Eq. (3), we here define covariance not between two random walks but between the squares of two random walks. We analyze the implications of this assumption. We consider two stationary processes y_k and y'_k, and we calculate

$$\langle (R_n - \langle R_n\rangle)^2 (R'_n - \langle R'_n\rangle)^2\rangle = \left\langle \left(\sum_k y_k\right)^2 \left(\sum_k y'_k\right)^2 \right\rangle, \tag{4}$$

where for simplicity we assume that $\langle y\rangle = \langle y'\rangle = 0$

$$\langle (R_n - \langle R_n\rangle)^2 (R'_n - \langle R'_n\rangle)^2\rangle \tag{5}$$

$$= \langle (y_1 + y_2 + \cdots + y_n)^2 (y'_1 + y'_2 + \cdots + y'_n)^2\rangle \tag{6}$$

$$= \langle [(y_1^2 + y_2^2 + \cdots + y_n^2) + 2(y_1 y_2 + y_2 y_3 + y_{n-1} y_n) \tag{7}$$

$$+\ 2(y_1 y_3 + y_2 y_4 + y_{n-2} y_n) + \cdots 2 y_{n-1} y_n] \tag{8}$$

$$\times\ [(y_1'^2 + y_2'^2 + \cdots + y_n'^2) + 2(y'_1 y'_2 + y'_2 y'_3 + y'_{n-1} y'_n) \tag{9}$$

$$+\ 2(y'_1 y'_3 + y'_2 y'_4 + y'_{n-2} y'_n) + \cdots 2 y'_{n-1} y'_n]\rangle. \tag{10}$$

We assume that y_k and y'_k are independent and that each series is power-law auto-correlated, where $C(k) \propto Ck^{-\gamma}$ and $C'(k) \propto C'k^{-\gamma'}$, but there are no cross-correlations between the series, i.e.,

$$\langle (R_n - \langle R_n\rangle)^2 (R'_n - \langle R'_n\rangle)^2\rangle \tag{11}$$

$$= \langle (R_n - \langle R_n\rangle)^2\rangle\langle (R'_n - \langle R'_n\rangle)^2\rangle \tag{12}$$

$$= \left[n\mathrm{Var}(Y) + 2\sum_{k=1}^{n-1}(n-k)C(k)\right]\left[n\mathrm{Var}(Y') + 2\sum_{k=1}^{n-1}(n-k)C'(k)\right] \tag{13}$$

$$= n^2\mathrm{Var}(Y)\mathrm{Var}(Y') + 2n\mathrm{Var}(Y)\sum_{k=1}^{n-1}(n-k)C'(k) + 2n\mathrm{Var}(Y')\sum_{k=1}^{n-1}(n-k)C(k) \tag{14}$$

$$+\ 4\sum_{k=1}^{n-1}\sum_{k'=1}^{n-1}(n-k)(n-k')C(k)C'(k') \tag{15}$$

$$\approx n^2\mathrm{Var}(Y)\mathrm{Var}(Y') + 2n\mathrm{Var}(Y)n^{2-\gamma'}C_1 + 2n\mathrm{Var}(Y')n^{2-\gamma}C_2 \tag{16}$$

$$+\ 4\sum_{k=1}^{n-1}(n-k)C(k)\sum_{k'=1}^{n-1}(n-k')C'(k'). \tag{17}$$

In this expression we approximate the sums by the corresponding integrals [3],

$$\langle (R_n - \langle R_n\rangle)^2 (R'_n - \langle R'_n\rangle)^2\rangle \tag{18}$$

$$\approx n^2\mathrm{Var}(Y)\mathrm{Var}(Y') + 2n\mathrm{Var}(Y)n^{2-\gamma'}C_1 + 2n\mathrm{Var}(Y')n^{2-\gamma}C_2 \tag{19}$$

$$+\ 4n^{2-\gamma'}n^{2-\gamma} \tag{20}$$

$$= n^2\mathrm{Var}(Y)\mathrm{Var}(Y') + 2\mathrm{Var}(Y)n^{3-\gamma'}C_1 + 2\mathrm{Var}(Y')n^{3-\gamma}C_2 \tag{21}$$

$$+\ 4n^{4-\gamma'-\gamma} \tag{22}$$

Asymptotically, the last term dominates

$$\langle (R_n - \langle R_n\rangle)^2 (R'_n - \langle R'_n\rangle)^2\rangle 4n^{4-\gamma'-\gamma} \propto n^{4\lambda}, \tag{23}$$

where γ and the DFA exponent are related as $\gamma = 2(1-\alpha)$. We obtain

$$\lambda = 1 - \frac{\gamma' + \gamma}{4} = \frac{\alpha' + \alpha}{2}. \tag{24}$$

Thus, if we analyze the correlations not between R_n and R'_n but between R_n^2 and $R_n'^2$, we find that the power law exists even if there are no cross-correlations between series y and y_k. As stated above, (a) if each series is power-law auto-correlated characterized by Hurst exponents H and H', generated by ARFIMA process with a common error term, $F_{\mathrm{DCCA}}(n) \propto n^{\lambda}$, where $\lambda \approx (H + H')/2$ [3,17], and (b) if Y and Y' are power-law auto-correlated with DFA exponents α and α', but not cross-correlated, $F_{\mathrm{DCCA}}(n)$ fluctuates around zero, which is a sign that there are no cross-correlations. For both (a) and (b), Eq. (23), defined with the expectation between R_n^2 and $R_n'^2$, gives the same result.

References

[1] L. Laloux, et al., Phys. Rev. Lett. 83 (1999) 1467.
[2] V. Plerou, et al., Phys. Rev. Lett. 83 (1999) 1471; Phys. Rev. E 66 (2002) 066126.
[3] B. Podobnik, H.E. Stanley, Phys. Rev. Lett. 100 (2008) 084102.
[4] S. Arianos, A. Carbone, J. Stat. Mech. P03037 (2009).
[5] T. Guhr, B. Kalber, J. Phys. A 36 (2003) 3009.
[6] I.T. Jolliffe, Principal Component Analysis, Springer-Verlag, New York, 1986.
[7] B. Le Baron, J. Business 65 (1992) 199.
[8] C.B. Erb, C.R. Harvey, T.E. Viscanta, Financ. Anal. J. 50 (1994) 32.
[9] B. Solnik, C. Boucrelle, Y. Le Fur, Financ. Anal. J. 52 (5) (1996) 17–34.
[10] G. Canarella, S.K. Pollard, Int. Rev. Econ. 54 (2007) 445.
[11] B. Podobnik, D. Fu, H.E. Stanley, P.C. Ivanov, Eur. Phys. J. B 56 (2007) 47.
[12] B. Podobnik, et al., Europhys. Lett. 90 (2010) 68001.
[13] D. Wang, B. Podobnik, D. Horvatic, H.E. Stanley, Phys. Rev. E 83 (2011) 046121.
[14] C.K. Peng, et al., Phys. Rev. E 49 (1994) 1685;
L. Xu, et al., Phys. Rev. E 71 (2005) 051101.
[15] B. Podobnik, et al., Proc. Natl. Acad. Sci. USA 106 (2009) 22079.
[16] J. Beran, Statistics for Long-Memory Processes, Chapman & Hall, New York, 1994.

[17] B. Podobnik, et al., Eur. Phys. J. B 71 (2009) 243.
[18] W.-X. Zhou, Phys. Rev. E 77 (2008) 066211.
[19] J.W. Kantelhardt, et al., Physica A 295 (2001) 441.
[20] G.F. Zebende, Physica A 390 (2011) 614.
[21] B. Podobnik, Zhi-Qiang Jiang, Wei-Xing Zhou, H. Eugene Stanley, Phys. Rev. E 84 (2011) 066118.
[22] A. Carbone, et al., Phys. Rev. E 69 (2004) 026105; Physica A 344 (2004) 267–271.
[23] Z.-Q. Jiang, W.-X. Zhou, Phys. Rev. E 84 (2011) 016106.
[24] Y. Liu, et al., Phys. Rev. E 60 (1999) 1390;
P. Cizeau, et al., Physica A 245 (1997) 441–445.
[25] B. Podobnik, et al., Proc. Natl. Acad. Sci. USA 107 (2010) 18325.
[26] M.H.R. Stanley, et al., Economics Lett. 49 (1995) 453.
[27] W. Li, F. Wang, S. Havlin, H.E. Stanley, Phys. Rev. E 84 (2011) 046112.
[28] S.V. Buldyrev, et al., Nature 464 (2010) 1025;
J. Gao, S.V. Buldyrev, H.E. Stanley, S. Havlin, Nature Phys. 8 (2012) 40–48.
[29] X. Huang, et al., Phys. Rev. E 83 (2011) 065101;
J. Gao, et al., Phys. Rev. Lett. 107 (2011) 195701.