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The role of inertia on fluid flow through disordered porous media

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Abstract

We study the fluid flow through disordered porous media by numerically solving the complete set of the Navier–Stokes equations in a two-dimensional lattice with a spatially random distribution of solid obstacles (plaquettes). We simulate viscous and non-viscous flow through these idealized pore spaces to determine the origin of the deviations from the classical Darcy's law behavior. Due to the nonlinear contribution of inertia to the transport of momentum at the pore scale, we observe a typical departure from Darcy's law at sufficiently high Reynolds numbers. Moreover, we show that the classical Forchheimer equation provides a valid phenomenological model to correlate the variations of the friction factor of the porous media over a wide range of Reynolds conditions.

The application of Darcy's law is the standard approach to characterize single-phase fluid flow in microscopically disordered and macroscopically homogeneous porous media [1–4]. Basically, one simply assumes that a global index, the permeability k, relates the average fluid velocity V through the pores, with the pressure drop ΔP measured across the system,

$$V = -\frac{k}{\mu}\frac{\Delta P}{L}\ , \qquad (1)$$

where L is the length of the sample in the flow direction and μ is the viscosity of the fluid. However, in order to understand the interplay between porous structure and fluid flow, it is necessary to examine local aspects of the pore space morphology and relate them with the relevant mechanisms of momentum transfer (viscous and inertial forces). This has been accomplished in previous studies [5,6] where computational simulations

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based on the knowledge of the pore space morphology have been quite successful in predicting permeability coefficients of real porous materials.

In spite of its great applicability, the concept of permeability as a global index for flow, which implies the validity of Eq. (1), should be restricted to viscous flow conditions or, more precisely, to low values of the Reynolds number, $\mathrm{Re} \equiv \rho V d_p/\mu$, where ρ is the density of the fluid and d_p is the grain diameter. Unlike the sudden transition from laminar to turbulent flow in pipes and channels, where there is a critical Reynolds number condition separating both regimes, experimental studies have shown that the passage from the linear (Darcy's law) to the nonlinear behavior in flow through porous media is more likely to be gradual (see [1] and references therein). It has then been argued that the contribution of inertial forces (convection) to the flow in the pore space should also be examined in the framework of the laminar flow regime before assuming that fully developed turbulence effects are already present and relevant to momentum transport in the system. Recently [7], the transport of momentum in two-dimensional porous structures generated at the critical percolation point [3] has been evaluated through the direct solution of the Navier–Stokes and continuity equations. It has been shown that, beyond the range of validity of Darcy's law, a nonuniversal behavior should be considered for the critical exponent t relating flow permeability coefficients and porosities, $k \propto (\varepsilon - \varepsilon_c)^t$, with ε_c being the critical percolation porosity. In the present work we show by numerical simulation of the Navier–Stokes equations in high porosity structures ($\varepsilon > \varepsilon_c$) that the departure from Darcy's law at sufficiently high Re numbers can be explained in terms of the inertial contribution to *laminar fluid flow* through the void space. In other words, we demonstrate that there is no need to include turbulence effects to model incipient deviations from linearity usually found in permeability experiments with various porous materials [1].

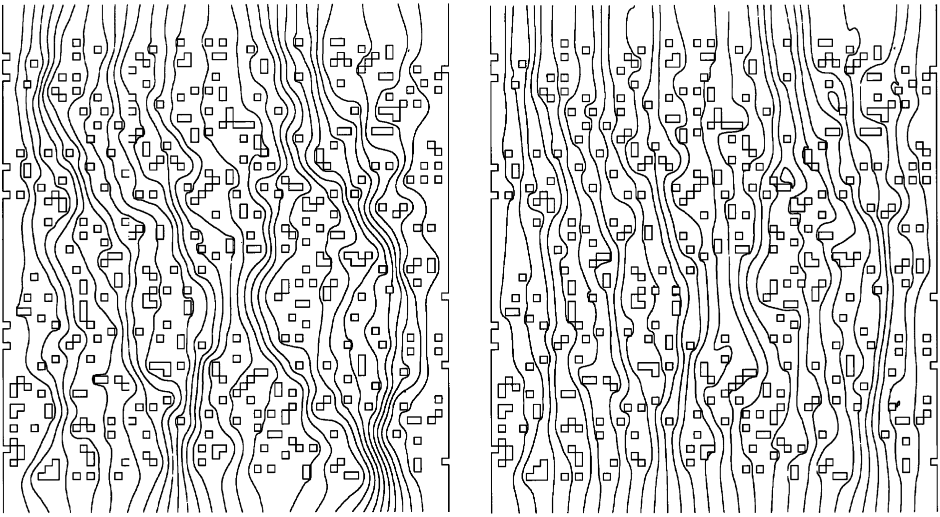
Our topological model for the pore connectivity is based on the general picture of site percolation disorder. Square obstacles are randomly removed from a 64×64 square lattice until a porous space with a prescribed void fraction ε is generated. The mathematical description for the detailed fluid mechanics in the interstitial pore space is based on the assumptions that we have steady state flow in isothermal conditions and the fluid is continuum, Newtonian and incompressible. Thus, the essence of our phenomenological description is the two-dimensional set of Navier–Stokes and continuity equations for momentum and mass conservation,

$$\rho \left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right] = -\frac{\partial P}{\partial x} + \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] ,$$

$$\rho \left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \right] = -\frac{\partial P}{\partial y} + \mu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] ,$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 . \tag{2}$$

Here u and v are the components of the velocity vector in the x- and y-directions, respectively. We use the nonslip boundary condition at the whole of the solid–fluid



(a) (b)

Fig. 1. (a) Contour plot of the stream function ψ for low Reynolds number conditions (Re = 0.0156). (b) Same as in (a), but for a higher Reynolds number (Re = 15.6).

interface. End effects of the flow field established inside the pore structure (particularly significant at high Re conditions) are minimized by attaching a header (inlet) and a recovery (outlet) region to two opposite faces arbitrarily chosen. At the inlet line, a constant inflow velocity in the normal direction to the boundary is specified whereas at the exit, the rate of the velocity change is assumed to be zero (gradientless boundary condition). Instead of periodic boundary conditions, we choose to close the remaining two faces of the system with two additional columns of obstacles. This insulating condition reproduces more closely the experimental setup usually adopted with real rocks and permeameters. For a given realization of the pore geometry and a fixed Re, the velocity and pressure fields in the fluid phase of the void space and ancillary zones are numerically obtained through discretization by means of the control volume finite-difference technique [8]. Finally, from the area-averaged pressures at the inlet and outlet positions, the overall pressure drop across the porous realizations can be readily calculated. [1]

Fig. 1a shows the contour plot of the stream function [2] ψ for a typical realization of a highly porous void space ($\varepsilon = 0.9$) subjected to low Reynolds conditions, Re = 0.0156. In spite of the highly connected pathway available for flow at this large porosity value, the predominant viscous forces in the momentum transport through the complex void geometry generates well defined "preferential channels" of fluid flow. As shown in Fig. 1b, the situation is quite different at high Reynolds conditions, where the degree

[1] The FLUENT (trademark of FLUENT Incorporated) fluid dynamics analysis package has been used in this study.

[2] The stream function ψ is defined for incompressible two-dimensional flows as $u \equiv \partial\psi/\partial y$ and $v \equiv -\partial\psi/\partial x$.

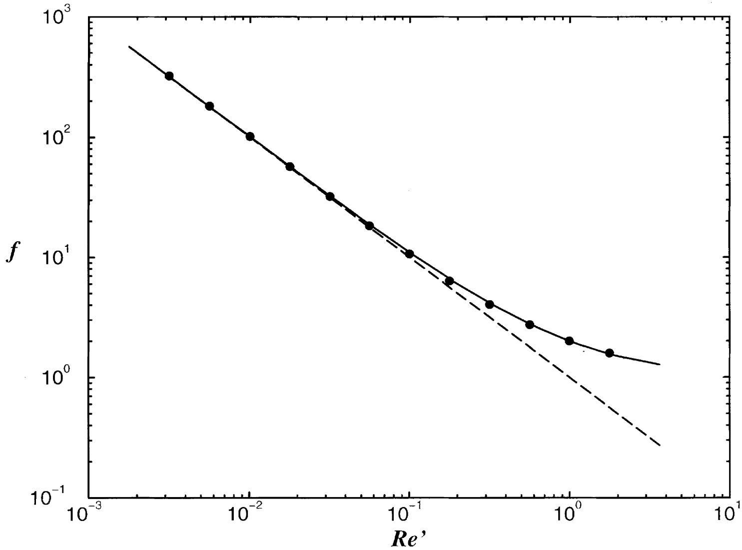


Fig. 2. Dependence of the generalized friction factor f on the modified Reynolds number Re′. The solid line is the best fit to the data of the Forchheimer equation. The dashed line is the best fit to the data at low Re′ of Darcy's law.

of channeling is clearly less intense than in Fig. 1a. In the case of Fig. 1b, due to the relevant contribution of inertial forces (convection) to the flow at the pore scale, the distribution of streamlines along the direction orthogonal to the main flux becomes more homogeneous. This nonlinear effect can be macroscopically quantified in terms of the so-called Forchheimer equation [1,3,4],

$$-\frac{\Delta P}{L} = \alpha\mu V + \beta\rho V^2 \ , \tag{3}$$

where the coefficient α should correspond to the reciprocal permeability of the porous material and β is usually named as an "inertial parameter". Eq. (3) with constant α and β parameters is not a purely empirical expression since it can be derived by an adequate average of the Navier–Stokes equation for steady and incompressible laminar flow of a Newtonian fluid in a rigid porous medium [1]. Rearranging the Forchheimer equation in the form:

$$f = \frac{1}{\mathrm{Re}'} + 1 \ , \tag{4}$$

where $f \equiv -\Delta P/L\beta\rho V^2$ and $\mathrm{Re}' \equiv \beta\rho V/\alpha\mu$, we obtain a *friction factor-Reynolds number* type of correlation which is presumably "universal". Indeed, Eq. (4) has been extensively and successfully used to correlate experimental data from a large variety of porous materials and a broad range of flow conditions [1]. In Fig. 2, we show the results of simulations performed with three realizations of the porous structure generated with $\varepsilon = 0.9$. After computing and averaging the overall pressure drops for all realizations at different Re numbers, we estimate the coefficients α and β and calculate the modified variables f and Re'. In agreement with real flow experiments, Eq. (4) also provides a satisfactory fit to all results of our computational simulations. Moreover, the point of departure from linear (Darcy's law) to nonlinear behavior in the range $10^{-2} < \mathrm{Re}' < 10^{-1}$ of modified Reynolds is also consistent with previous experimental observations.

In an attempt to characterize the influence of inertial forces on the flow behavior of a single fluid in highly porous structures, we demonstrate here, by direct simulation of the Navier–Stokes equations, that incipient deviations from Darcy's law observed in several experiments, can be satisfactorily modeled in the laminar regime of fluid flow, without including turbulence effects. The results of our computational simulations corroborate numerous experimental data which display a gradual transition at high Re from linear to nonlinear flow in the pore space. In addition, we show that the physical description underlying the classical Forchheimer equation provides a legitimate correlation for the global friction factor of the porous media over a wide range of Reynolds number conditions. In summary, whether true turbulence effects have been detected or not in real flows through porous media, our calculations with the Navier–Stokes equations indicate that the Forchheimer model with constant α and β parameters remains valid for rather high Re numbers, even when convective nonlinearities can significantly affect the momentum transport at the pore scale, in comparison to viscous forces. These facts might be relevant to understand the role played by convection in several technical applications involving consolidated and nonconsolidated porous materials.

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